THE COMPLETE GUIDE TO PERSPECTIVE DRAWING

From One-Point to Six-Point

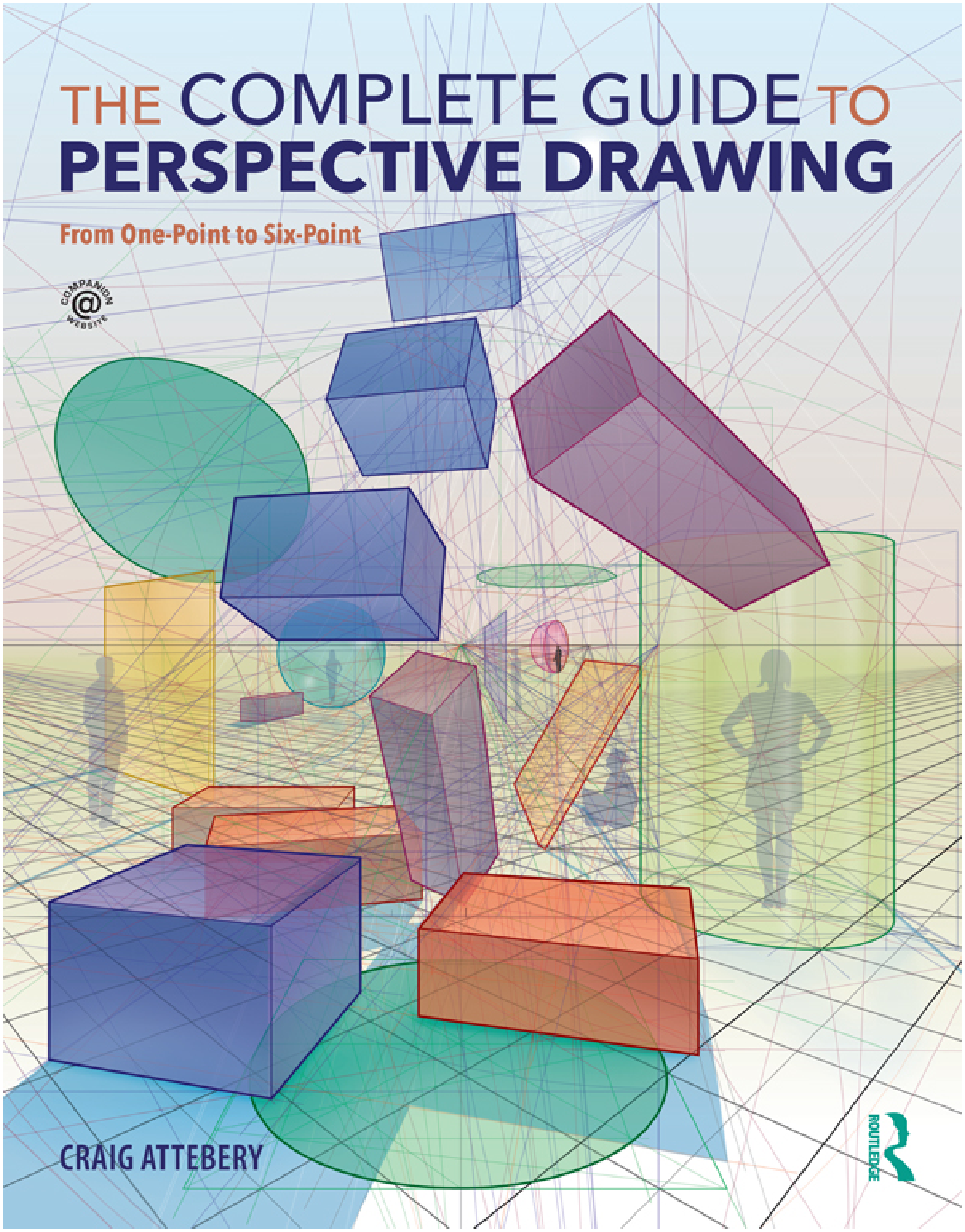
COMPANION WEBSITE

CRAIG ATTEBERY

ROUTLEDGE

THE COMPLETE GUIDE TO PERSPECTIVE DRAWING

From One-Point to Six-Point



CRAIG ATTEBERY

ROUTLEDGE

The Complete Guide to Perspective Drawing

Computers can calculate perspective angles and create a drawing for us, but the spontaneity of mark-making, the tactile quality of a writing surface, the weight of a drawing instrument, and the immediacy of the human touch are sensations that keep traditional drawing skills perpetually relevant. The sensuality and convenience of the hand persists and will survive as a valuable communication tool, as will the need to accurately express your ideas on paper. As a professional, understanding the foundations of drawing, how we process images, and how we interpret what we see are principal skills. Understanding linear perspective enables artists to communicate their ideas accurately on paper. *The Complete Guide to Perspective Drawing* offers a step-by-step guide for the beginner as well as the advanced student on how to draw in one-point through six-point perspective and how to make scientifically accurate conceptual illustrations from simple to complex situations.

Craig Attebery is a native southern Californian. He received a BFA with Honors from ArtCenter College of Design, USA, in 1980, and an MFA from Otis/Parsons Art Institute, USA, in 1984. Craig has worked as a freelance illustrator for advertising agencies, science books, and the entertainment industry, as well as creating conceptual art for JPL/NASA and the aerospace industry. Craig's illustrations have appeared in many publications including *Newsweek and Time magazines*. In addition to his commercial work, he has participated in exhibitions at galleries and museums throughout the country and internationally, including the Fry Museum (Seattle, WA), the Arnot Museum (Elmira, NY), the Art Museum of South Texas (Corpus Christi, TX), and the Oceanside Museum (Oceanside, CA). His work is in the permanent collection of the de Young Museum (San Francisco, CA). Craig is a faculty member at ArtCenter College of Design where he has taught perspective for over fifteen years.

"This is the comprehensive book on how to see and utilize perspective that educators and serious students of the subject have been waiting for! It covers every aspect of the perspective problems creative artists might encounter, providing multiple solutions and concise explanations illustrated with hundreds of easily read drawings. The reader is connected to the historical background of perspective in art and science, taught how to transfer what is seen or imagined into two dimensions, and delivered a source of reference that will endure for decades."

—F. Scott Hess, artist and Associate Professor of Painting for Laguna College of Art + Design's MFA and BFA programs

The Complete Guide to Perspective Drawing

From One-Point to Six-Point

Craig Attebery

Routledge
Taylor & Francis Group
NEW YORK AND LONDON

First published 2018
by Routledge
711 Third Avenue, New York, NY 10017

and by Routledge
2 Park Square, Milton Park, Abingdon, Oxon, OX14 4RN

Routledge is an imprint of the Taylor & Francis Group, an informa business

Library of Congress Cataloging-in-Publication Data
Names: Attebery, Craig, author.
Title: The complete guide to perspective drawing : from one-point to six-point / Craig Attebery.
Description: New York : Routledge, 2018. | Includes bibliographical references and index.
Identifiers: LCCN 2017042917| ISBN 9781138215603 (hb : alk. paper) | ISBN 9781138215627 (pb : alk. paper) | ISBN 9781315443560 (ebook)
Subjects: LCSH: Perspective—Technique.
Classification: LCC NC750 .A88 2018 | DDC 741.2—dc23
LC record available at https://lccn.loc.gov/2017042917

ISBN: 978-1-138-21560-3 (hbk)
ISBN: 978-1-138-21562-7 (pbk)
ISBN: 978-1-315-44356-0 (ebk)

Typeset in Univers
by Florence Production Ltd, Stoodleigh, Devon, UK

Visit the companion website: www.routledge.com/cw/attebery

Contents

Preface

I tried to look relaxed as I sat across from the interviewer—fresh out of my sophomore year at art school, hoping to land a summer job doing artwork of some kind, any kind; I wasn't picky. The interviewer looked tired. His questions were delivered with a fatigued voice; a voice that seemed to have echoed the same question to countless interviewees. "We are looking for someone who can do perspective drawings. Can you do perspective drawings?" he asked. It was a straightforward question that solicited a straightforward answer. I knew any hesitation would belie an affirmative reply. I had to react quickly. My choice was clear: tell the interviewer I was up to the task, or tell the truth. Knowing the latter reply would end the interview, and feeling I had nothing to lose, I decided to … well, *stretch* the truth. I struggled for the first few years, trying to teach myself the finer aspects of perspective and not get fired in the process. A decade later I was still working for the same company, still doing perspective drawings, and finally feeling secure in my perspective abilities. So much so that when I was asked to teach a class in perspective, I was confident that I knew my stuff.

But then came the students' questions. And with them came the realization that I was not as knowledgeable about perspective as I had thought. My students asked challenging questions. Trying to find answers to these questions in publications and online sources led me nowhere. Sources that answered elemental perspective concepts were ubiquitous, but— beyond the basics—information was scarce. Once again, I was on my own figuring it out.

Perspective theory can be challenging. Having a detailed publication to refer to is invaluable. At the college where I teach, ArtCenter College of Design, the students are known for their passion and resolve. I knew a basic perspective text would not answer their questions—or be up to their standard. So, I made my own set of handouts. It was only a few pages at first, covering some of the more complicated aspects, the aspects that would solicit the greatest number of questions from my students. Writing a book was the furthest thing from my mind. Whenever a student was having difficulty applying a procedure, I would create a handout addressing that subject. The number of handouts grew; the small set of handouts soon became a stack. After fifteen years of teaching perspective, I had created several hundred handouts. When I showed them to a colleague he said, "These are not handouts—this is a *book*!"

Well, it wasn't really a book, at least not yet. It still took some additional coaxing from students, and a timely call from Routledge Press, before the handouts were at last assembled, finalized, and transformed into this publication: *The Complete Guide to Perspective Drawing*.

I want to thank all my students for their patience, thirst for knowledge, and continual appeals for additional handouts. A special thanks to Nancy Tsai for being so generous with her time, Tanya Preston for sorting out my prose, my family for tolerating my countless hours staring at a computer screen, and my dog for intently listening to my ramblings about perspective.

Abbreviations

AUX. HL	Auxiliary Horizon Line
AUX. MP	Auxiliary Measuring Point
AUX. VP	Auxiliary Vanishing Point
CV	Center of Vision
EL	Eye Level
GL	Ground Line
GLMP	Ground Line Measuring Point
GLVP	Ground Line Vanishing Point
HL	Horizon Line
HML	Horizontal Measuring Line
LA	Light Angle
LAP	Left Axis Point
LAVP	Light Angle Vanishing Point
LMP	Left Measuring Point
LRL	Left Reference Line
LRP	Left Reference Point
LVP	Left Vanishing Point
LSP	Left Station Point
ML	Measuring Line
MP	Measuring Point
PP	Picture Plane
RAP	Right Axis Point
RMP	Right Measuring Point
RP	Reference Point

RRL	Right Reference Line
RRP	Right Reference Point
RSP	Right Station Point
RVP	Right Vanishing Point
SP	Station Point
VAP	Vertical Axis Point
VML	Vertical Measuring Line
VMP	Vertical Measuring Point
VP	Vanishing Point
VRL	Vertical Reference Line
VRP	Vertical Reference Point
VSP	Vertical Station Point
VVP	Vertical Vanishing Point
XAP	*x*-Axis Point

Introduction

How important is it to learn **linear perspective**? Computers can calculate perspective angles and create a drawing for us, so what need is there to learn it traditionally?

To begin with, pencils and paper are not going away soon. The freedom and spontaneity of mark-making, the tactile quality of a writing surface and drawing instrument, and the immediacy and convenience of the human touch will forever remain seductive. The sensuality of the hand persists and will survive as a valuable aid to visual communication, as will the need to place your ideas accurately on paper. Furthermore, knowledge is empowering. As a professional, understanding the foundations of drawing, how we process images, and how we interpret what we see are principal skills. This knowledge transfers directly to your drawings, giving them an air of confidence. If you understand the geometry of lines you have a powerful tool to create believable images.

You can avoid learning perspective—but only for a while. Those pesky drawing problems will continue to surface: the misguided lines, the trapezoidal buildings, the awkward ellipses, the floating figures, the shapes that, well, just look "off." The problems seem endless. You realize it is time to end your procrastination. It is time to learn perspective. This realization is typically accompanied by a heavy sigh, for learning perspective can be overwhelming. Take a deep breath. Give yourself time to let the material sink in. It takes practice. It also takes an abnormal amount of left-brain thinking; at least, more than most artists like to do. Approach the material one problem at a time, from the simple to the complex, step by step. Have a solid understanding of the basics before you progress to the advanced.

I have tried to strike a balance between showing and explaining, so the descriptions and the images work together. Some readers connect with written descriptions, others (like me) connect with images. Both are needed to some extent. Describing the diagrams in prose is often a difficult task. I have tried to avoid describing the obvious; unnecessary wordiness attributes to confusion and tedium. In some cases, the reader may not need the descriptions at all—the drawings may tell the story.

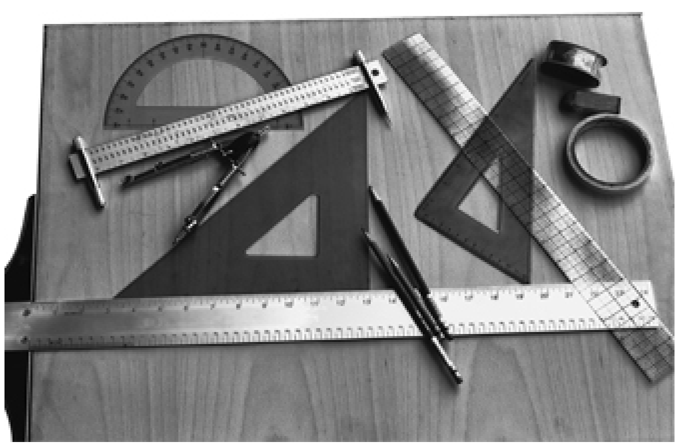
Start at the beginning of this book, as the information builds on previous chapters. Without the foundations supplied in the earlier sections, the rest of the book may be perplexing. There are step-by-step guides for each drawing. The instructional illustrations use basic geometric shapes as placeholders for real world objects. Depending on its proportion and scale, a **cuboid** can represent a building, a car, or a person. All objects can be reduced to simple geometric forms. You may wish to use colored pencils to color-code the procedures. Practice by using the worksheets (available as a download from the Routledge website). As you advance through the book it may be helpful to review previous sections. This will ensure you don't forget what you have learned. Reviewing the material also assists in gaining a deeper understanding of the procedures. Keep in mind that there are many solutions to any given problem. There is no one correct procedure. There is, however, only one correct answer, one correct result—just different ways to achieve it.

After becoming well-versed in perspective theory, you will be able to find various solutions to any given problem. These solutions become evident when you understand the "whys" as well as the "hows." I want this book to illustrate how to draw accurately, but also to explain why the procedures work the way they do. I have attempted to create a book for beginners and for the advanced. I want to tackle the difficult problems as well as the basic problems, to create (as much as possible) a complete perspective book. That being said, it is impossible to include solutions to every scenario. The purpose of this book is to give you the information needed to extrapolate from the given samples, and to find a resolution to specific problems not addressed here. Remember: with knowledge, there is nothing too difficult to draw.

Tools of the Trade

Here is a list of equipment you will find useful. Perspective drawing requires precise angles and dimensions. Having the proper tools and understanding how to use them is important to creating successful images. You will need:

- A drawing/drafting board. The type with metal edges designed to accommodate a T-square.
- A T-square. A T-square is designed to draw parallel horizontal lines.
- 45° and 30°/60° triangles. In addition to creating these angles, they are also used to draw vertical lines.
- A protractor. Useful to draw angles other than those drawn by the triangles.
- A ruler. One made out of transparent material is best.
- Color pencils. Perspective drawings can become complicated. Color coding your procedures is a helpful technique.
- A sharpener. Keeping a nice point on your pencils is important.
- Tracing paper. Working with overlays is another method to keep elements in your drawing organized.
- A compass. A beam compass is also useful for making circles and arcs that are too large for a standard compass.
- Drafting tape. Keep your paper securely fastened to your drawing board.
- An eraser. For that rare occasion when you make a mistake.



1
Basic Perspective Terms

> *A painting is the intersection of a visual pyramid at a given distance with a fixed center.*
>
> —Leon Battista Alberti, *On Painting*, 1435

This quote is far from the romantic verse commonly used to describe creative endeavors. Nonetheless, Alberti's clinical delineation encapsulated a revolutionary transformation in art production: a revolution based on science. No longer did artists need to base their images on speculation and assumption, on convention and estimation. Artists could now depend on verifiable data. Art and reason became allies.

As perspective's value became apparent, so did its deft requirements. Perspective procedures can be challenging. The alternative—drawing without perspective techniques— is merely guesswork. Perspective's most valued asset is its ability to portray objects accurately, to assess dimensions, and to project those dimensions spatially. Perspective produces an uncompromised image, sometimes a surprising image. Drawing is often anti-intuitive, and shapes can appear different than expected. It is not a requirement to plot all images using perspective guidelines, but by practicing perspective techniques, a sense of how **foreshortened** shapes present themselves can be developed. Then, when the artist is sketching from observation or imagination, the skillset learned from studying perspective becomes a valuable tool to base their estimations on.

The word perspective derives from the Latin *perspirere*—to look through. Alberti's definition arose from the realization that, when a sheet of glass is placed between the viewer and the world and then the view is traced, the perspective is flawless. This vision—as self-evident as it may seem—changed art production forever. Alberti's statement is concise but his terms are abstract, so they will be examined further.

The equation consists of a viewer, a sheet of glass, and an object to be drawn. The viewer is Alberti's "fixed center." It is fixed because the viewer must remain stationary. A drawing cannot begin from one point of view and be finished from another, because a different location results in a different image.

Light makes the world visible. Rays of light reflect from objects and project onto the retina, converging at the viewer's eye. This is Alberti's "visual pyramid." The rays "intersect" the sheet of glass (known as the **picture plane**). The intersection of these rays on the picture plane creates the projected image seen—a perfect representation of the world (Figure 1.1).

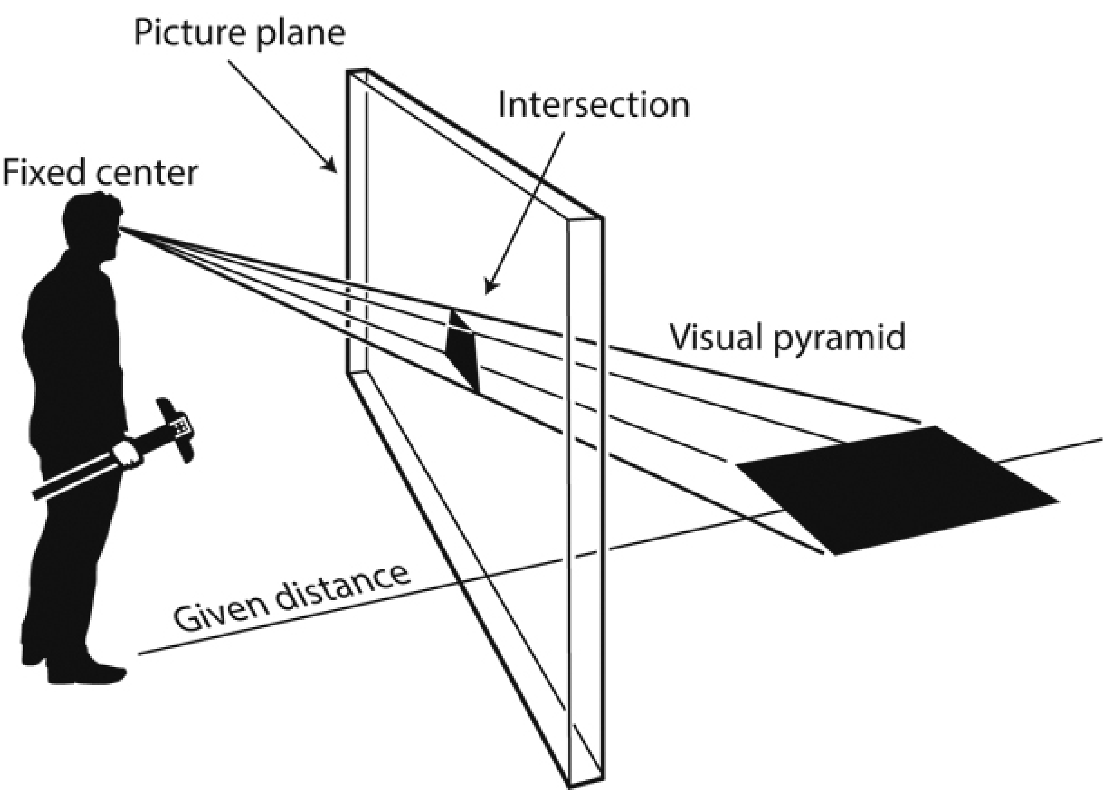


Figure 1.1 The intersection of the picture plane within Alberti's "visual pyramid."

From its inception, perspective was met with resistance. Change is difficult; artists had been creating images for hundreds of years without perspective. Painters found this new method of spatial organization difficult, confusing, and, frankly, unnecessary. But some early converts (one of the most noted being Masaccio, 1401–1428) embraced the new technology with breathtaking results. Others waited, but eventually the popularity of these new and exciting images forced those holding out to convert. The procedures were daunting. But to compete as an image maker in this new world required a new prerequisite: perspective proficiency. Alberti's procedures will be explored further in Chapter 15.

Perspective has evolved over several hundred years to the modern approach used today, based on geometry. The only knowledge the artist requires for perspective drawing is how to read a ruler, that there are 360° in a circle, and what an **isosceles triangle** is (a triangle with two sides of equal length). Mastering the thirteen books of *Euclid's Elements* is not required to understand perspective.

As the understanding of geometry and its relationship to perspective evolved, so did the methods. Perspective terms have also changed since 1413. For example, 600 years ago the term **vanishing point** did not exist, it was called a centric point. The language of perspective has evolved—as all language does—and today the term centric point has vanished. Likewise, the term distance point was once used for what is now called a **measuring point (MP)**. Variations in terminology still exist. When perusing publications that discuss perspective, different terms may be used to describe the same thing.

Basic Terms

To begin, some basic perspective terms will be defined. These terms are used throughout the book, so it is important that their meaning and function is understood.

Station Point (SP)

The **station point** represents the viewer, specifically the viewer's eye. A perspective drawing is created using only one eye. Creating a perspective drawing using a pair of eyes would result in two slightly different images. A stereographic (3-D) image is made using two station points. A single image requires a single station point. It is called a station point because it must remain stationary (the station point is Alberti's "fixed center").

Eye Level (EL)

The **eye level** is the distance from the ground to the viewer's eye.

Horizon Line (HL)

The **horizon line** is the edge of the earth, where ground meets sky.

The edge of the earth is aligned with the viewer's eye level. Whether flying in a plane or sitting on the ground, the horizon is always at eye level. Why is this? The following exercise will explain. Stand in front of a piece of glass, place a small object on the ground, and trace its position on the glass. Then place the object farther away and trace its new position. It is now higher on the glass (closer to eye level). The farther away an object is, the higher it will appear on the glass. Objects move up the glass as they move farther away. At some point, depending on its size, the object will no longer be able to be seen. Larger objects can be seen from a greater distance and are thus higher on the glass. They appear closer to eye level. If something is *very* large, and is *very* far away, it will disappear *at* eye level. For example, the edge of the Earth disappears at eye level, at infinity (Figure 1.2).

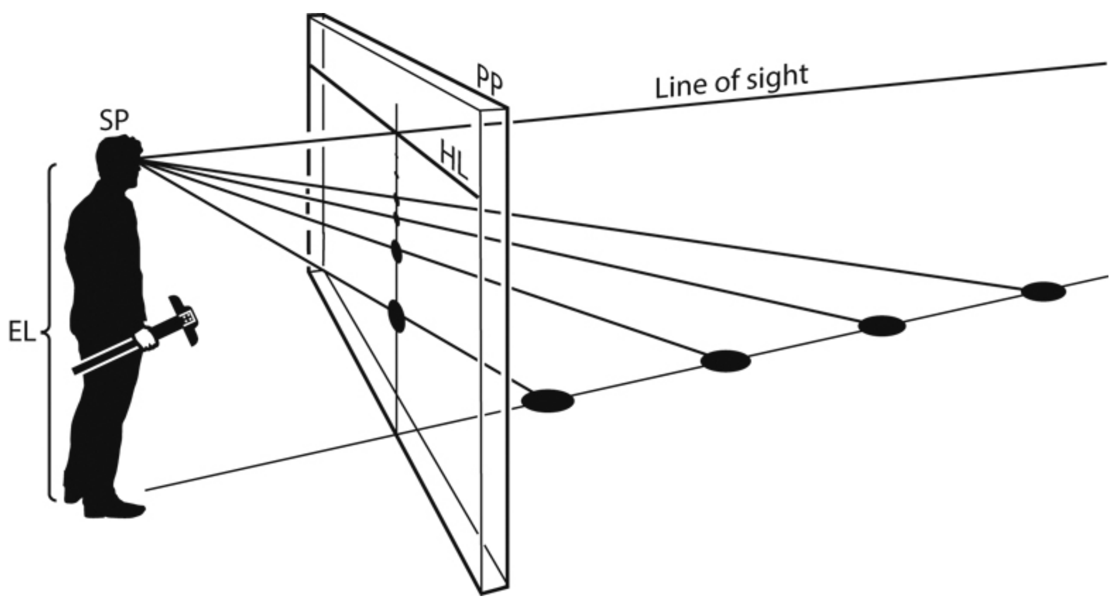


Figure 1.2 As the dots move farther from the picture plane, they become closer to the eye level. Objects at infinity, like the horizon line, are depicted at eye level.

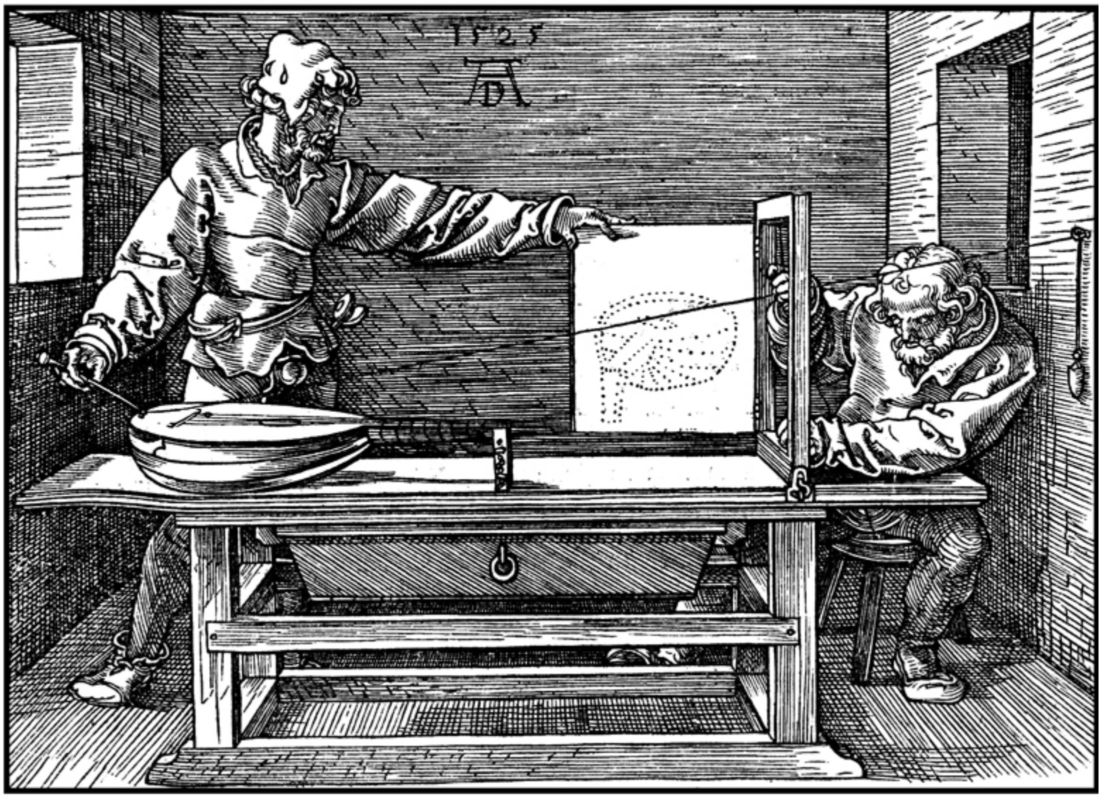


Figure 1.3 Albrecht Dürer, *Underweysung*, 1525. This etching shows the picture plane (the frame), the station point (the hook attached to the wall), and the visual pyramid (the string attached to the lute).

Picture Plane (PP)

The picture plane is an imaginary window positioned between the viewer and the world (Figure 1.2). It is always 90° to the **line of sight** (the exception being anamorphic perspective). The orientation and shape of the picture plane defines the type of perspective. If the picture plane is **perpendicular** to the ground, objects are in **one**- or **two-point perspective**. If the picture plane is angled to the ground, objects are in **three-point perspective**. In **four**-, **five**-, and **six-point perspective** the picture plane is curved.

Albrecht Dürer created a perspective machine that demonstrated Alberti's theory and how the picture plane, station point, and visual pyramid function (Figure 1.3). One end of a string was attached to the wall (fixed center), and another to the object (in this example, a lute). The string represented the visual pyramid. Using movable cross hairs fixed to a frame, the intersection of the string to the picture plane was plotted. The frame represented the picture plane. When the cross hairs were in position, the string was removed. The hinged door was closed, and a dot placed where the cross hairs aligned. The door was then opened, the string was attached to a different spot on the lute and the process was begun again. This was not only tedious, but apparently a two-person job.

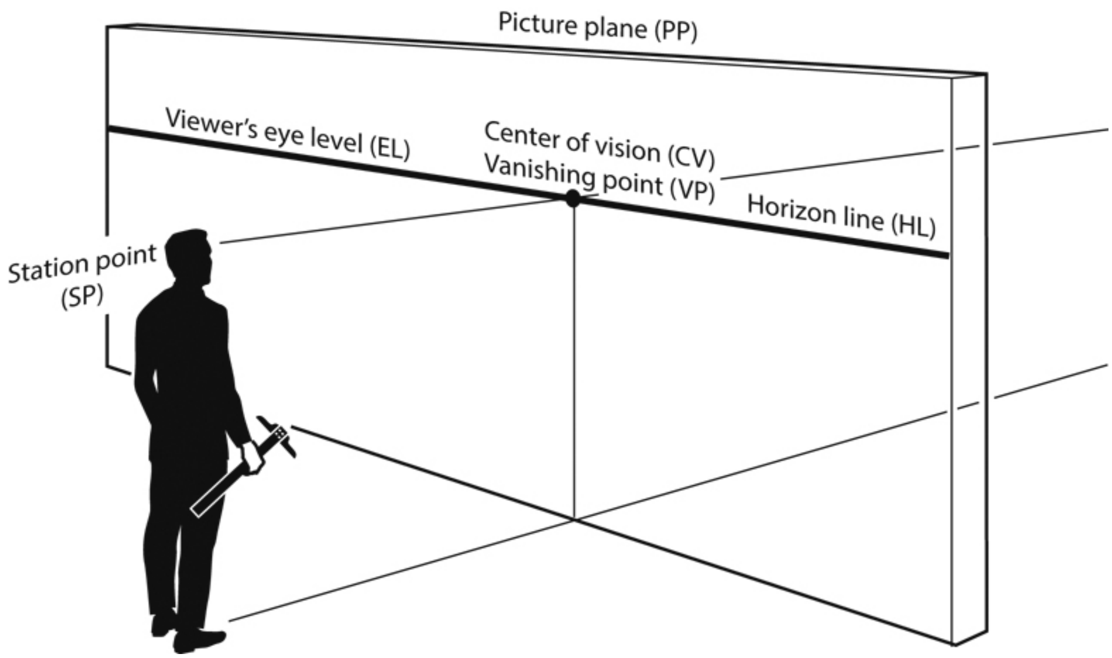


Figure 1.4 This illustration shows the relationship between the station point, picture plane, eye level, horizon line, and vanishing point.

Center of Vision (CV)

The **center of vision** is where the viewer is looking (also known as the focal point).

In one- and two-point perspective the line of sight is parallel with the **ground plane**, and the center of vision is on the horizon line. In three-point perspective the line of sight is angled to the ground plane and the center of vision is above or below the horizon line.

In day-to-day activities, a person's focus darts from place to place as they assess their surroundings. A perspective drawing, however, is from a specific focus point. Because the viewer can only look at one place at a time, they can only have one center of vision.

Vanishing Point (VP)

The vanishing point is at infinity. Objects get smaller as they recede in space, therefore at infinity all objects disappear. Parallel lines of infinite length appear as **converging lines** and connect to the same vanishing point. Vanishing points can be on the horizon line, above the horizon line, or below the horizon line. There can be an unlimited number of vanishing points, but there is only a single one-point vanishing point. This single point is always located at the center of vision. Figure 1.4 demonstrates how the vanishing point and previously discussed terms relate to each other.

Cone of Vision

A person can focus on only a small area, about 2° of the surrounding environment. **Peripheral vision**, however, is quite large, beyond 180°. The **cone of vision** lies between them (Figure 1.5). The cone of vision is a 60° circle that defines the viewer's image area (the area to be drawn within). The center axis of this circle is the focal point. Imagine looking through a cone with a 60° angle centered along its axis. Its intersection with the picture plane creates a circle that defines the size of the cone of vision (Figure 1.6). The farther away the viewer is from the picture plane, the larger the cone of vision and the circle become. Beyond the confines of the 60° cone, distortion becomes problematic. Shapes look stretched, tilted, and corners no longer look like **right angles**. The cone of vision is drawn to warn the artist that excessive distortion waits beyond its border. Inside the 60° cone, distortion still exists, but is less apparent.

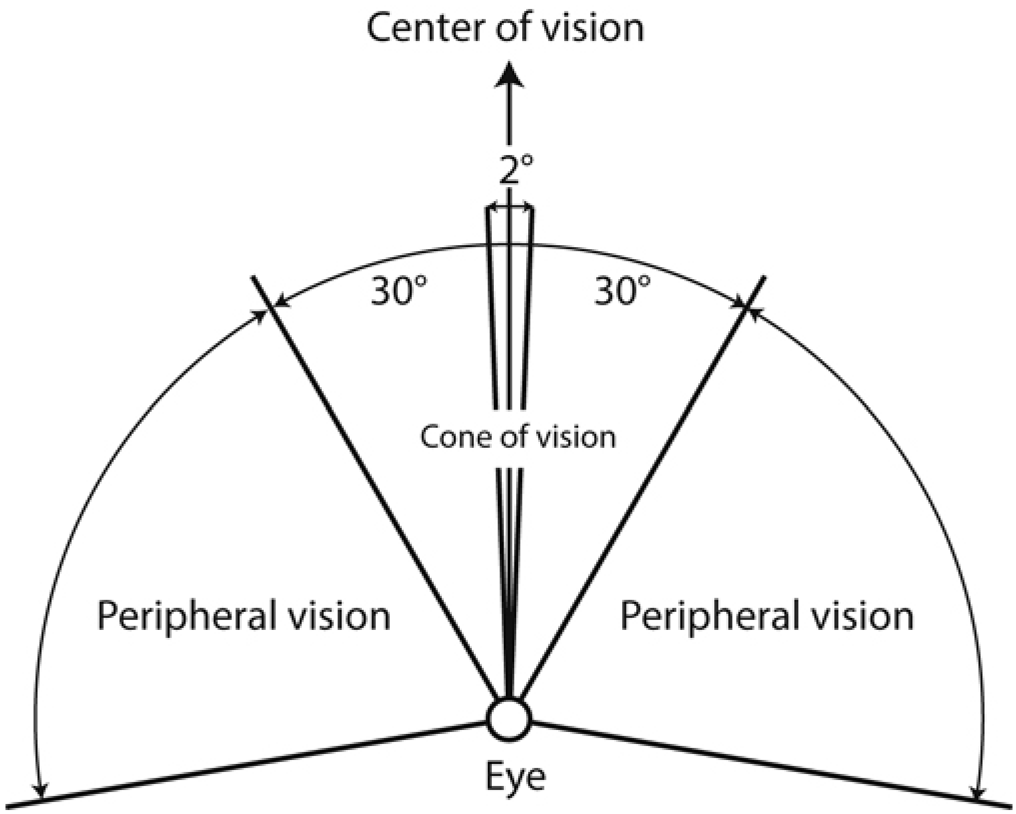


Figure 1.5 **Peripheral vision is beyond 180°. The cone of vision is 60°, and the focal area is 2°.**

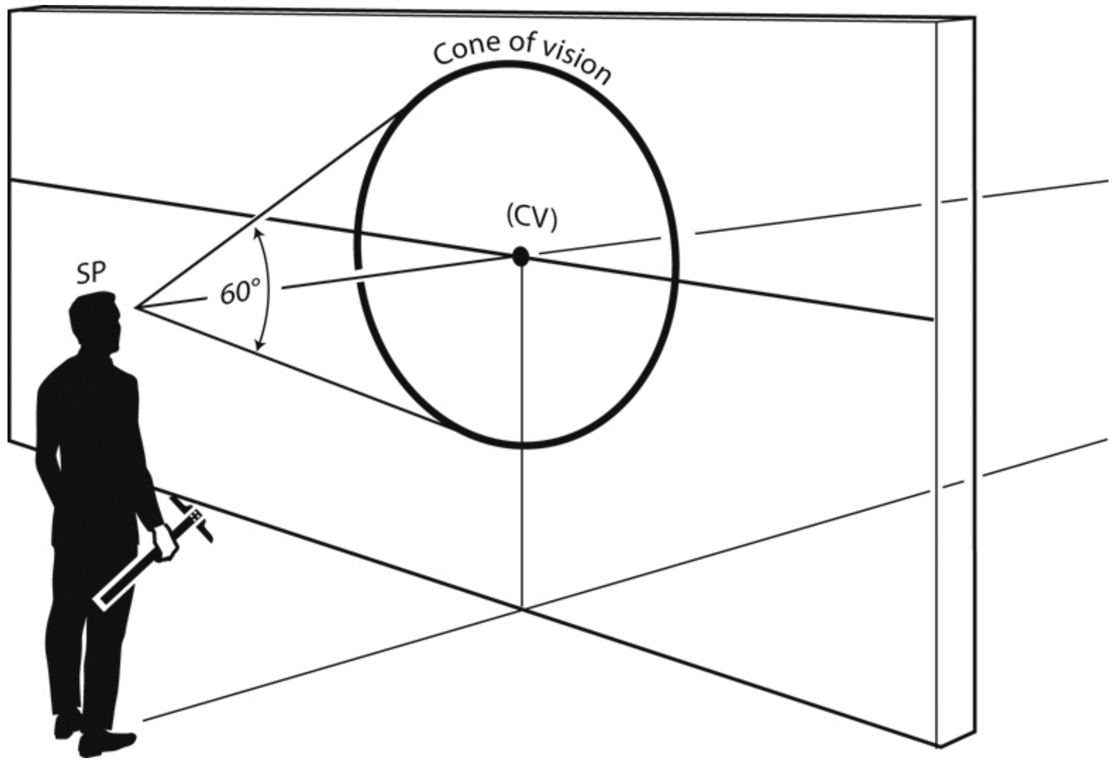


Figure 1.6 **The cone of vision is 60° from the station point.**

Measuring Point (MP)

The measuring point is a tool used to measure foreshortened lines. Every vanishing point has a dedicated measuring point. Before perspective, measuring was a guessing game. Understanding how math relates to drawing enabled artists to draw accurate dimensions. The placement of a measuring point is specific and determined by geometry. Calculating measuring points is discussed further in Chapter 3.

Measuring Line (ML)

The **measuring line** is the ruler and determines the drawing's scale. A 1:1 scale creates a drawing that is actual size. Every foot or meter of the object being drawn equals that dimension on the paper. Drawing a house using a 1:1 scale would require a piece of paper as large as the house. To avoid this, the scale can be changed. A 1:2 scale means every unit on the drawing equals 2 units in the real world; therefore, the drawing is half scale. The size of the paper and the subject being drawn determine the scale to use.

The measuring line is typically placed on the picture plane at the ground level. In this position, the measuring line not only determines the scale, it also determines the height of the viewer. Since the horizon line is at the viewer's eye level and the measuring line is on the ground, the distance between the horizon line and measuring line equals the distance between the viewer's eyes and the ground plane (Figure 1.7).

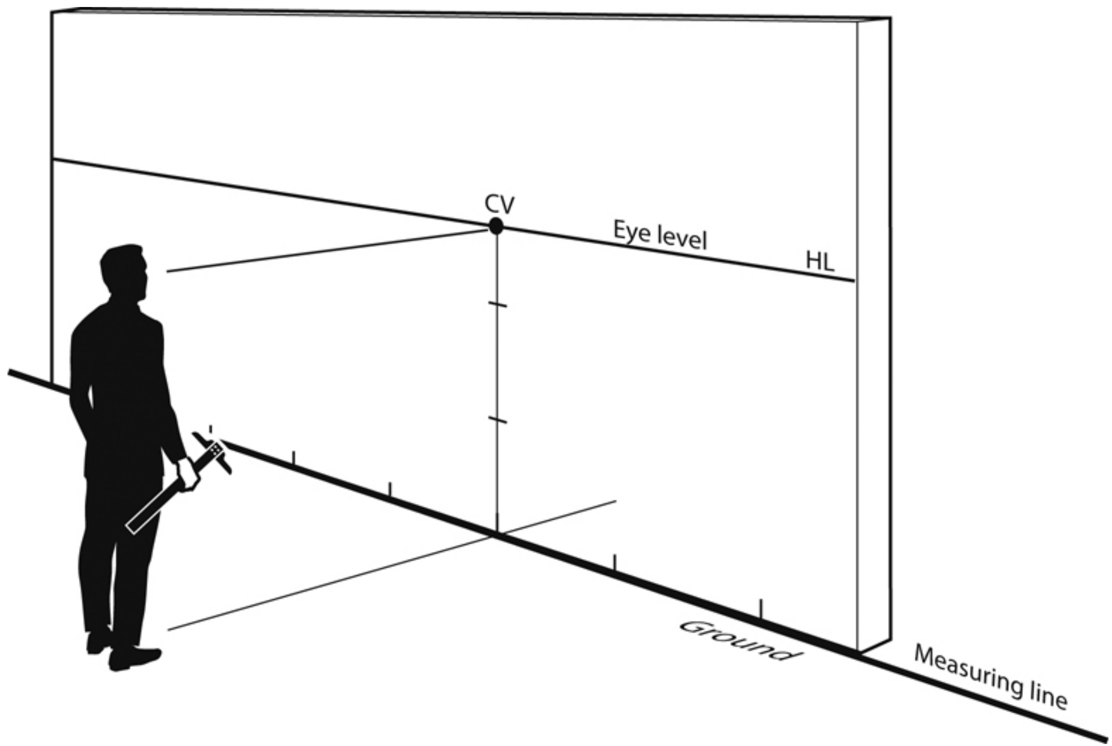


Figure 1.7 The viewer's eye level equals the distance from the measuring line to the horizon line. The placement of the measuring line determines the height of the viewer.

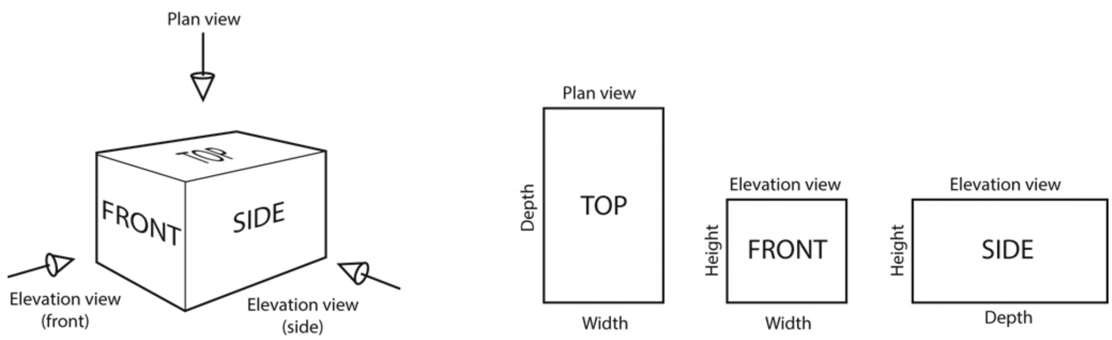


Figure 1.8 Plan and elevation views show only two dimensions.

Plan View

A **plan view** is an **orthographic** drawing from above. A plan view has no perspective; it shows only two dimensions: width and depth (Figure 1.8).

Elevation View

An **elevation view** is an orthographic drawing from the front, back, or side. An elevation view has no perspective. Like plan view, only two dimensions are shown: height and width (front or back elevation view), or height and depth (side elevation view) (Figure 1.8).

Station Point and Picture Plane Dynamics

When a viewer sees an image, the shape perceived is determined by the relationship between three elements: the station point, the picture plane, and the object. When the relationship between these three elements changes, the object seen also changes. To gain a better understanding of this phenomenon, each of these elements will be examined more closely.

Station Point

The station point can be close to the picture plane, or far away. The farther away the viewer is from the picture plane, the more foreshortened depth appears. When the viewer moves closer to the picture plane, the foreshortening is less severe. Width, however, is not affected (Figure 1.9).

Picture Plane

The picture plane can be positioned anywhere between the station point and the horizon line. The closer the picture plane is to the viewer, the smaller the object appears. The farther the picture plane is from the viewer, the larger the image appears. The perspective does not change. The images are identical in shape, but are different sizes (Figure 1.10).

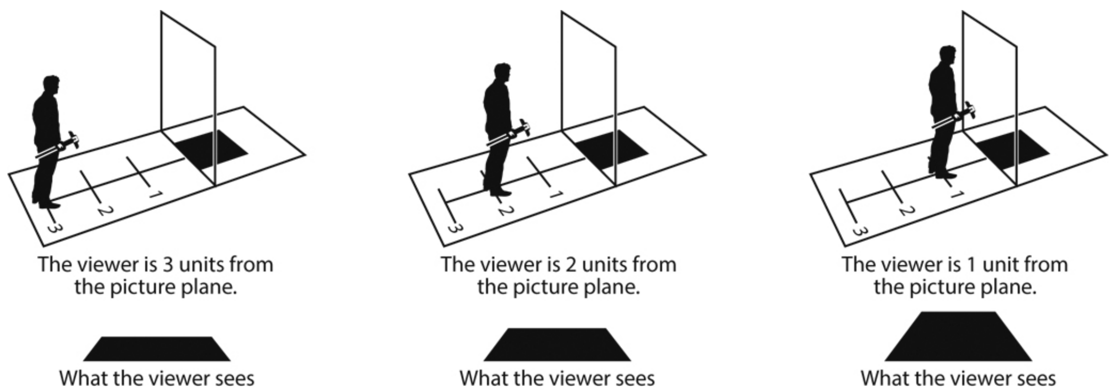


Figure 1.9 Moving the viewer changes the object's shape but not the object's size. It remains the same width.

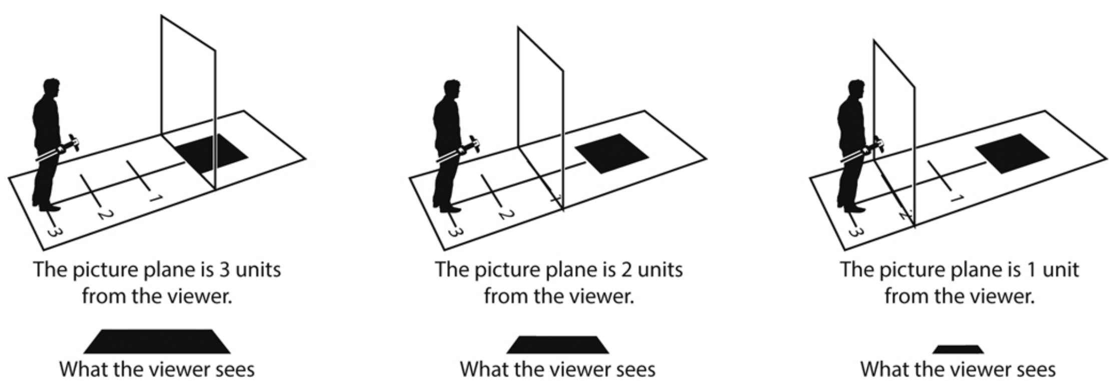


Figure 1.10 Moving the picture plane changes the object's size but not the shape.

Object

When the image is closer to the picture plane it is larger and less foreshortened. As the image moves farther from the picture plane it becomes smaller and more foreshortened. The object's size *and* shape are affected (Figure 1.11).

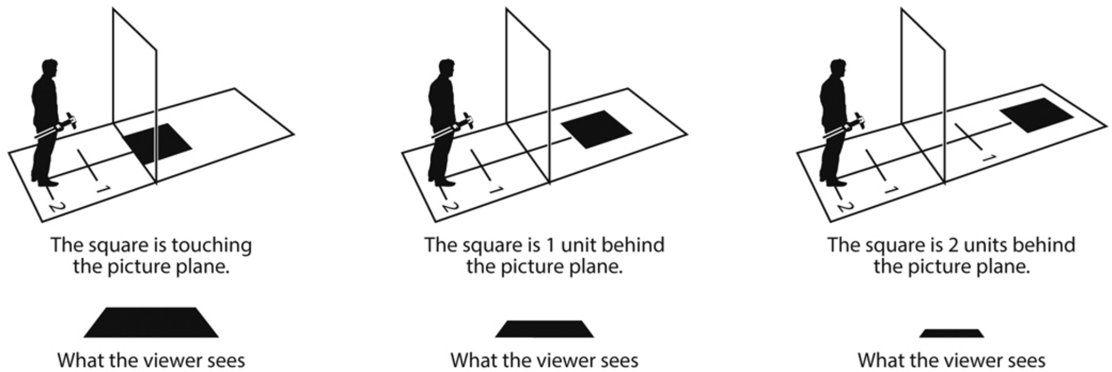


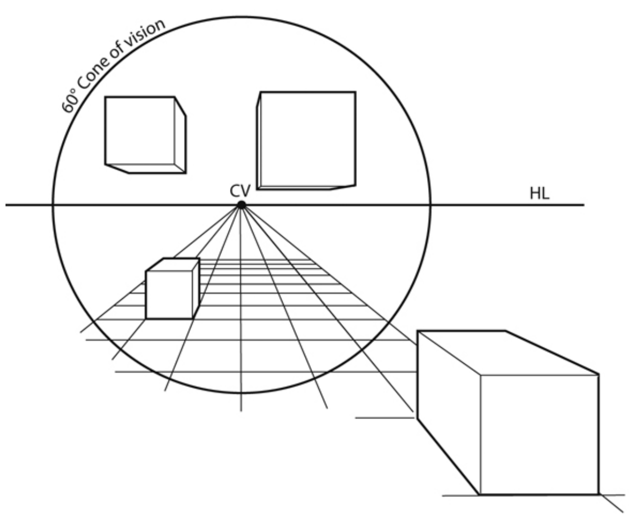
Figure 1.11 **Moving the object changes the size and shape**.

While it may seem as though there are many variables to be remembered when drawing in perspective, in reality these effects happen automatically when using perspective techniques.

2
One- through Six-Point Perspective, an Overview

In the world of perspective, there are six ways a viewer can be oriented to the *mise-en-scène*. Each orientation results in a different diagram, and each diagram depends on the relationship of the viewer to the picture plane and to the object being drawn. The relationship of these three items (as well as the shape of the picture plane) determines the diagram used. Perspective diagrams can be from one-point all the way up to six-point perspective. Detailed instructions are given for these orientations in subsequent chapters, but first, a brief overview of them follows before examining their finer aspects.

One-Point Perspective



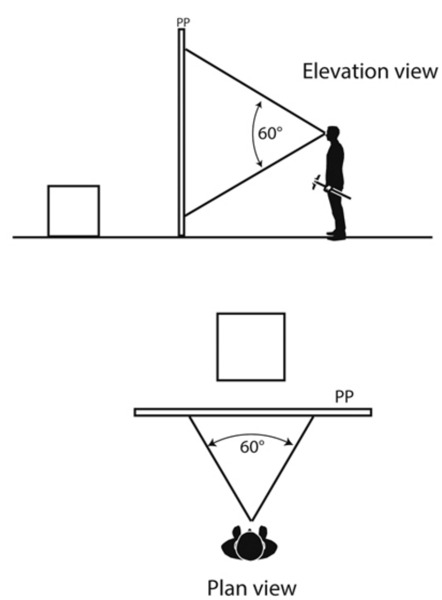


Figure 2.1 In one-point perspective vertical and horizontal lines are parallel with the picture plane. Objects outside of the cone of vision are distorted. A cube will look more like a rectangle.

In 1435, Leon Battista Alberti published *On Painting*, the first book to diagram perspective— specifically, one-point perspective. In one-point perspective, vertical and horizontal dimensions are parallel with the picture plane. Vertical lines are drawn perpendicular to the horizon line, and horizontal lines are drawn parallel with the horizon line. Depth is foreshortened. These foreshortened lines are oriented 90° to the picture plane and connect to the center of vision (Figure 2.1).

Two-Point Perspective

Objects drawn in two-point perspective appear early in the sixteenth century (Figure 2.2). In two-point perspective, horizontal lines are angled to the picture plane, and thus foreshortened. They connect to a left vanishing point (LVP) or right vanishing point (RVP). Only vertical dimensions are parallel with the picture plane and are drawn as true vertical lines (Figure 2.3).

Figure 2.2 Albrecht Dürer, *St. Jerome in His Study* (engraving), 1514, London, British Museum. The chair to the right of St. Jerome is an early example of two-point perspective.

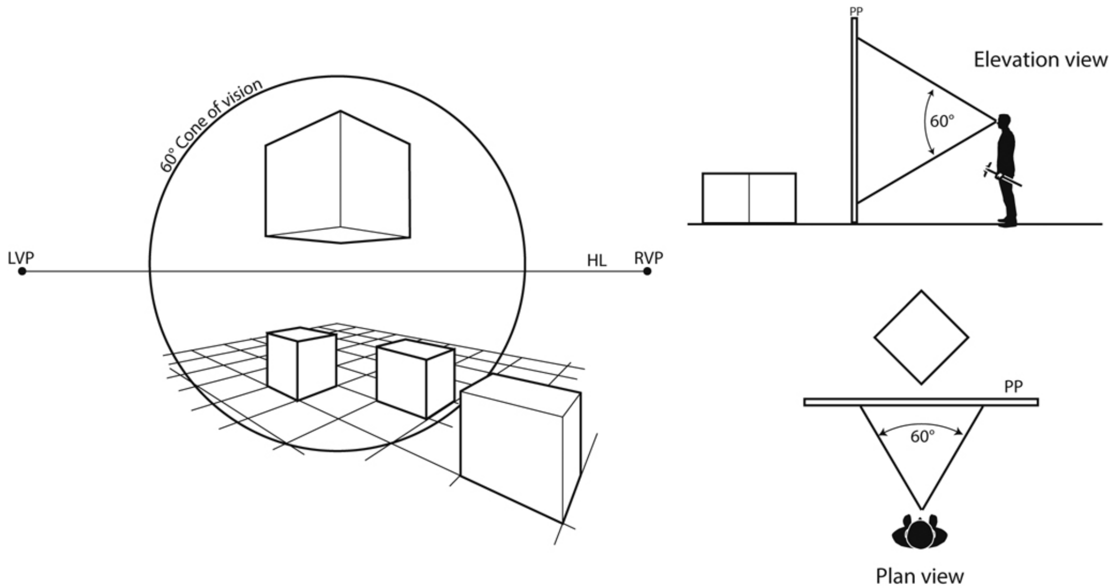
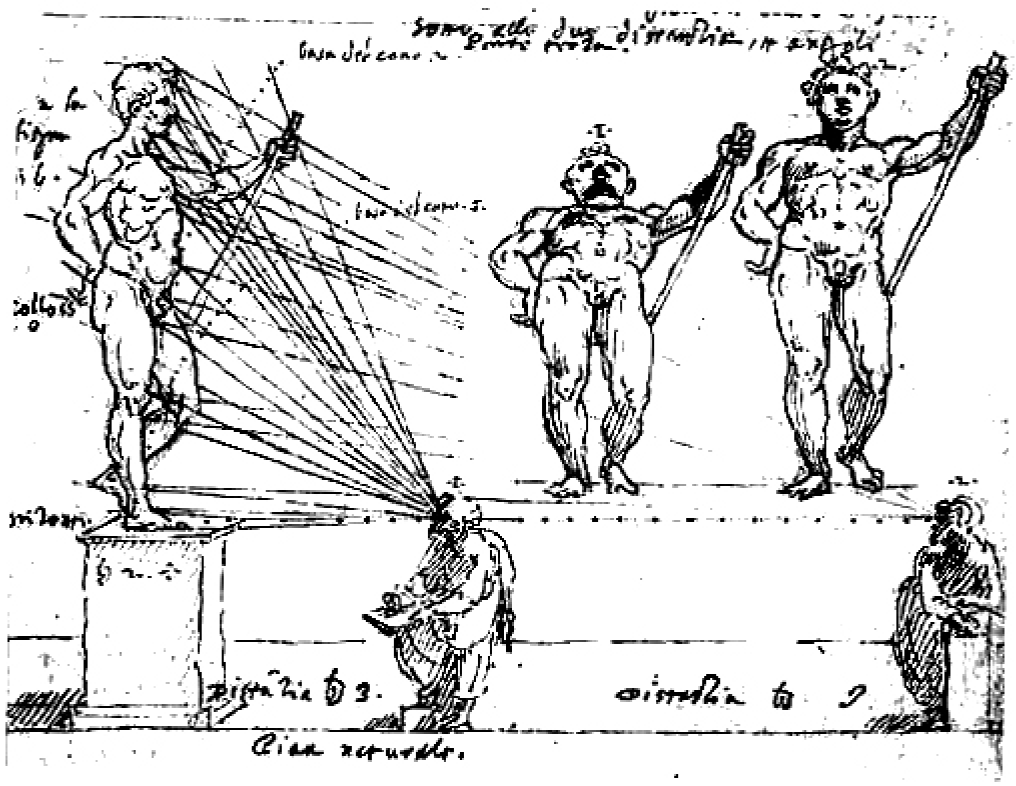


Figure 2.3 **In two-point perspective, vertical dimensions are parallel with the picture plane and horizontal dimensions are angled to the picture plane. Objects outside the cone of vision are distorted; cubes look rhomboid, their corners not appearing to be right angles.**

Three-Point Perspective



[Figure 2.4](#) **Carlo Urbino, Detail from the *Codex Huygens*, Morgan Library & Museum, NY.**

A rudimentary understanding of three-point perspective appears in examples from the *Codex Huygens*, a sixteenth-century Renaissance manuscript once thought to be the work of Leonardo da Vinci. This manuscript demonstrates the foreshortening effect of looking up: a **worm's-eye view** (Figure 2.4). In three-point perspective, no lines are parallel with the picture plane. The picture plane is angled to the ground, because the viewer is looking up or down at the image. The center of vision is above or below the horizon line. There is a left, right, and vertical vanishing point (VVP), and all lines are foreshortened (Figure 2.5).

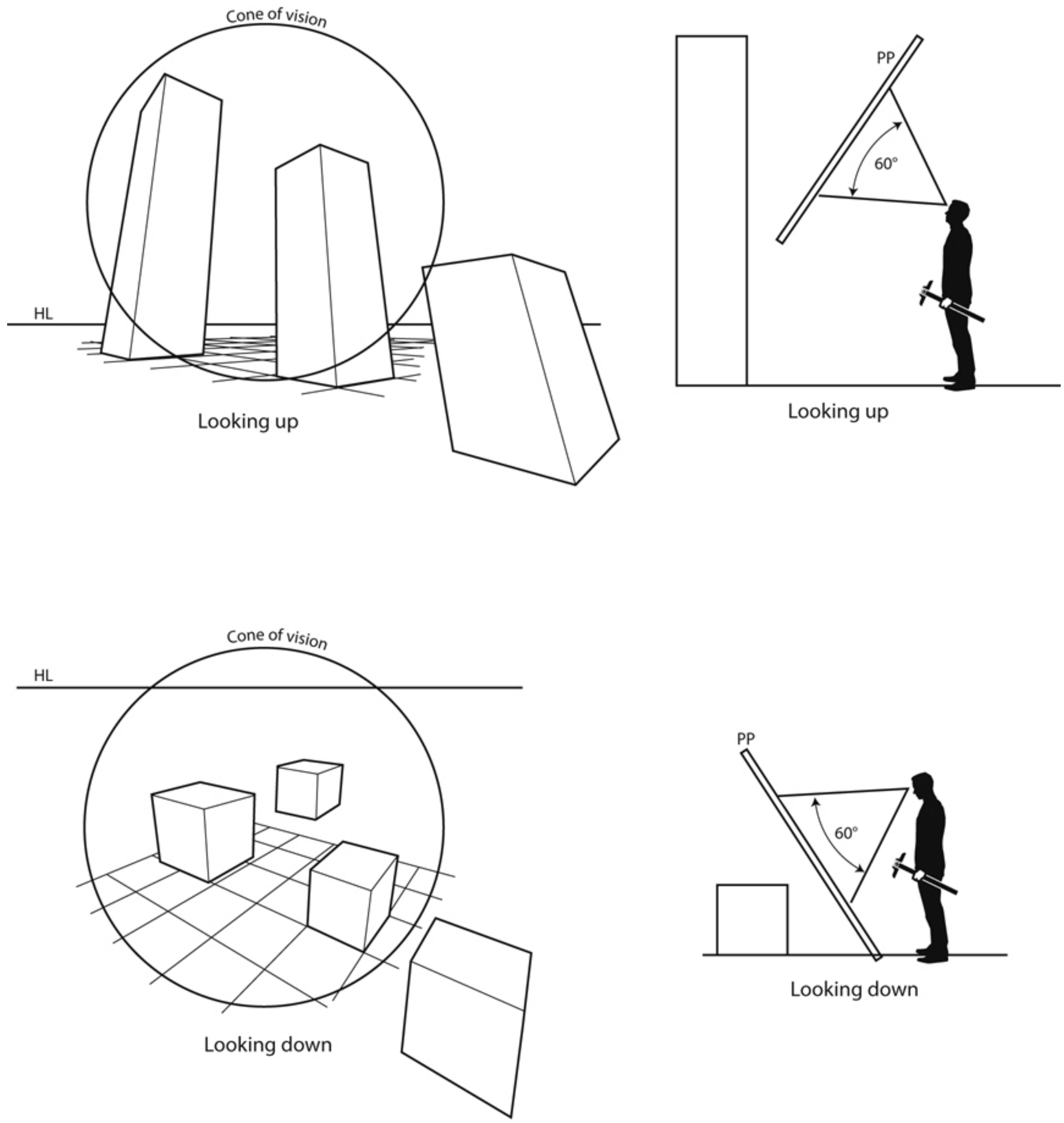


Figure 2.5 In three-point perspective no lines are parallel with the picture plane. Objects outside the cone of vision are distorted. They look stretched and tilted.

Four-Point Perspective

Before artists understood the cone of vision, many unwittingly attempted wide-angled views— with unfortunate results. They had no way to compensate for the effects of distortion caused by a flat picture plane.

Four-point perspective is a panorama view. It displays information normally unseen. There are four vanishing points, and each are 90° apart. Horizontally, the picture plane is curved and surrounds the viewer like a **cylinder**. This creates a 360° image area. Vertically, the picture plane is flat. Therefore, to prevent distortion, the vertical image area remains at 60°. Vertical lines are parallel with the picture plane and are drawn straight. Horizontal lines are not parallel with the picture plane, and are drawn curved (Figure 2.6).

A variation on the panorama theme is to turn the picture plane 90° (a vertical cylinder). This orientation creates a four-point view from zenith to nadir.

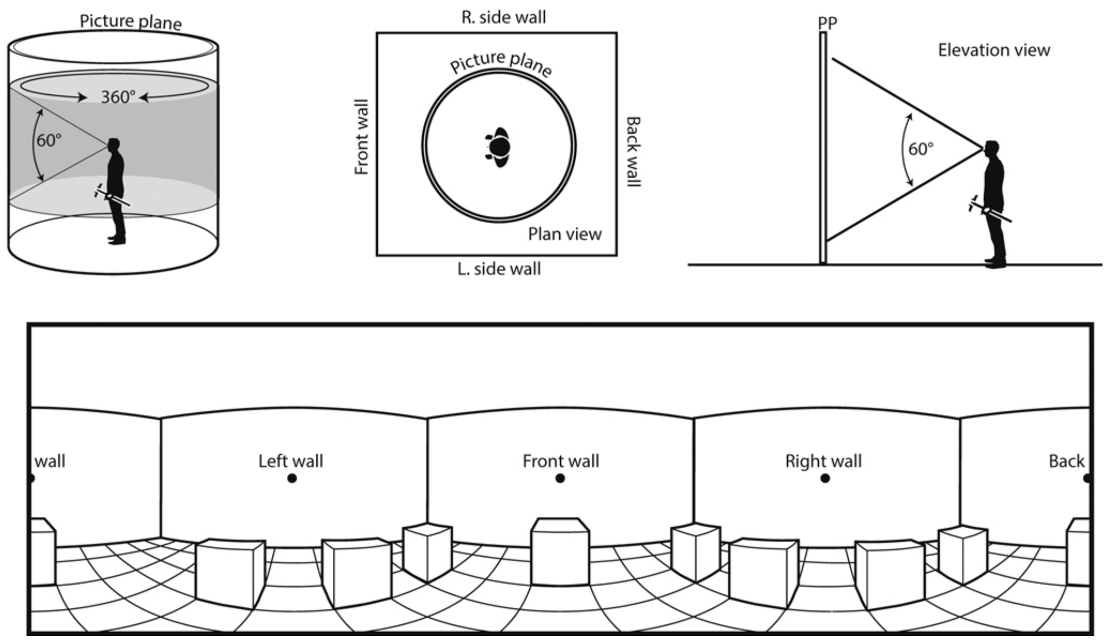


Figure 2.6 In four-point perspective vertical lines are straight and horizontal lines are curved.

Five-Point Perspective

Curvilinear perspective, sometimes called "fish-eye," can be traced to the infancy of perspective. Jan van Eyck's painting, *The Arnolfini Portrait*, depicts a reflection on a curved mirror (Figure 2.7). The science surrounding curvilinear perspective was not fully understood, but artists had many opportunities to study the appearance of lines reflected on curved surfaces. Polished stones used as mirrors have been found dating back to 6000 BC.

Early curvilinear perspective was confined to reflected images. However, curvilinear perspective can be applied to real objects. In five-point perspective, the picture plane is a **hemisphere**. Five-point perspective creates a 180° image. Everything is depicted from east to west and from north to south. There is a vanishing point at the top of the hemisphere and one at the bottom, one to the left and one to the right. The fifth vanishing point is at the center of vision (Figure 2.8).

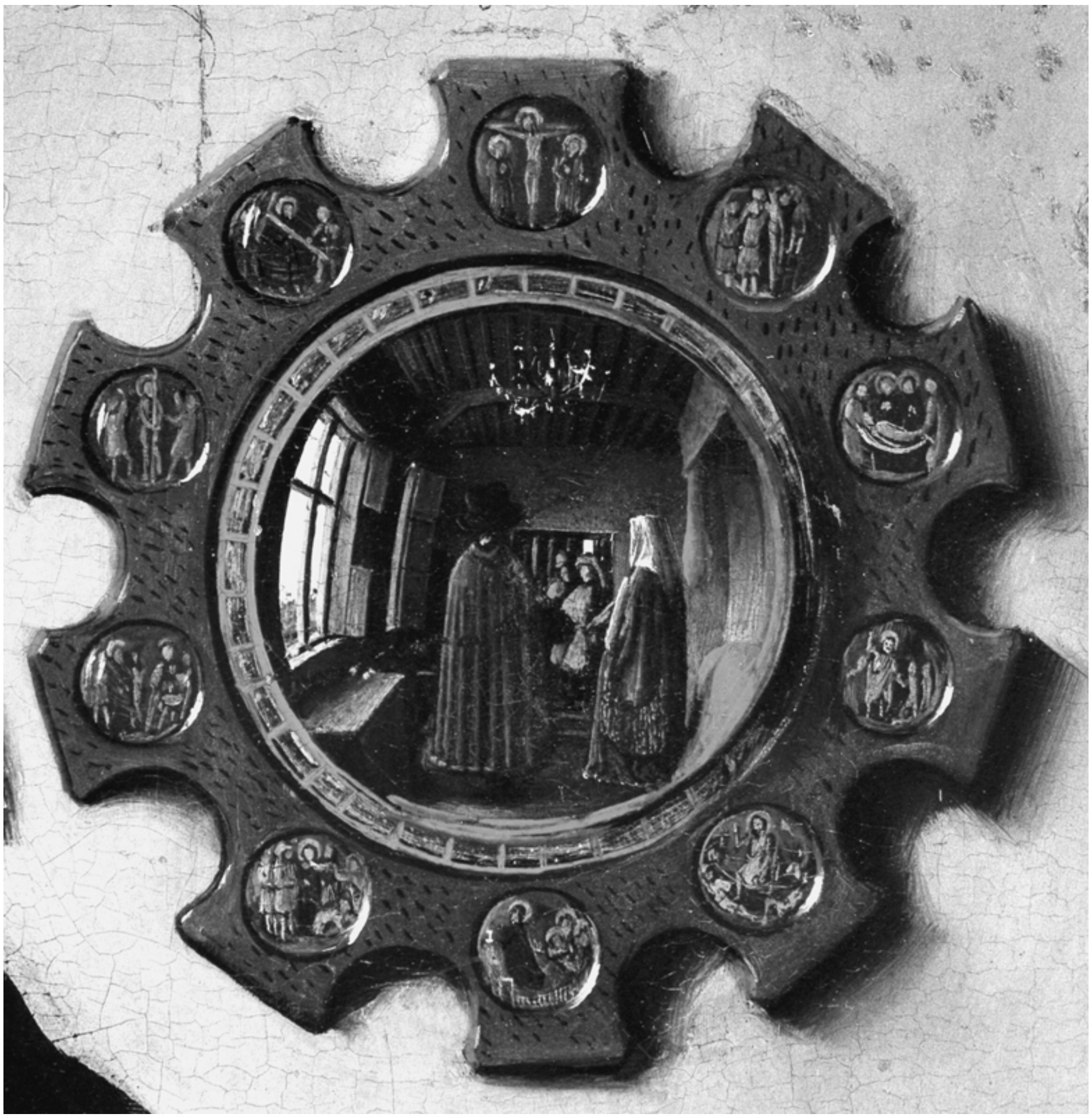


Figure 2.7 Jan van Eyck, *The Arnolfini Portrait* (detail), 1434, National Portrait Gallery, London.

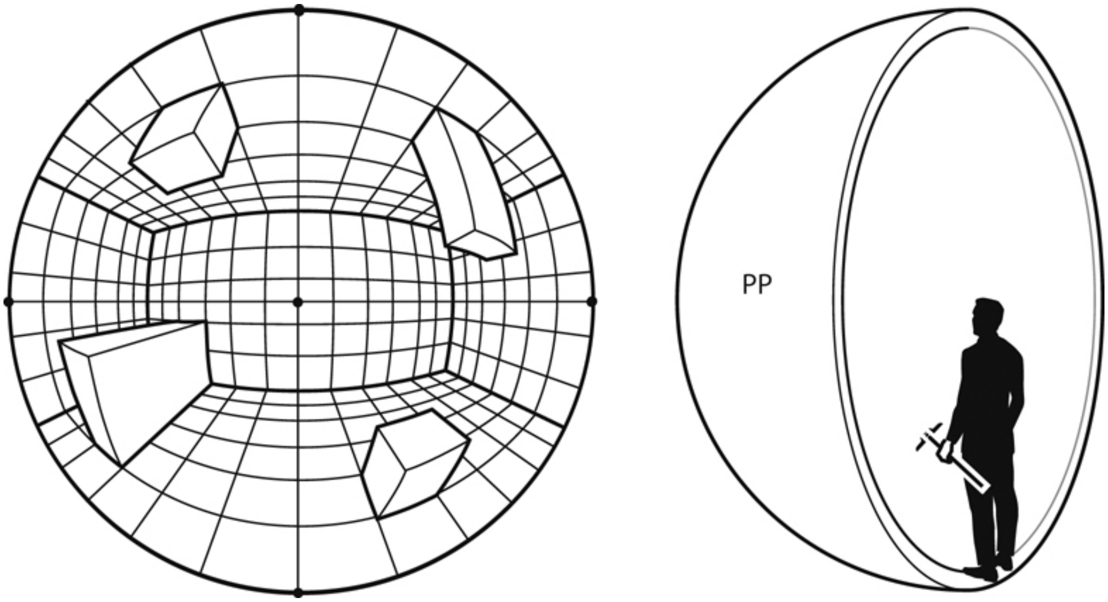


Figure 2.8 In five-point perspective, the picture plane is a hemisphere.

Six-Point Perspective

A six-point image displays everything in front of the viewer, as well as everything behind them—a 360° image. The picture plane is a **sphere**. The two images are typically displayed side-by-side (Figure 2.9).

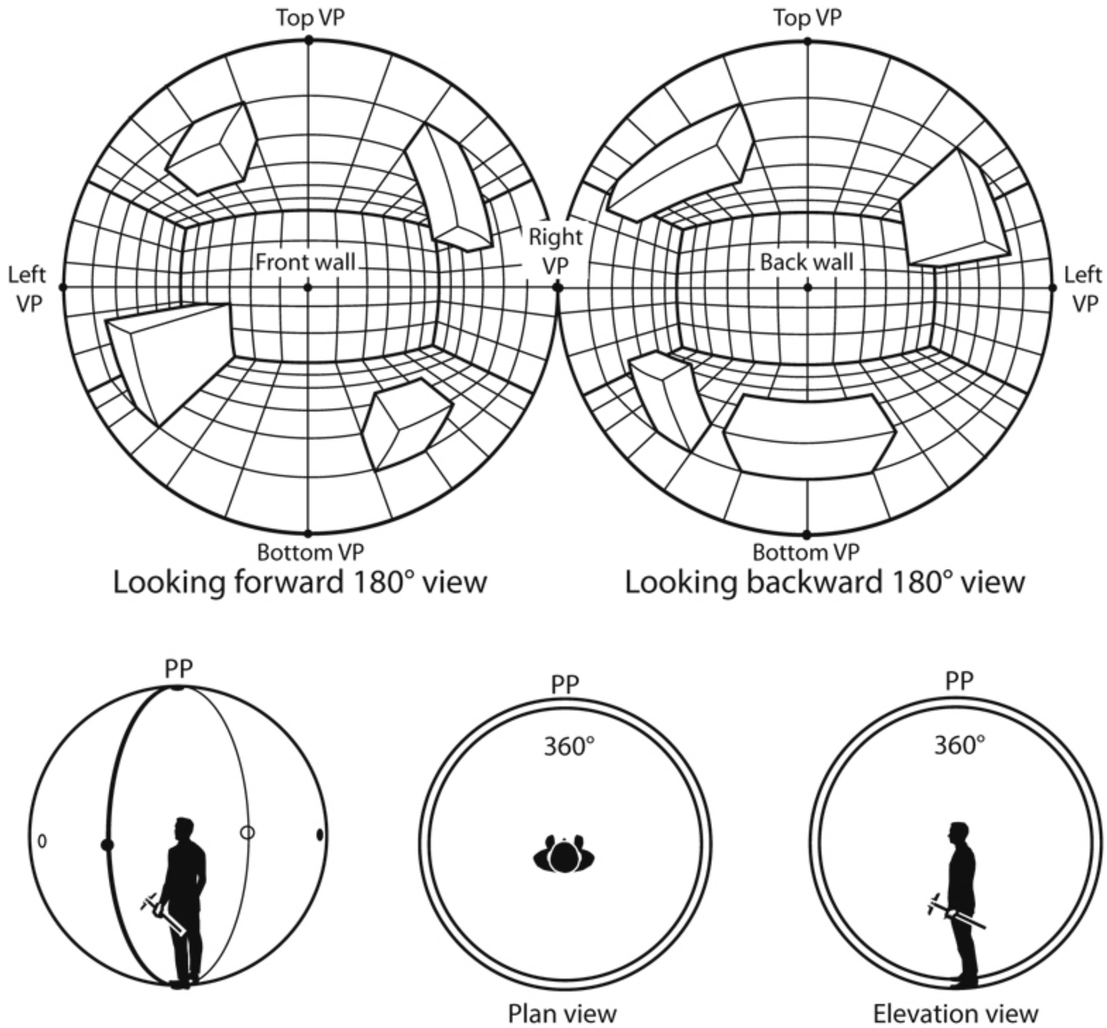


Figure 2.9 **In six-point perspective, the picture plane is a sphere with the station point at its center.**

3
One-Point Perspective

Before a perspective drawing can be created, a perspective diagram needs to be constructed. The diagram is the foundation for the image, the basal element of perspective. It establishes the infrastructure and key elements necessary for accurate representation.

Creating a One-Point Perspective Diagram

Here is a list of the principal components of the diagram, and how to arrange them.

Horizon Line (HL)

Begin by drawing a horizontal line. This line represents the horizon. It can be drawn anywhere, although it is usually best to place it somewhere in the center of the page. The horizon line is always at the viewer's eye level.

Station Point (SP)

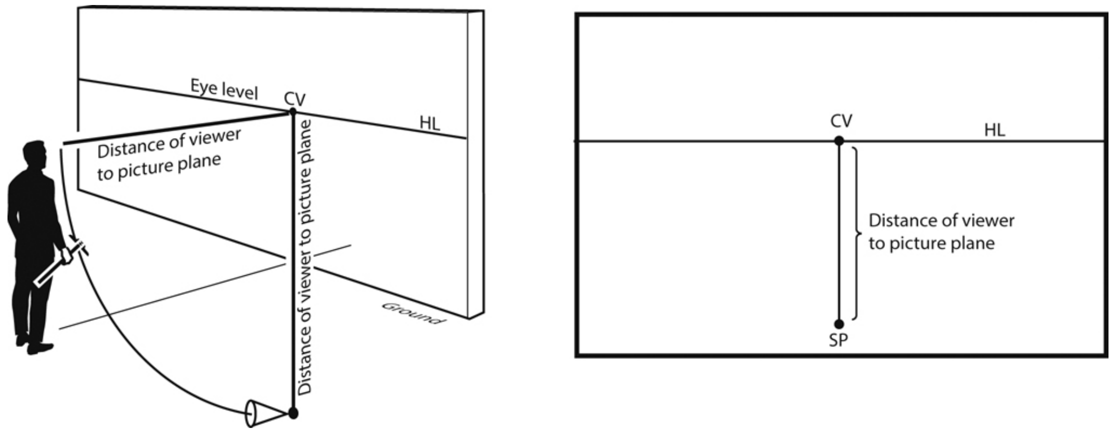


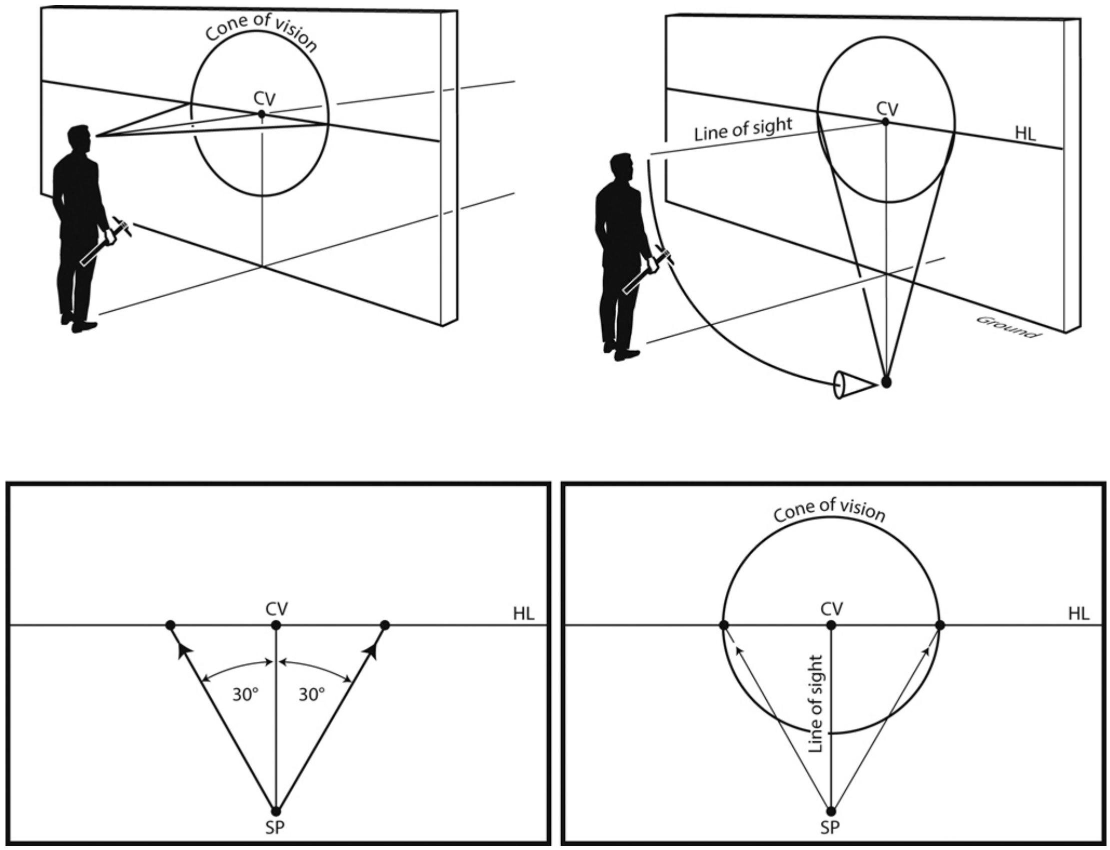
Figure 3.1 Establishing the distance from the viewer to the picture plane.

The placement of the station point defines the distance from the viewer to the picture plane. The station point's location is in the third dimension—however, the paper has only two dimensions. This creates an obvious problem. This obstacle is circumvented by drawing the distance from the viewer to the picture plane vertically. The distance from the center of vision to the station point represents the distance from the viewer to the picture plane. Decide how far the viewer is from the picture plane (the greater the distance, the larger the cone of vision). Then, from the center of vision, draw a line down to the station point (the station point and the center of vision are always aligned). There is no foreshortening to this line (Figure 3.1).

For example, if the distance is 10 units long and the scale is 1:2, then the viewer is 20 units from the picture plane. It is usually a good idea to put the station point as far away as comfortable. The farther away it is, the larger the cone of vision will be, therefore allowing for a larger drawing area.

Cone of Vision

From the station point, project a 30° angle on each of the lines of sight. Then, centered on the focal point, draw a circle (Figure 3.2, bottom). Before beginning the drawing, it is prudent to establish the cone of vision. It's helpful to know the size of the image area at the outset.



[Figure 3.2](#) The cone of vision is a circle created from a 60° cone projected from the station point.

Measuring Points (MP)

From the station point, project two 45° angles to the horizon line (Figure 3.3). These are the measuring points. Lines drawn from either of these points create 45° angles in perspective. These points are used to measure foreshortened lines—lines drawn to the center of vision.

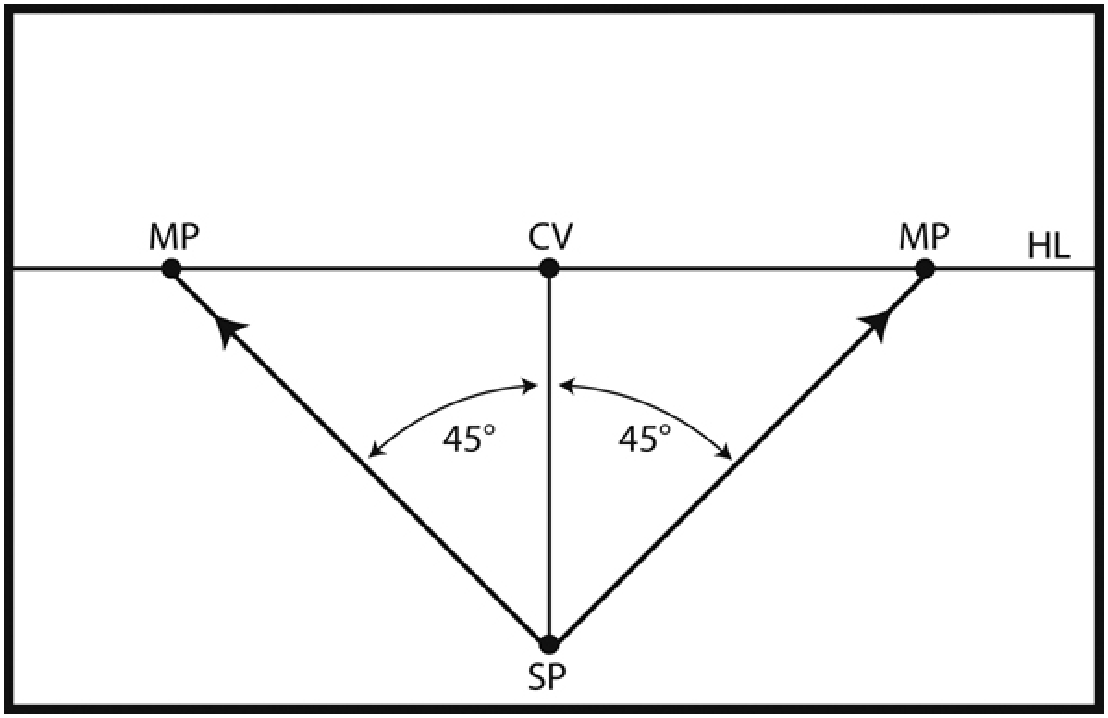


Figure 3.3 One-point measuring points are 45° from the station point.

Measuring Line (ML)

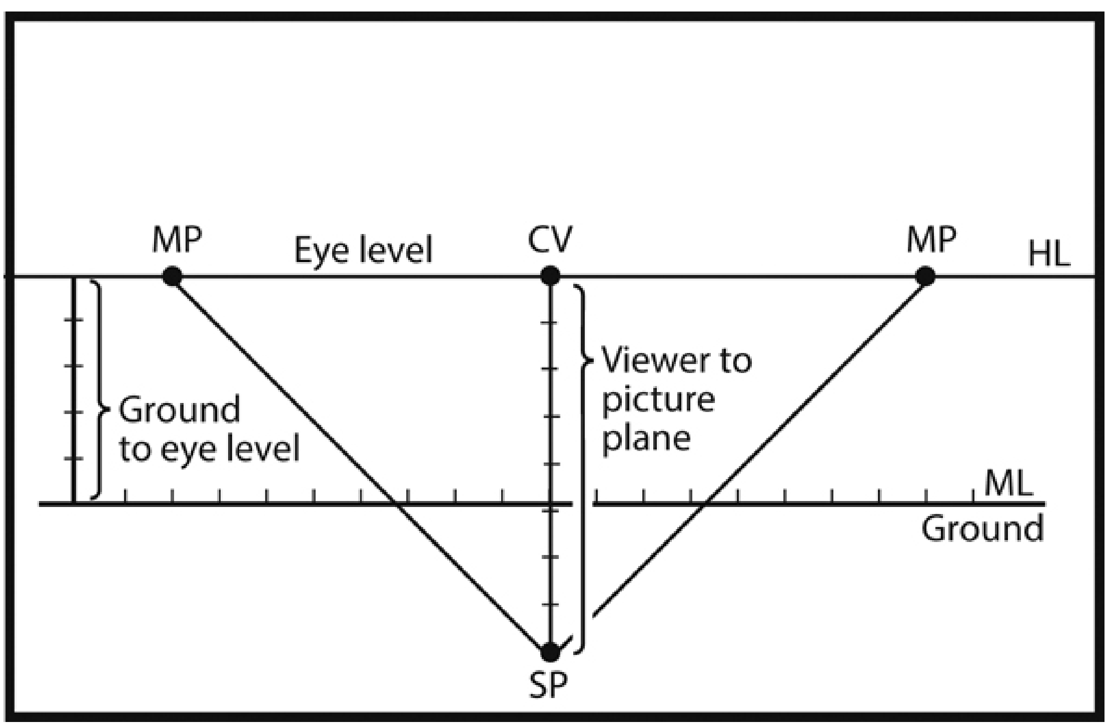


Figure 3.4 **The measuring line defines the scale. The distance from the horizon line to the measuring line defines the height of the viewer (5 units tall in this illustration). The distance from the center of vision to the station point is how far the viewer is from the picture plane (8 units in this illustration).**

Place the measuring line below the horizon. The measuring line is on the ground plane. The horizon line is at the viewer's eye level, therefore the distance from the measuring line to the horizon line equals the distance from the ground to the viewer's eye. The lower the measuring line is drawn, the taller the viewer.

Divide the measuring line into units (Figure 3.4). Throughout this book, distances are referred to as units. A unit can represent any distance—one inch or one centimeter, ten miles or 10,000 meters.

The one-point perspective diagram is now complete, and the drawing can be started.

Measuring in One-Point Perspective

In one-point perspective objects are oriented to the picture plane in a specific way. Height and width are parallel with the picture plane, and depth is perpendicular to the picture plane. Each of these dimensions will be examined in turn.

Measuring Width

The line of sight is the direction in which the viewer is looking. Objects can be to the left or to the right of the line of sight. Measure this distance by counting units along the measuring line (Figure 3.5).

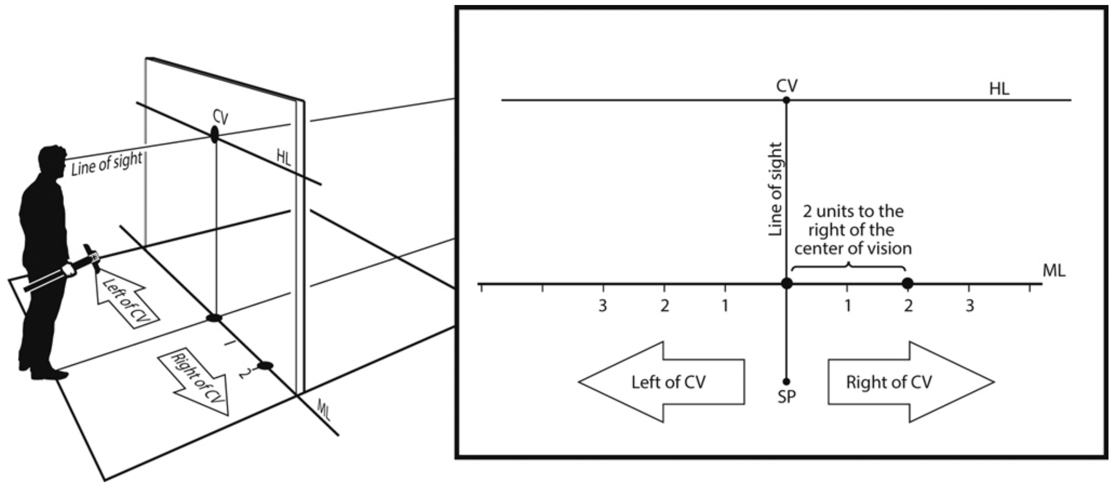
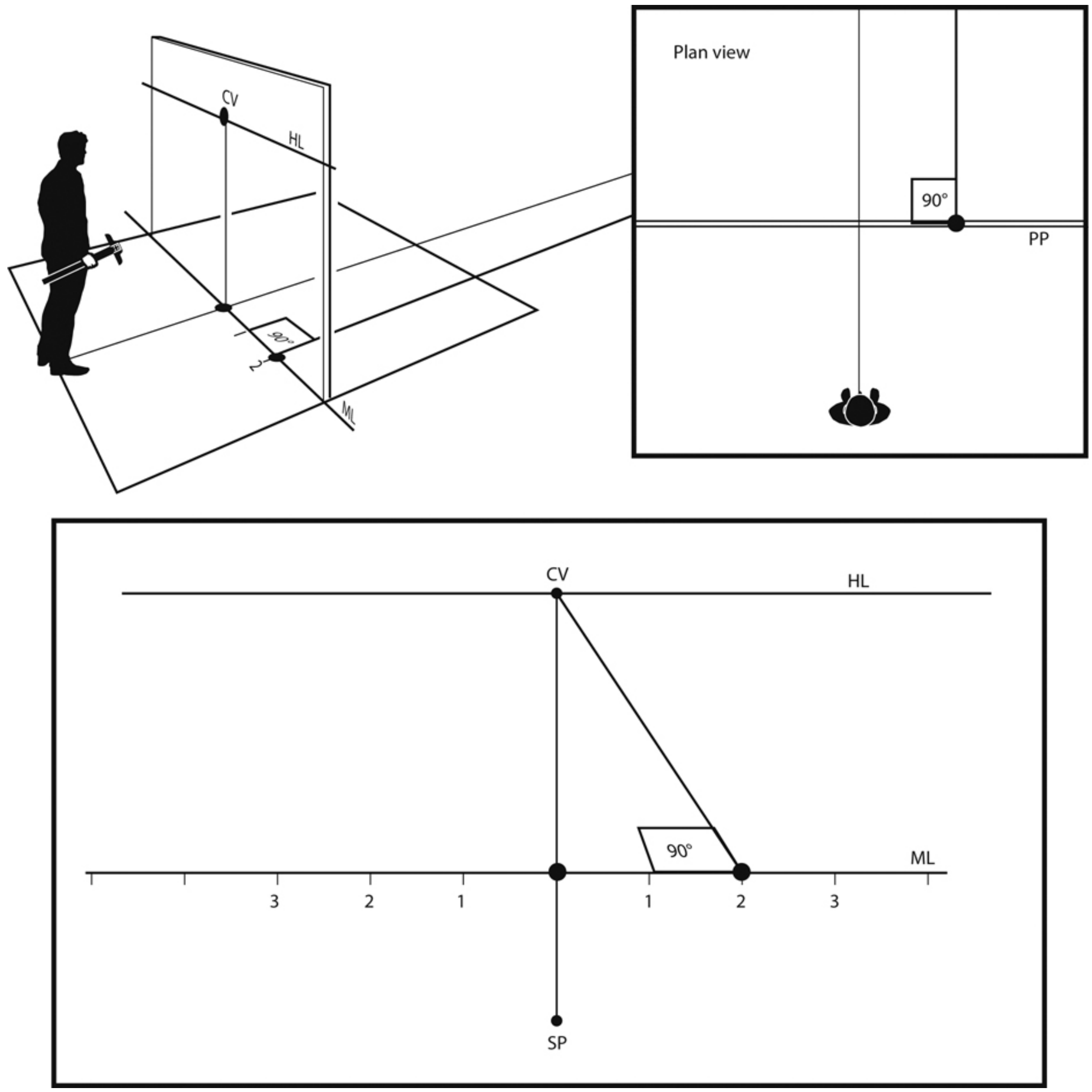


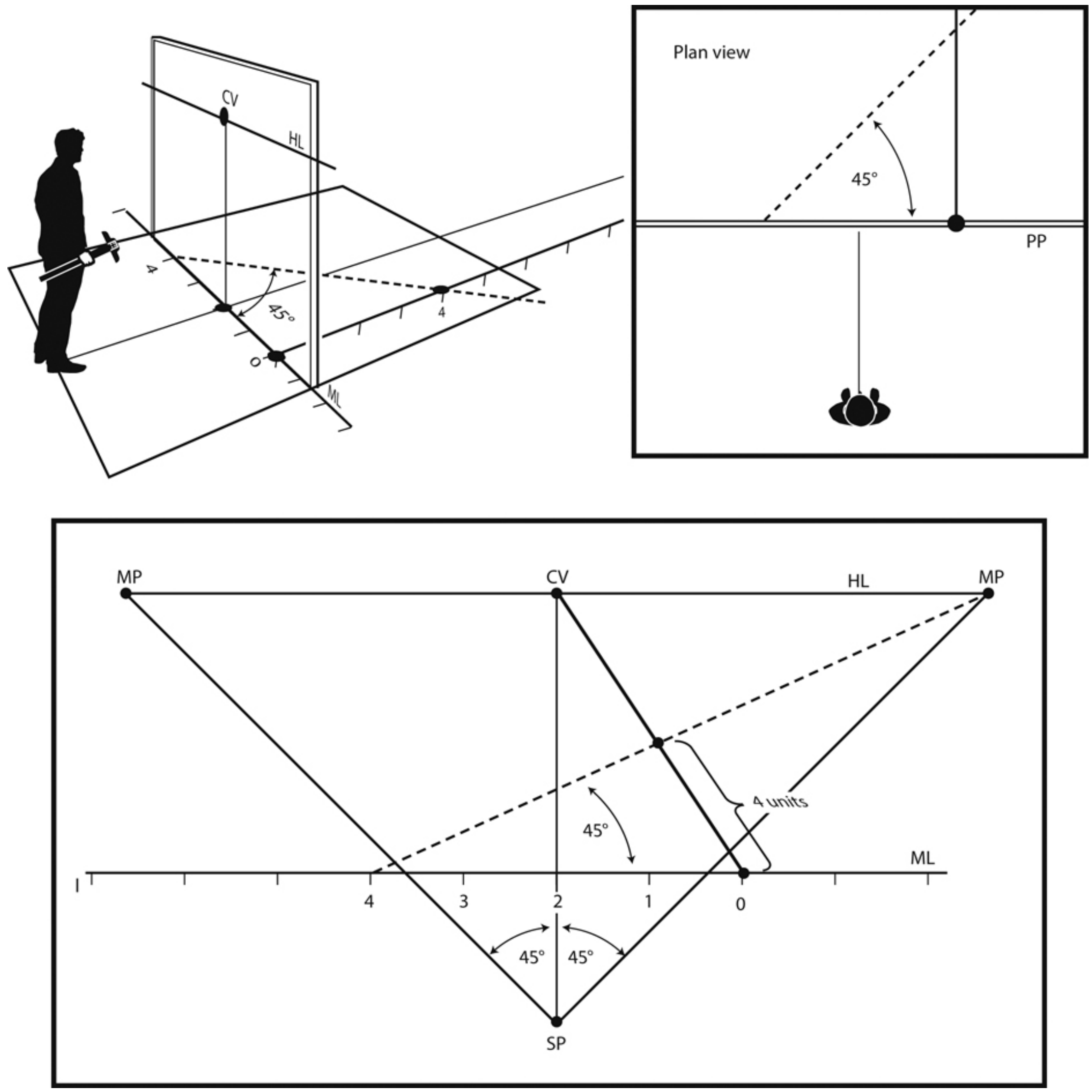
Figure 3.5 Measure width by counting units to the left or right of the line of sight.

Measuring Depth

Lines that are 90° to the picture plane connect to the center of vision (Figure 3.6). These lines are foreshortened, and can't be measured directly with a ruler. The process of measuring foreshortened lines involves some simple geometry—no equations are required. The angle between opposing corners of a square is 45°. By drawing a 45° angle, a square can be drawn. The measuring point draws 45° angles in perspective. The measuring point transfers a horizontal distance to a foreshortened line. When measuring in one-point perspective, half a square is drawn—a right-angle isosceles triangle (Figure 3.7). See Chapter 6 for supplemental information on measuring.



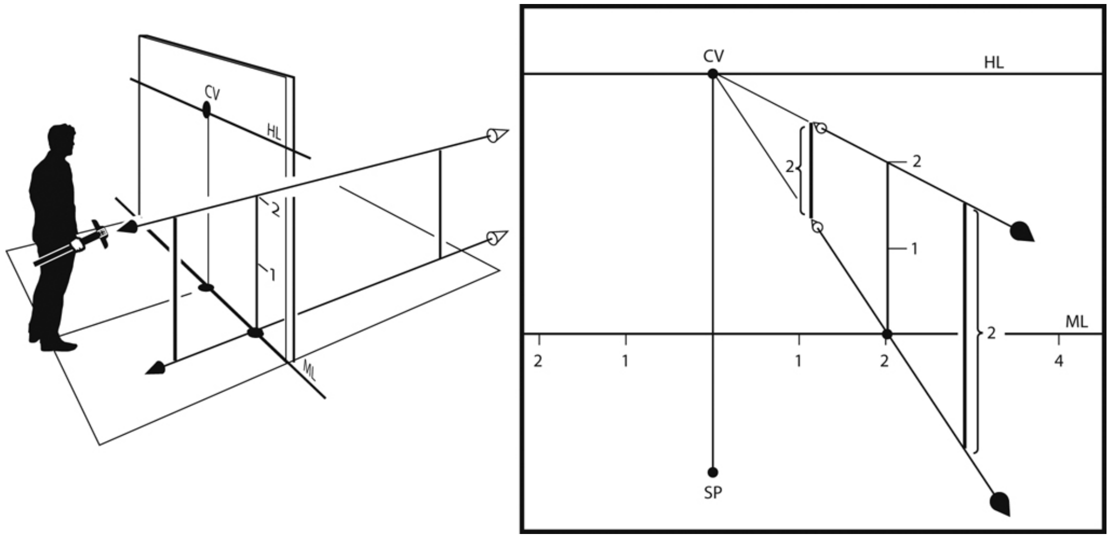
[Figure 3.6] **Lines that are 90° to the picture plane connect to the center of vision.**



[Figure 3.7](#) Use a measuring point to measure lines connecting to the center of vision. The measuring point draws a foreshortened isosceles triangle.

Measuring Height

To measure vertically, turn the measuring line 90°. Using the same scale, measure up from the ground to the desired height ([Figure 3.8](#)). The height can be projected forward or backward using the center of vision.



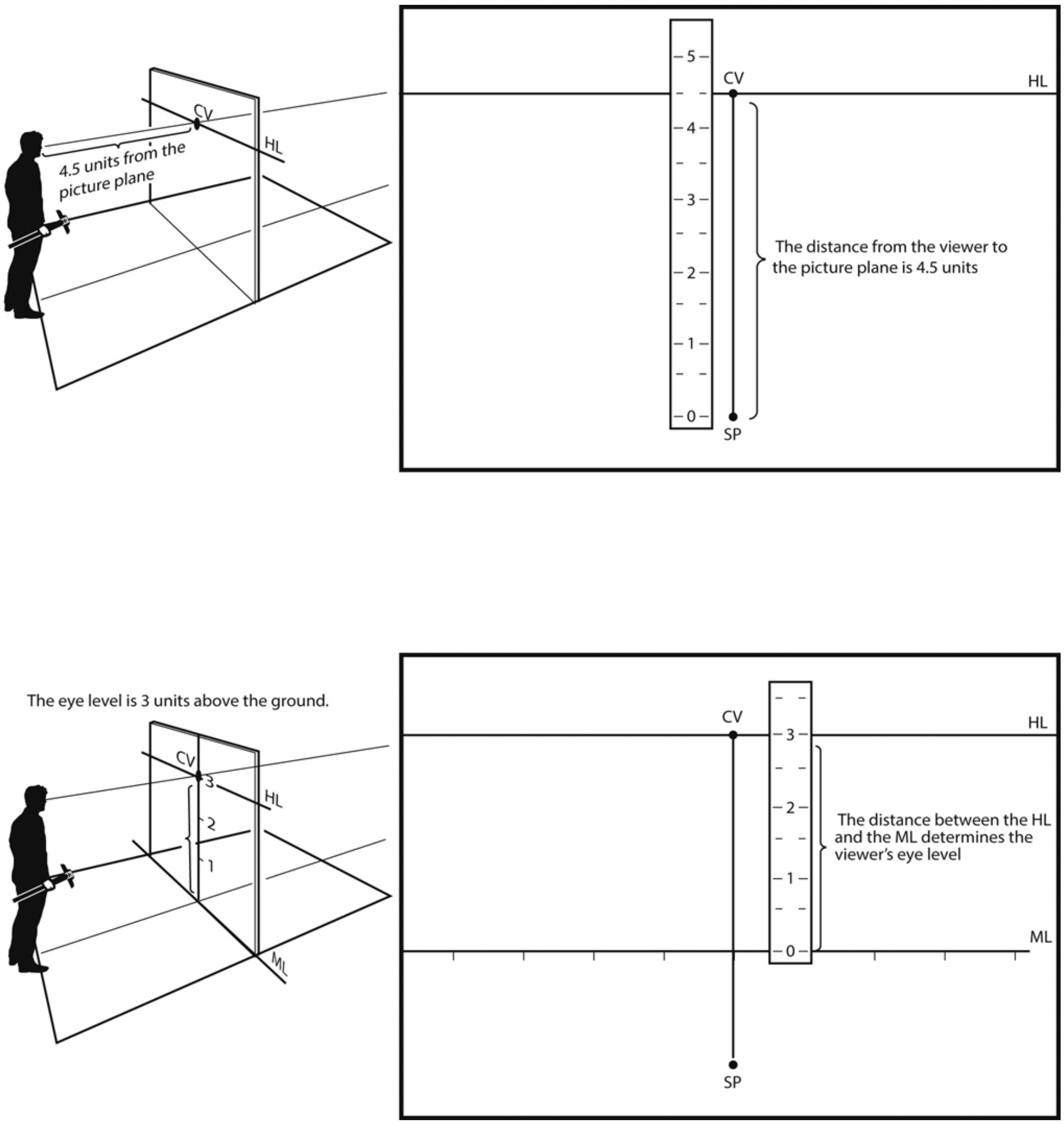
[Figure 3.8](#) **Measure height by turning the measuring line vertically. Height can be projected forward or backward using the center of vision.**

Drawing a One-Point Perspective Box

Now height, width, and depth can be measured, these three skills can be combined to draw a box—a one-point perspective, right-angled cuboid.

Create the Diagram

First, draw the diagram. The dimensions used are random, and can, of course, be any desired number. Make the viewer 4.5 units from the picture plane, with an eye-level 3 units above ground (Figure 3.9).



[Figure 3.9](#) **Defining the viewer's height and distance from the picture plane.**

Place the Box

The box is 1 unit to the left of the center of vision and 2 units behind the picture plane ([Figure 3.10](#)).

Measure the Box

The box is 2 units wide, 3 units deep, and 2 units high. Measure each dimension one line at a time (Figures 3.11–3.14).

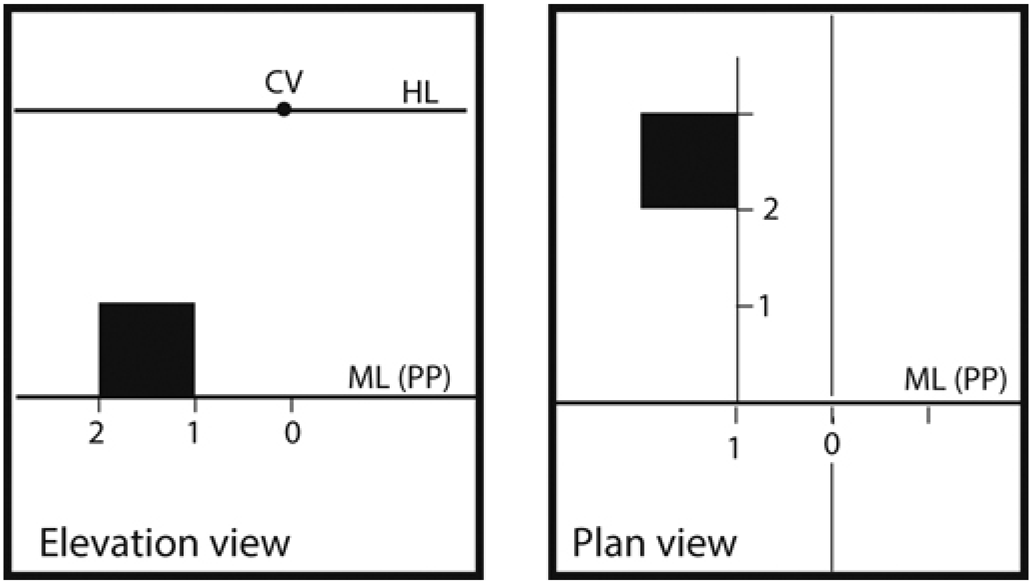
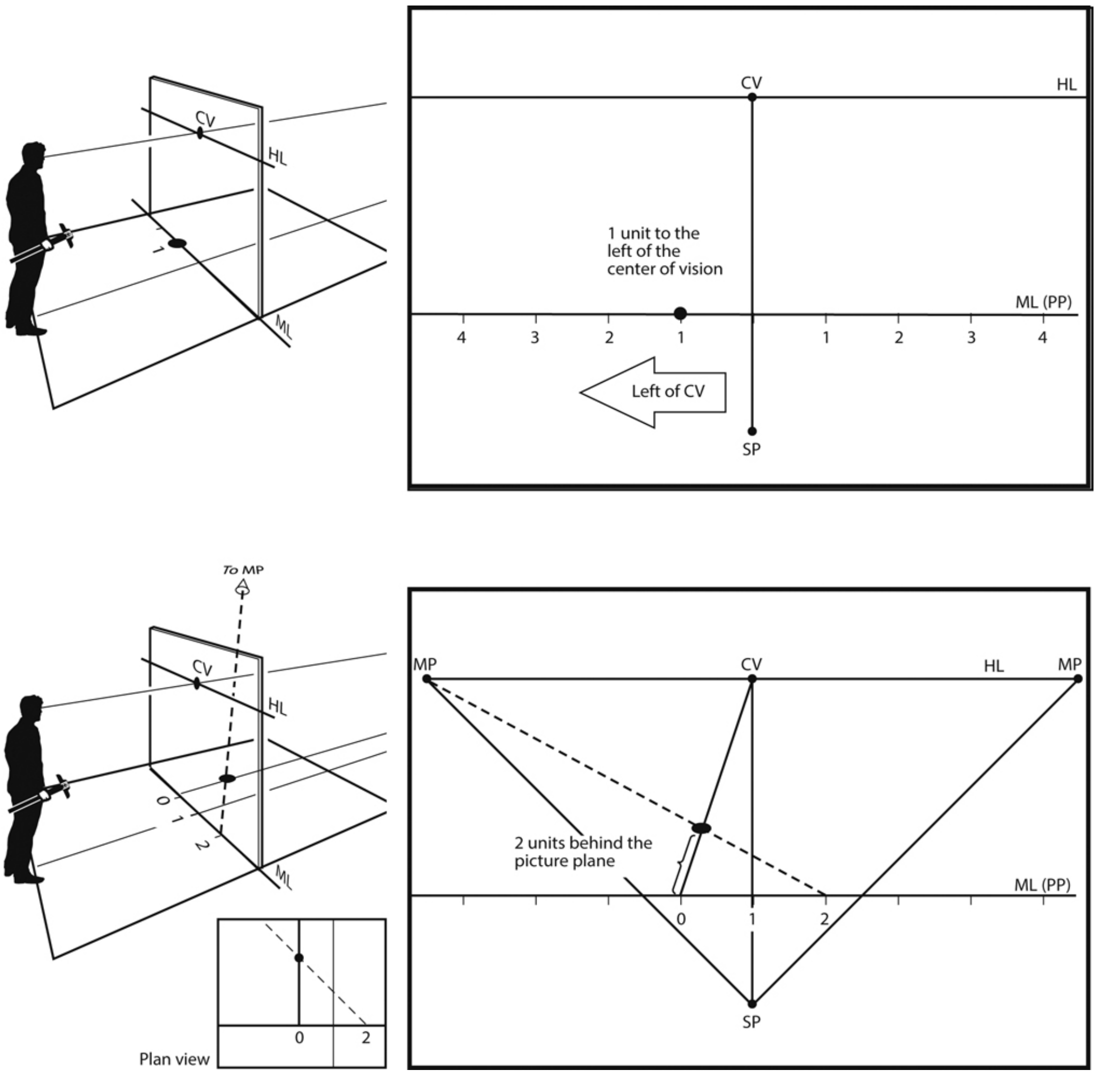


Figure 3.10 The elevation view shows the box 1 unit to the left of the center of vision. The plan view shows the box 2 units behind the picture plane.



[Figure 3.11](#) **(BELOW) The dot represents the front right corner of the box. It is 1 unit to the left of the center of vision, and 2 units behind the picture plane.**

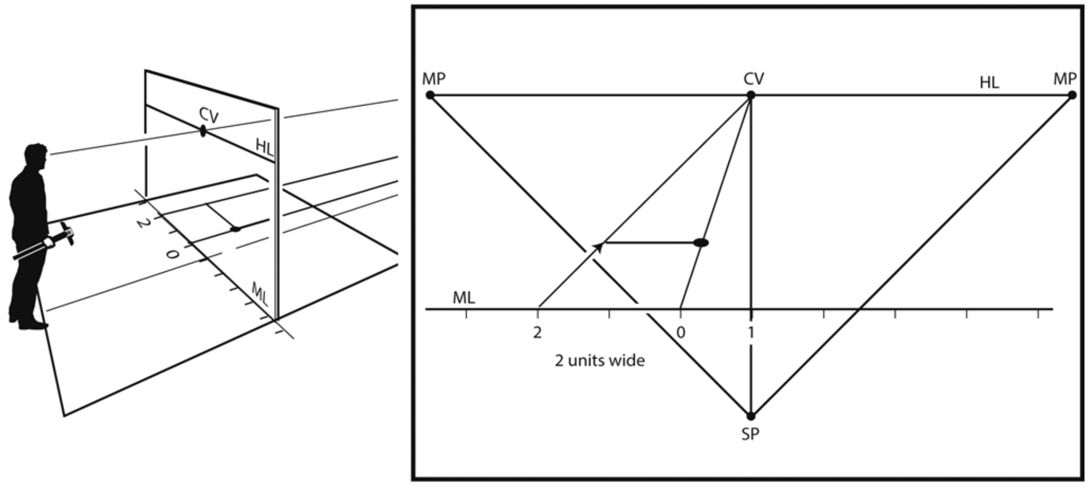


Figure 3.12 Measure the width of the box along the measuring line, and project the dimension backward using the center of vision.

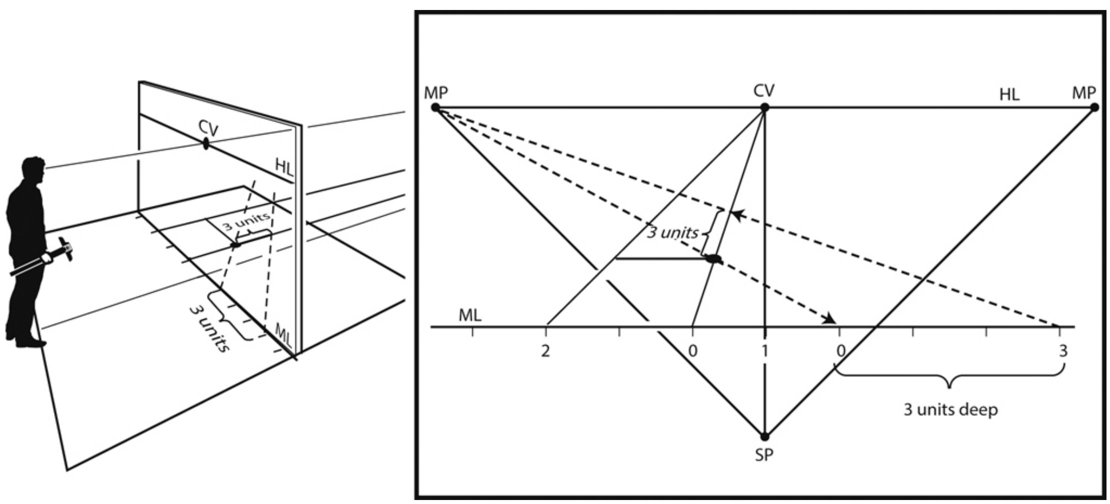
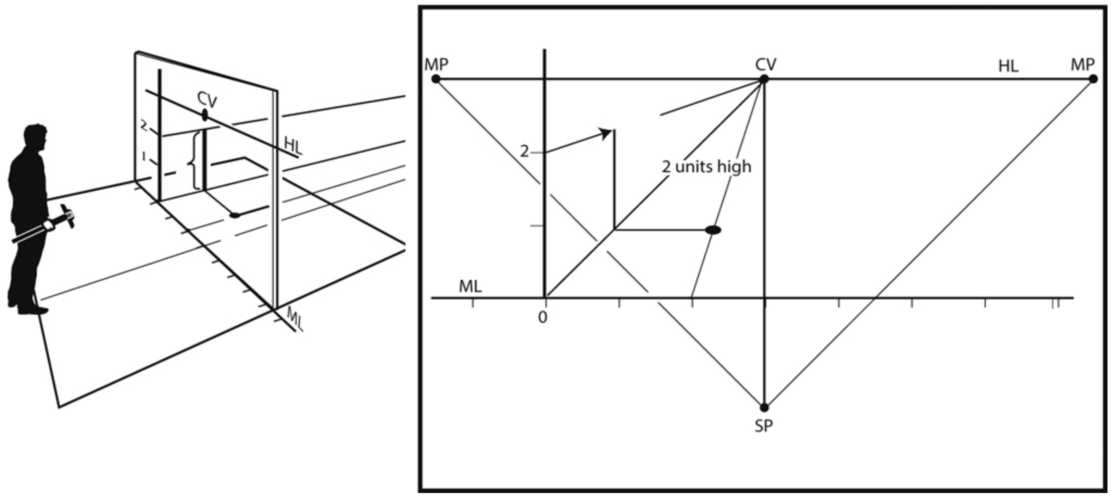


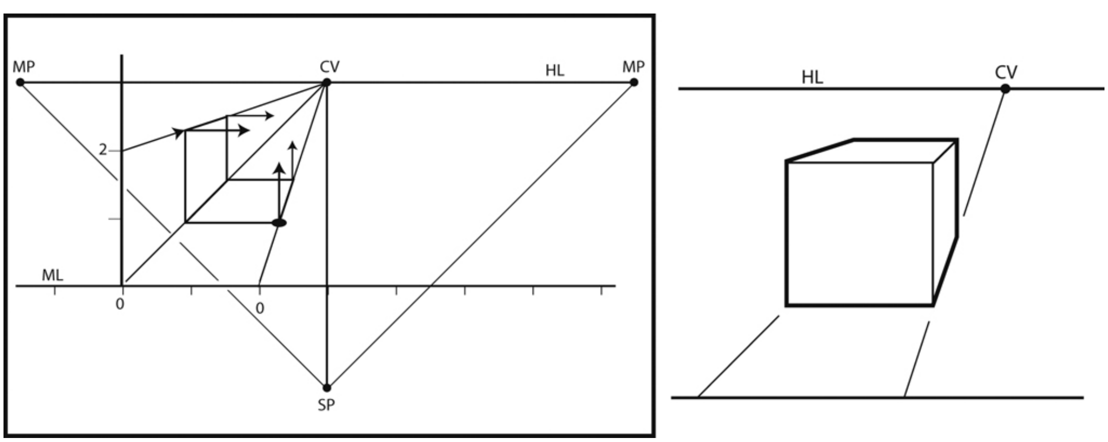
Figure 3.13 Use the measuring point to determine depth.



[Figure 3.14](#) **Measure up from the measuring line, then project the height back to the desired location using the center of vision.**

Complete the Box

Once the height, width, and depth have been measured, find the intersections of these lines to create the corners of the box. Vertical lines are perpendicular to the horizon line. Horizontal lines are parallel with the horizon line. Lines that recede in space connect to the center of vision ([Figure 3.15](#)).



[Figure 3.15](#) **The completed box.**

Continue to practice. Try varying the dimensions, or changing the viewer's eye level and distance to the picture plane. Draw several objects on one page. Become comfortable with one-point perspective before progressing to two-point.

Measuring in Front of the Picture Plane

The previous box was behind the picture plane (between the picture plane and the horizon line). Depth was measured by projecting backward from the measuring line (toward the measuring point) (Figure 3.16).

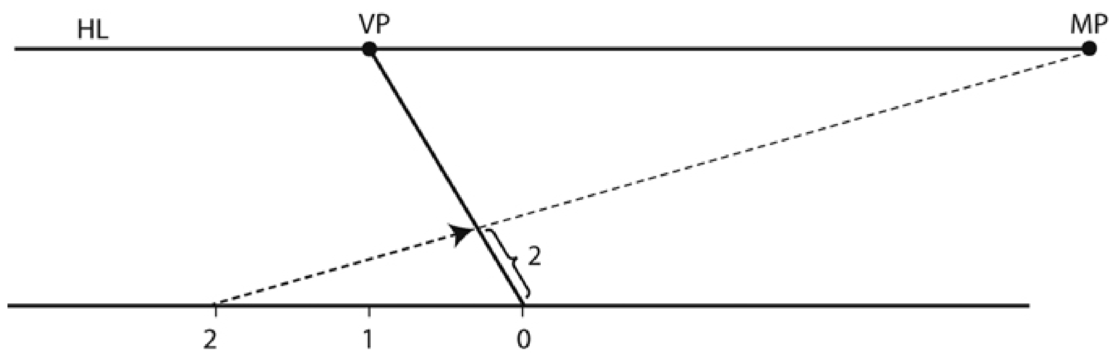
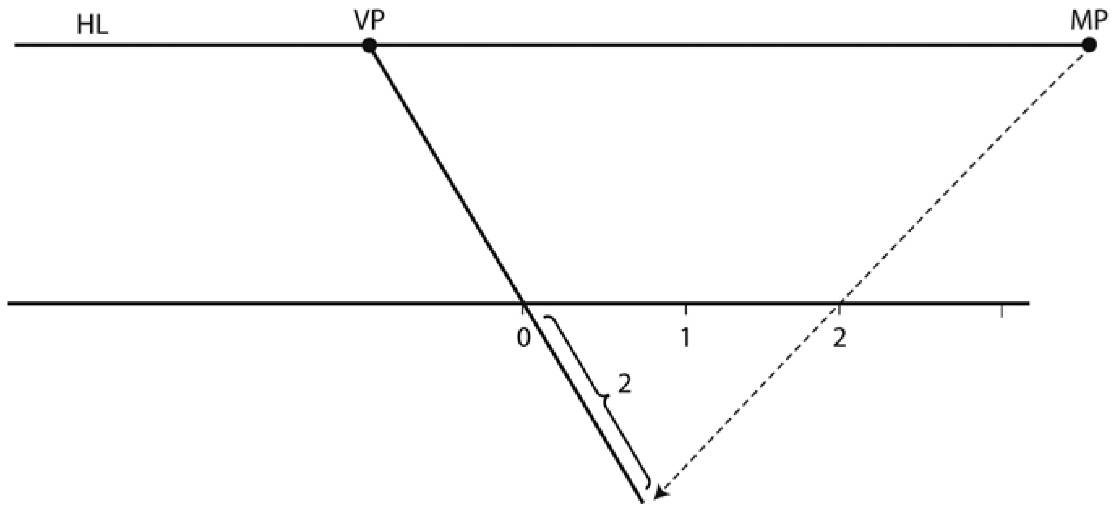


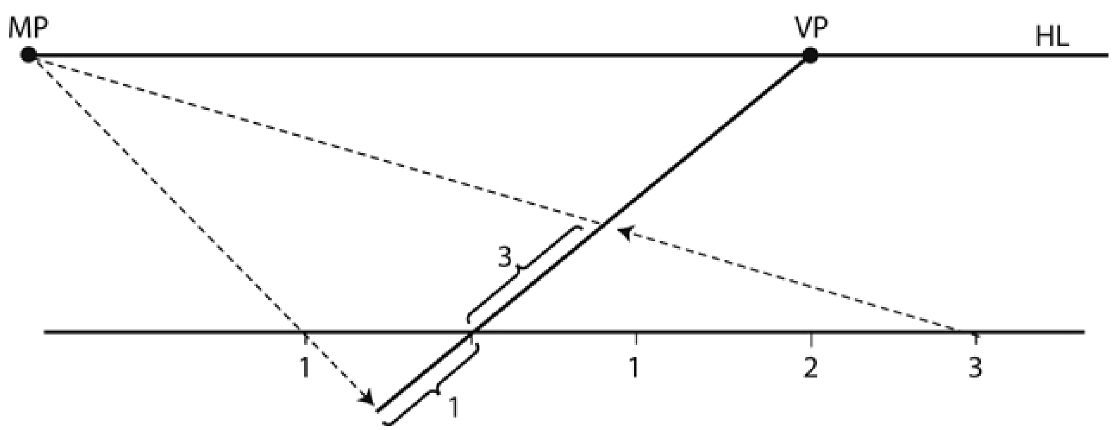
Figure 3.16 Measuring 2 units behind the picture plane.

If an object is in front of the picture plane (between the picture plane and the viewer), measure depth by projecting forward (away from the measuring point) (Figure 3.17).



[Figure 3.17](#) **Measuring 2 units in front of the picture plane.**

If an object is straddling the picture plane, depth is measured by projecting backward *and* forward from the measuring point ([Figure 3.18](#)).



[Figure 3.18](#) **Measuring a line that straddles the picture plane. This line is 4 units long (1 unit in front of the picture plane, and 3 units behind the picture plane).**

This dynamic will be explored further with an example. Draw a room with the measuring line placed between the back wall and the viewer (the picture plane **bisects** the room) ([Figure 3.19](#)). Measurements behind the picture plane are projected backward, toward the measuring point ([Figure 3.20](#)). Measurements in front of the picture plane are projected forward, away from the measuring point. The room surrounds the viewer, so the complete room cannot be drawn. The front of the room is beyond the cone of vision, as well as beyond the edge of the page (Figure 3.21). Placing a grid on the floor may help to visualize the space. Each square represents a half unit, which leads to the next topic: grids (Figure 3.22).

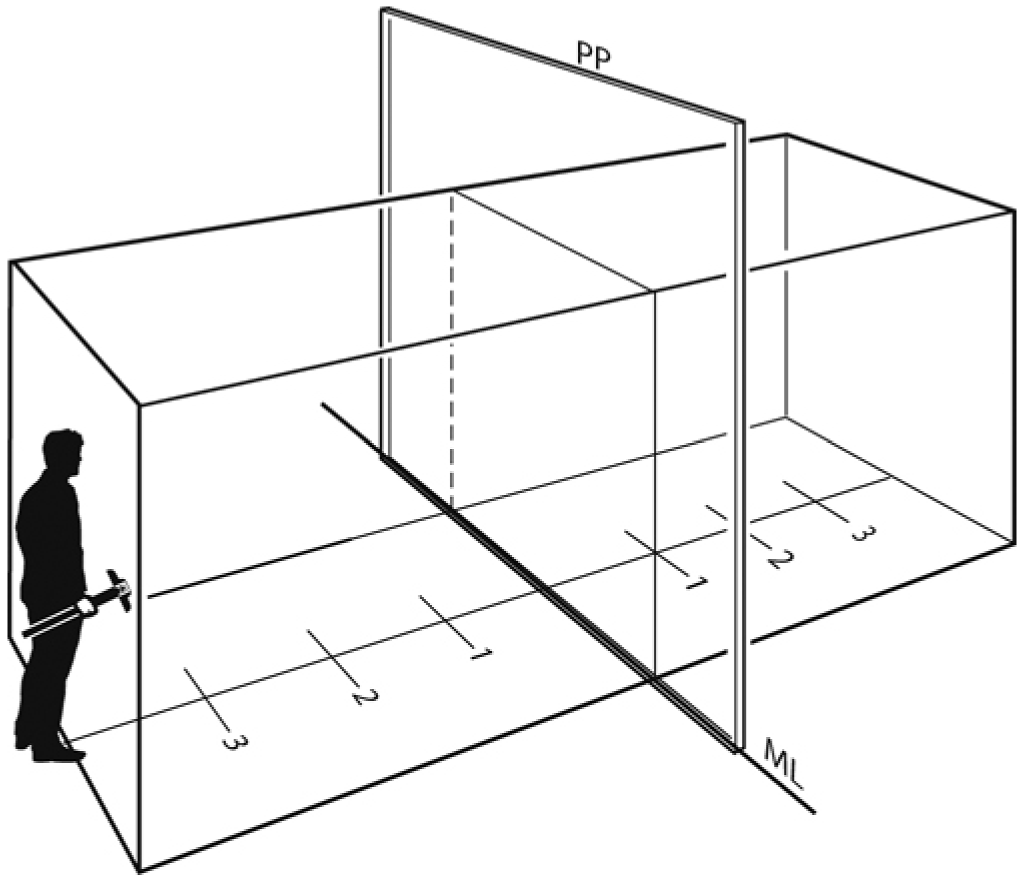
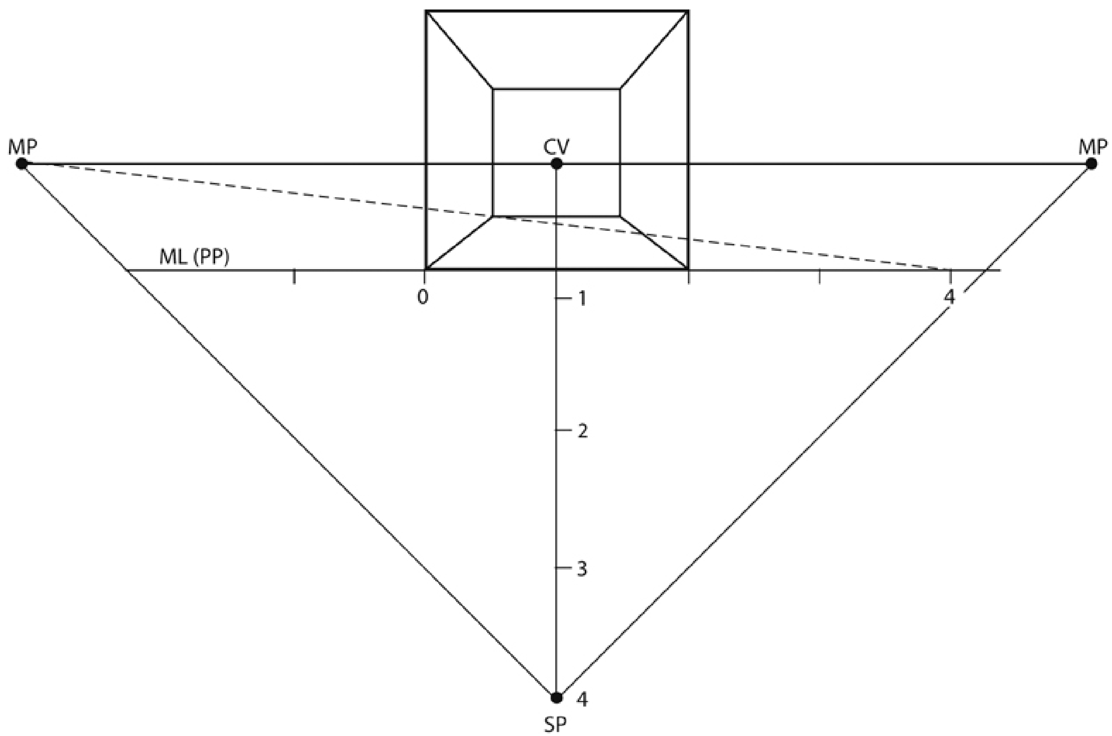
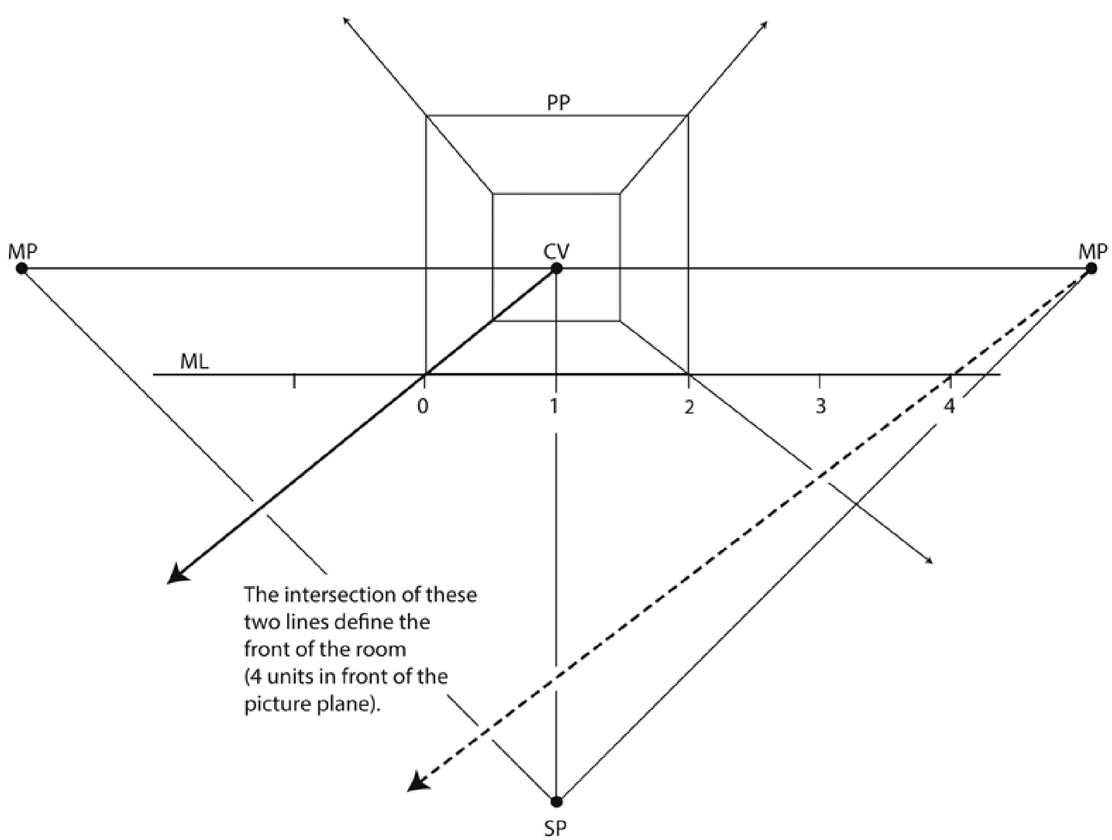


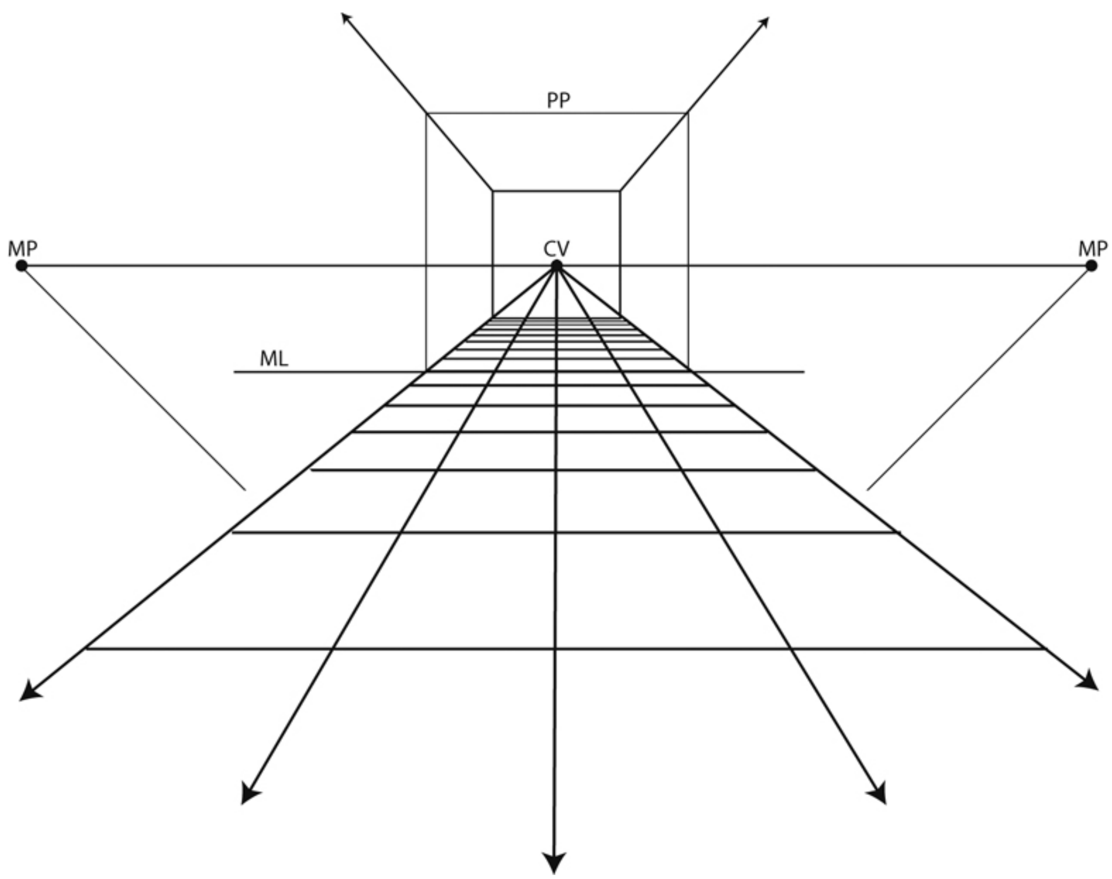
Figure 3.19 The viewer is 4 units in front of the picture plane. The back of the room is 4 units behind the picture plane.



[Figure 3.20](#) **Measuring 4 units behind the picture plane.**



[Figure 3.21](#) The beginning of the room is 4 units in front of the picture plane. The viewer is also 4 units in front of the picture plane. This creates a distance far beyond the cone of vision, and far beyond what is practical to plot.



[Figure 3.22](#) Placing a grid on the floor helps to visualize the space.

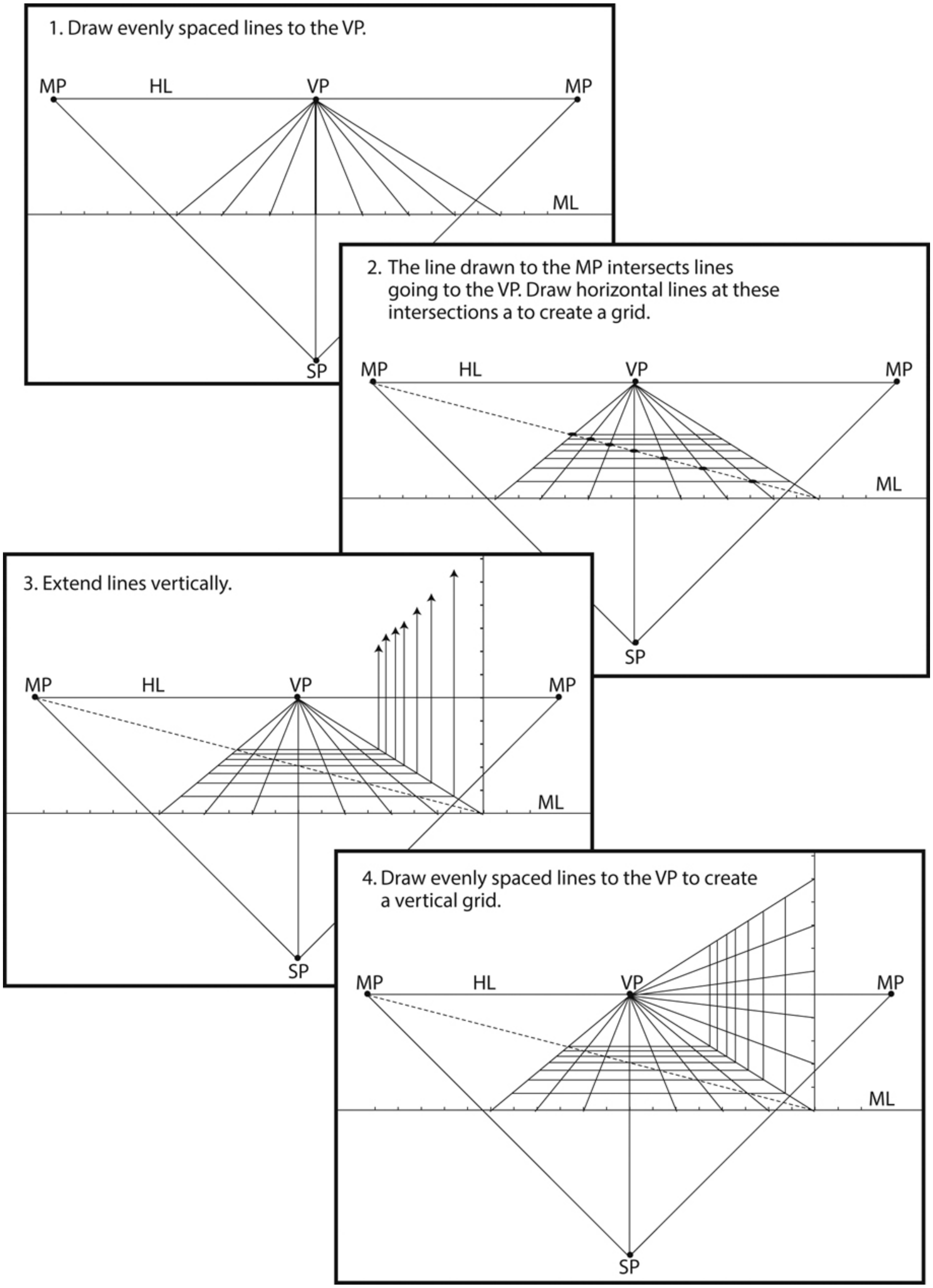


Figure 3.23 Follow these steps to create a one-point perspective grid.

Drawing a One-Point Grid

Drawing a grid is a straightforward task. However, take care to not be overly dependent on grids. If perspective—and the geometry—is understood, then the grid is superfluous. Drawing without a grid is faster and more versatile, but it does have a steeper learning curve. A grid takes time to draw, and is awkward for depicting objects that do not conform to its pattern. However, grids—once they are established—conveniently guide the direction of lines and assist in establishing dimensions. Learning the grid system can be a good starting point for those new to perspective, and there are some instances where establishing a grid is the best solution to a problem.

Drawing a grid involves creating a series of squares. The size of each square, and the number of squares created, are determined by the image. More detailed drawings suggest a smaller, tighter grid. Because of the superabundance of lines, grids are usually used as an underlay. Each square represents 1 unit of measurement (Figure 3.23). A horizontal grid is used to measure width and depth, and a vertical grid is used to measure height (Figure 3.24).

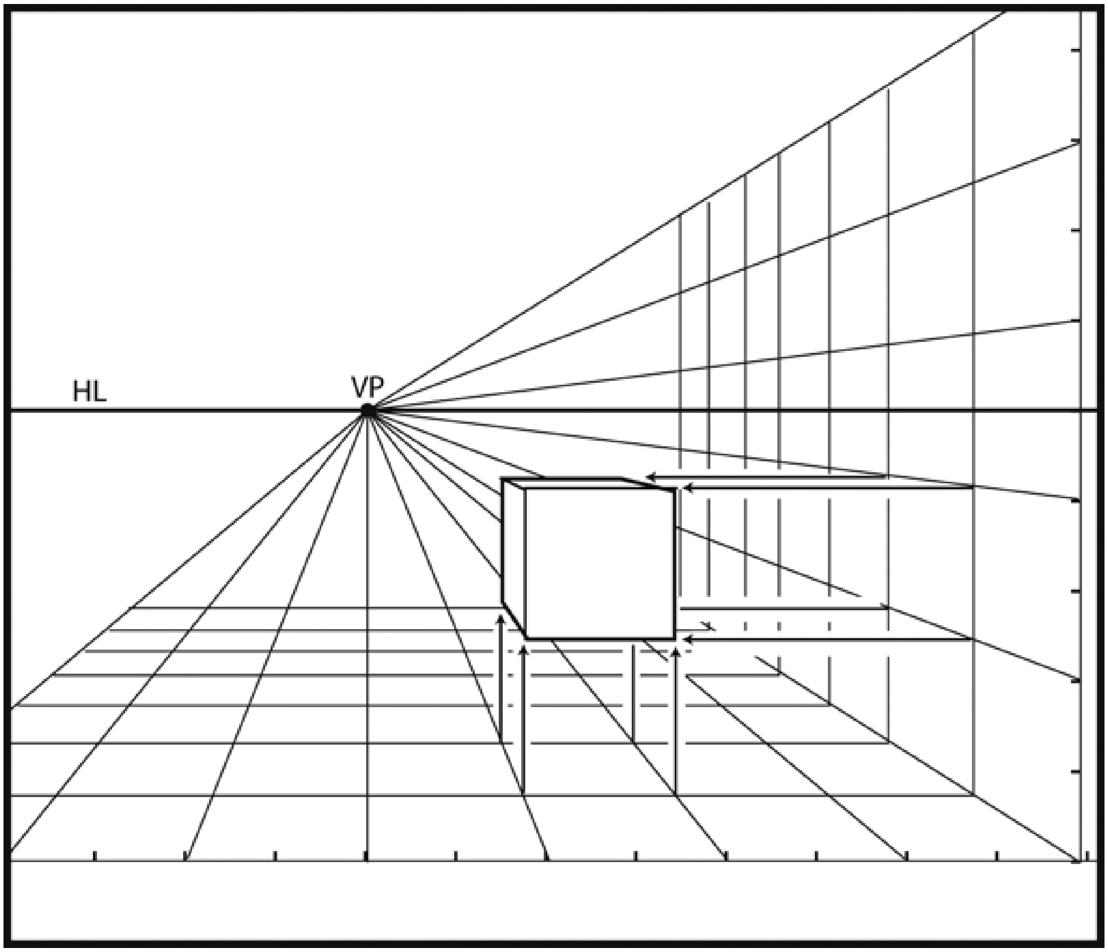
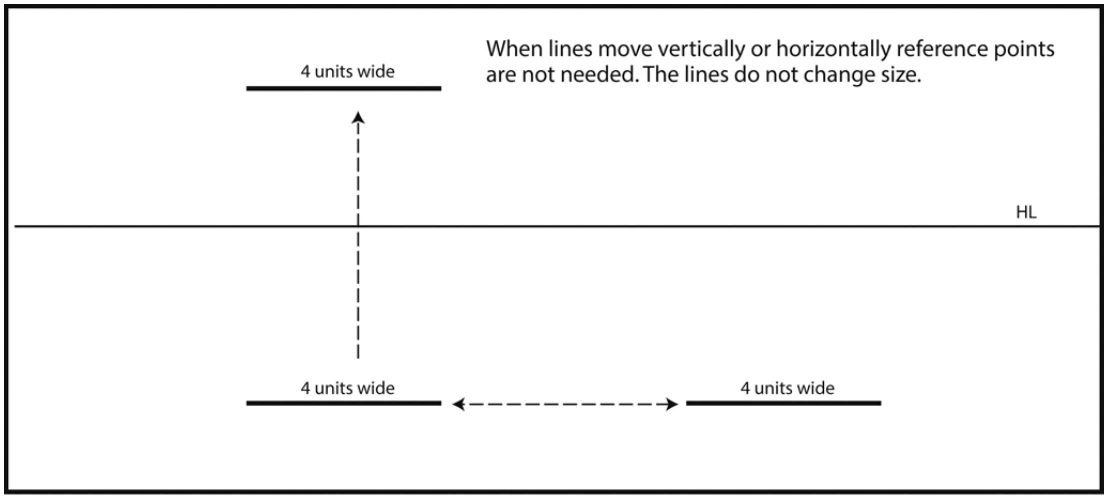


Figure 3.24 Using a grid to measure.

4
Reference Points

Reference points (RP) look like vanishing points, but they differ in function. Vanishing points are used to *draw* objects; reference points are used to *move* objects. While vanishing points are specific in their location, any dot can be used as a reference point. When drawing multiples of an object (a crowd of people, for example), reference points are convenient tools. There are, however, two important caveats: the lines being moved must be on the same horizontal plane, and they must be parallel with each other. A point on the horizon line creates lines parallel with the ground plane. Therefore, a reference point can't be used to move a line that is *on* the ground to a position *above* the ground.

Reference points are not needed to move lines up, down, or from side to side. Lines moving in any of these directions do not change size (Figures 4.1–4.3, top). Reference points are only used to move lines forward or backward in space (Figures 4.1–4.3, bottom).



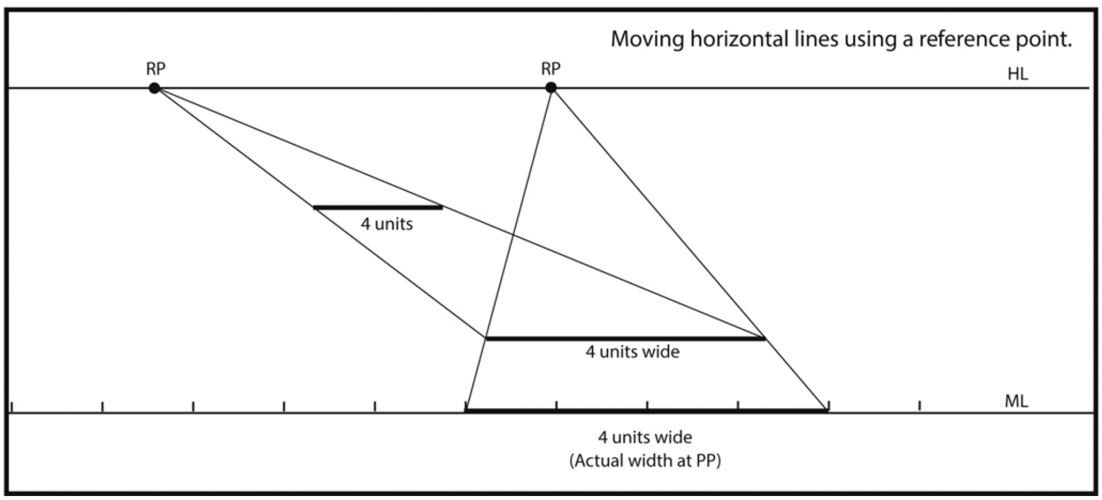


Figure 4.1 Reference points are not needed when moving lines from side to side or up and down (top). Use reference points to move lines forward or backward in space. The lines being moved must be parallel with each other and on the same horizontal plane (bottom).

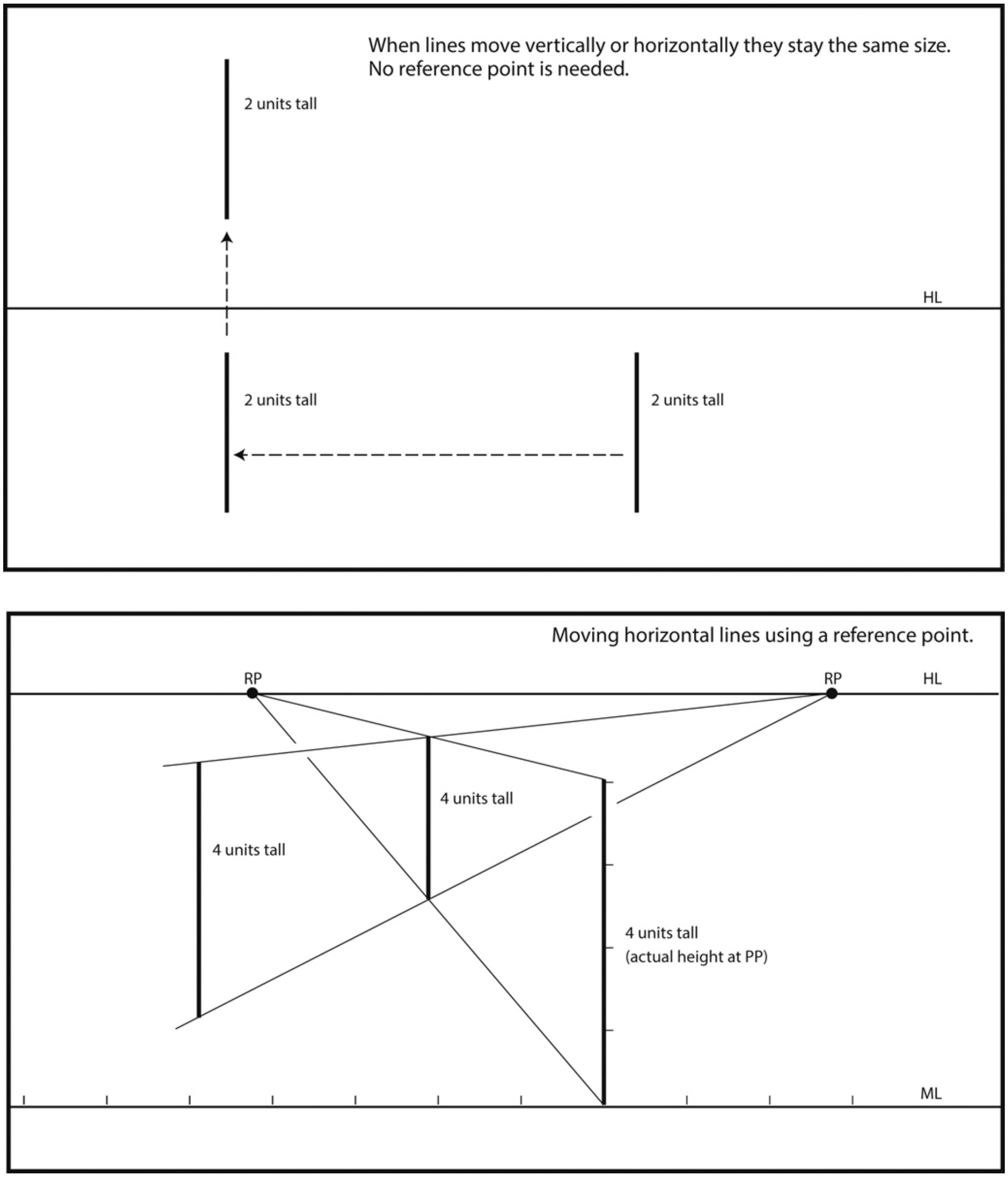


Figure 4.2 Use reference points to move vertical lines. The lines being moved must be parallel with each other and on the same horizontal plane.

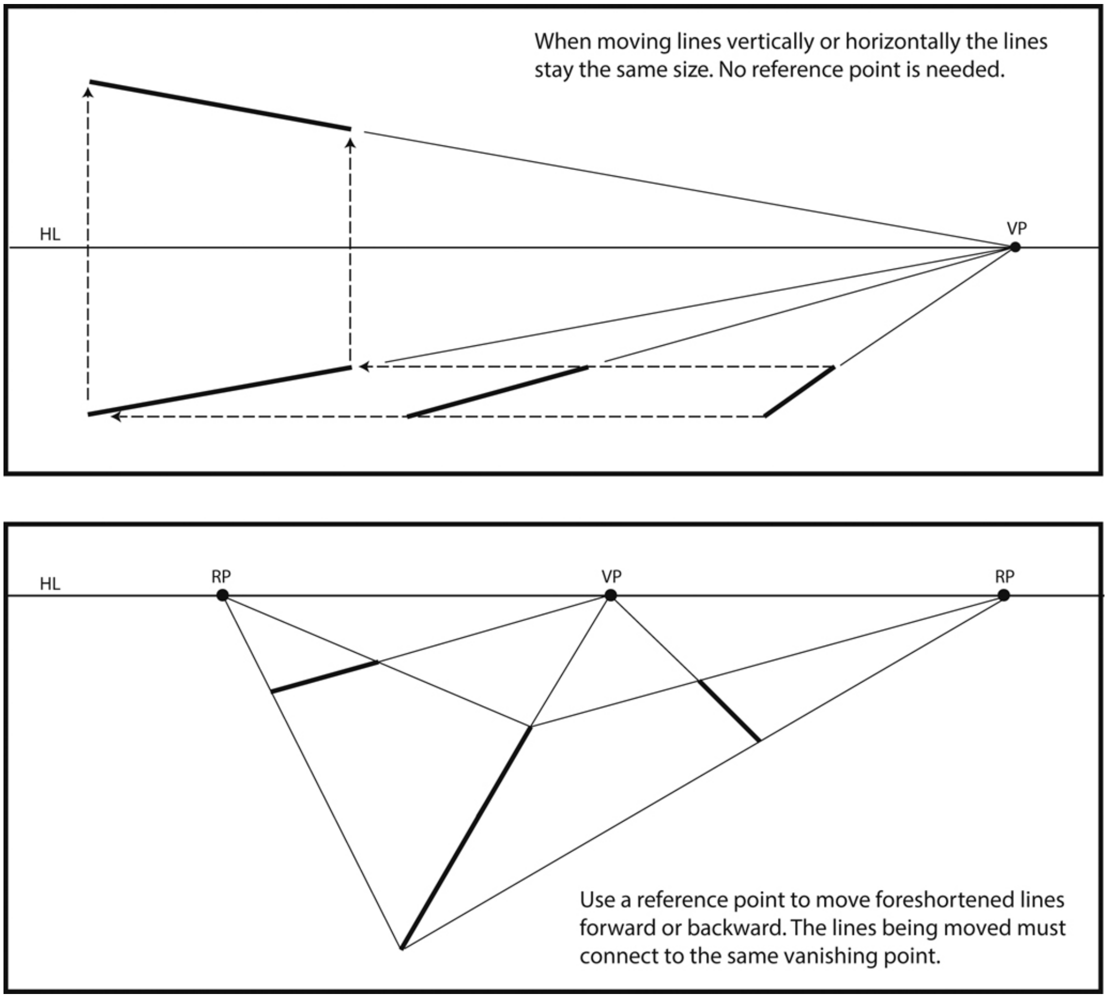


Figure 4.3 Use reference points to move foreshortened lines. The lines moved must remain parallel with each other (connect to the same vanishing point), and be on the same horizontal plane.

5
Two-Point Perspective

When an object is viewed at an angle, when only the vertical dimension is parallel with the picture plane, and when both width and depth are foreshortened, the object is in two-point perspective. Predictably, two-point perspective has two vanishing points: a left vanishing point (LVP) and a right vanishing point (RVP). The location of the vanishing point depends on the angle of the object being drawn.

Two-Point Perspective Diagram

Vanishing Points

Most objects have 90° corners. To create 90° corners in perspective, the left and right vanishing points must be 90° apart (subsequent chapters will discuss how to create other angles).

Using a triangle, place a true 90° angle at the station point, then project that angle to the horizon line. The resulting points (the right and left vanishing points) will draw 90° angles in perspective. Any 90° angle projected from the station point creates two vanishing points that draw that same angle in perspective (Figure 5.1).

Understanding perspective is to understand angles. The station point is a powerful tool. Angles placed at the station point mirror the perspective angles in the drawing. To draw an object at a specific angle to the picture plane, draw that angle from the station point to the horizon line; the vanishing points created will draw those same angles in perspective (Figure

5.2). Multiple objects at various angles can be created using this technique, as discussed further in Chapter 7 (Figure 7.2).

Measuring Points

Each vanishing point has a dedicated measuring point. The left vanishing point has a left measuring point (LMP) and the right vanishing point has a right measuring point (RMP). The left measuring point measures lines connecting to the left vanishing point. The right measuring point measures lines connecting to the right vanishing point.

The placement of measuring points is specific. The distance from the measuring point to the vanishing point is the same as the distance from the station point to the vanishing point. There are two ways to find the correct placement of the measuring point, using a compass or a ruler. When using a compass, put the stationary arm of the compass on the vanishing point and draw an arch from the station point to the horizon line. The same result can be achieved using a ruler (Figure 5.3).

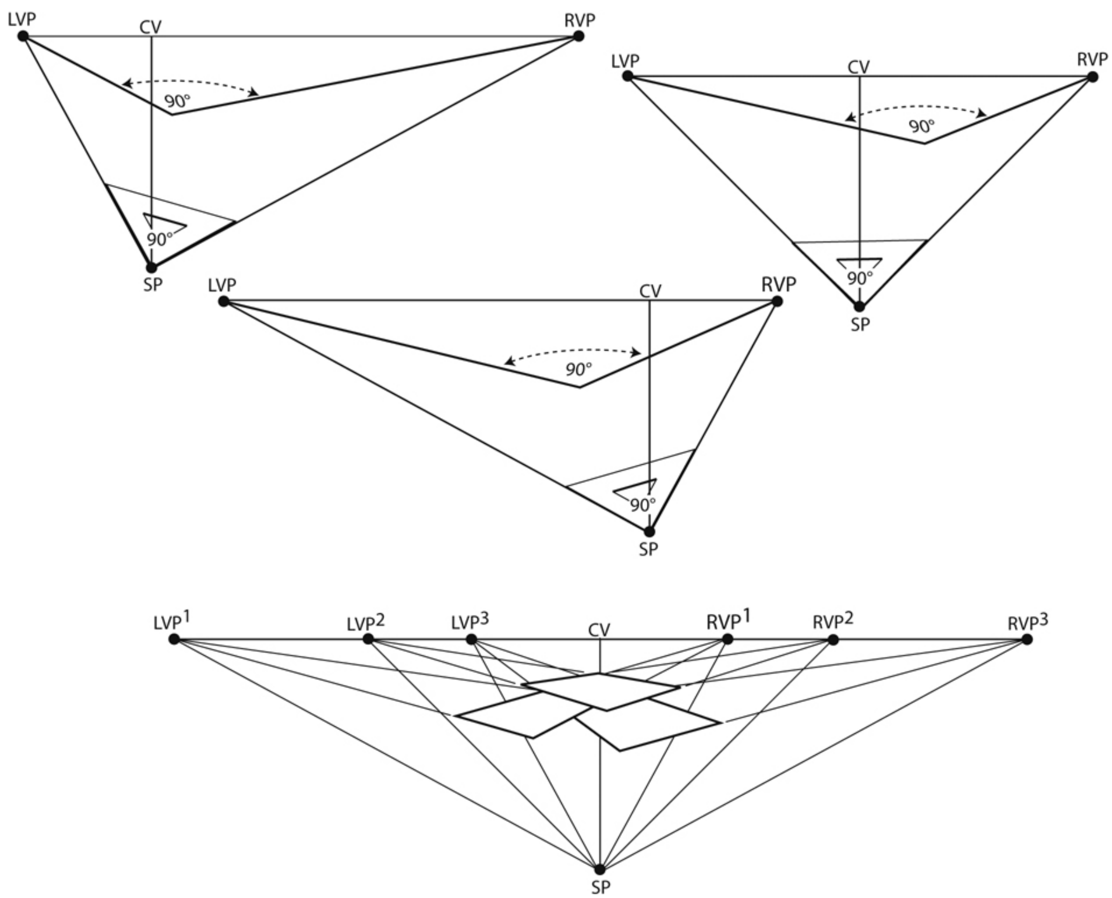


Figure 5.1 True angles are found at the station point. A true 90° angle drawn from the station point creates left and right vanishing points that draw 90° angles in perspective. There can be as many pairs of vanishing points as there are objects.

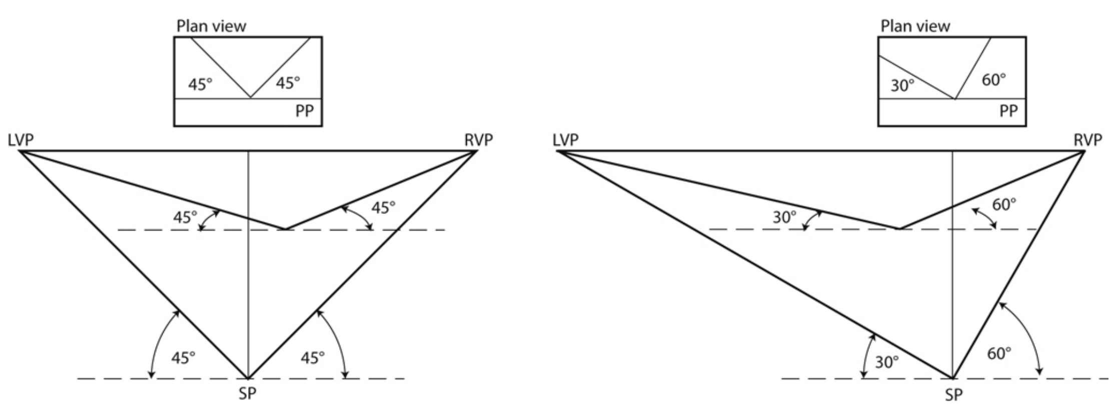


Figure 5.2 Use the station point to draw objects at specific angles. The left example shows the object turned 45° to the picture plane. The right example shows the object turned 30°/60° to the picture plane. Any angle can be created by projecting it from the station point to the horizon line.

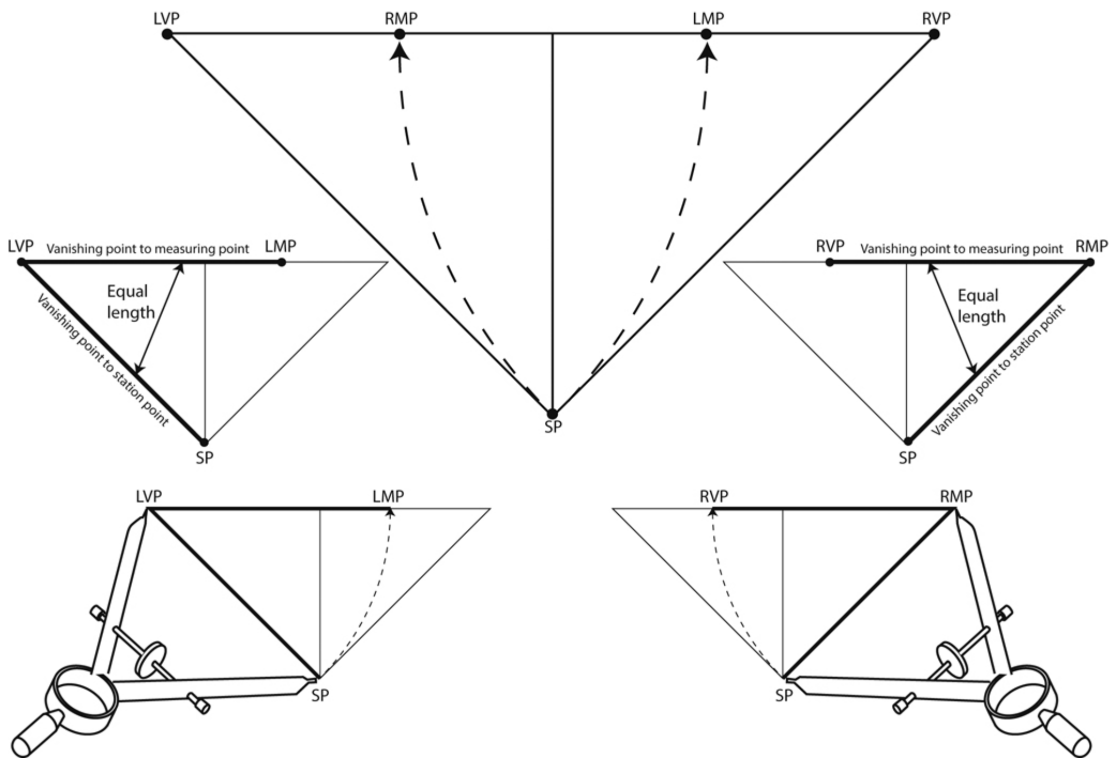
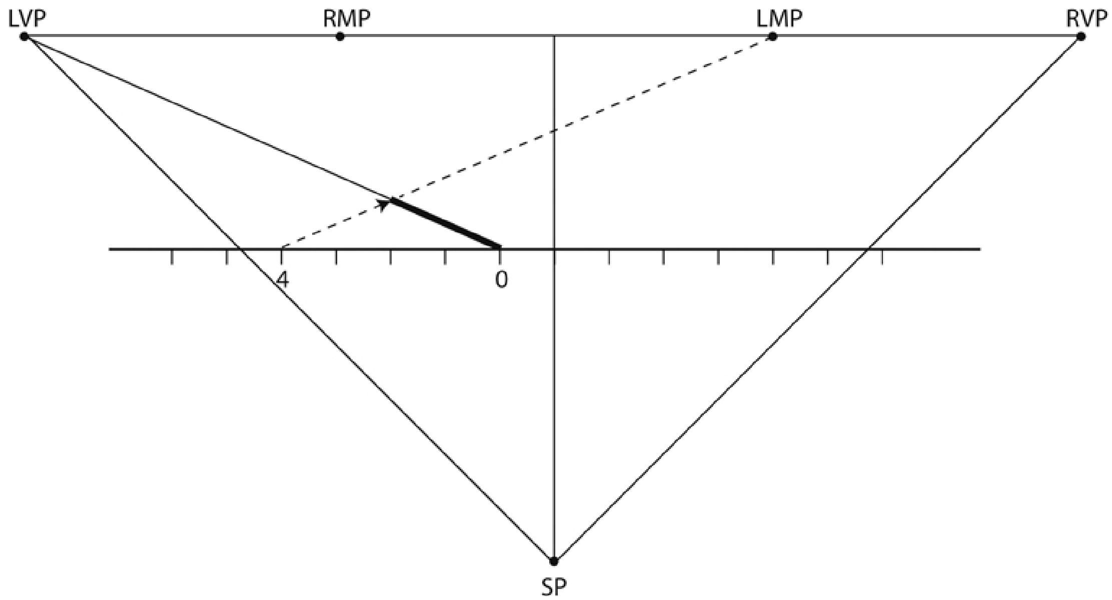


Figure 5.3 **Plot the location of the left and right measuring points using a compass or a ruler.**

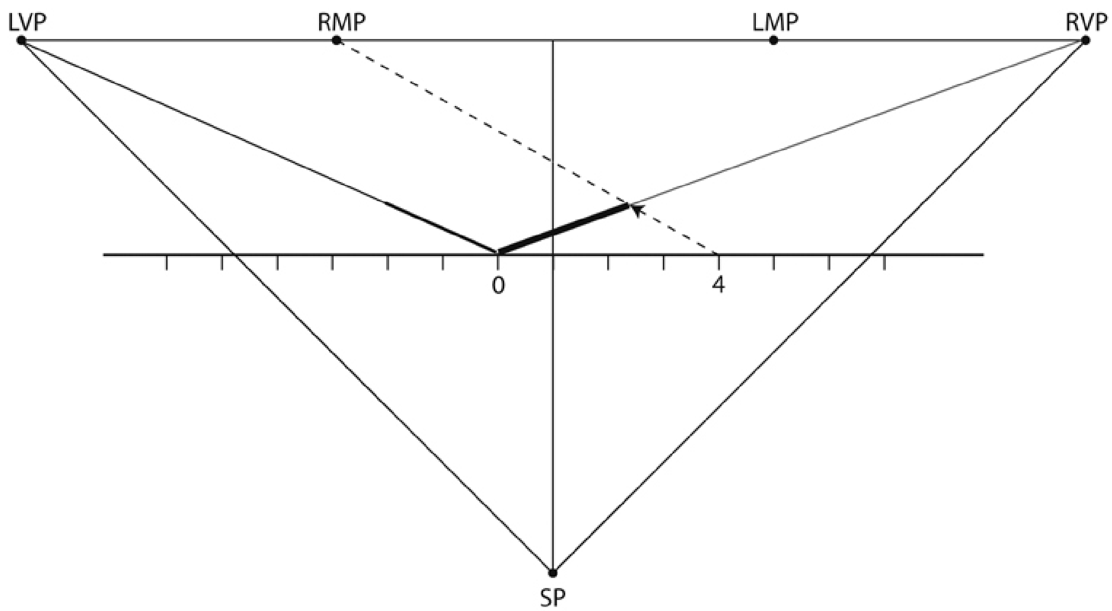
Measuring Depth

Measuring in two-point perspective follows the same procedures as one-point. It is, however, a little more complicated, as there are now two measuring points. The more points there are on the horizon line, the harder it is to keep track of them. Color-coding the perspective layout keeps mistakes to a minimum. For example, label lines from the right vanishing point and right measuring point in one color, and label lines from the left vanishing point and left measuring point in another.

Use the left measuring point to measure lines connecting to the left vanishing point (Figure 5.4). Use the right measuring point to measure lines connecting to the right vanishing point (Figure 5.5).



[Figure 5.4](#) Use the left measuring point to measure lines that connect to the left vanishing point.



[Figure 5.5](#) Use the right measuring point to measure lines that connect to the right vanishing point.

Completing the Shape

After measuring the left and right side, complete the shape by connecting the corners to vanishing points. Take care to connect the lines to *vanishing points*. Do not connect lines to measuring points. Measuring points are only for measuring, they are not part of the physical object. Lines connecting to measuring points are phantom lines; they are invisible. This mistake can usually be spotted quickly, as the shape's corners will not look square (Figure 5.6).

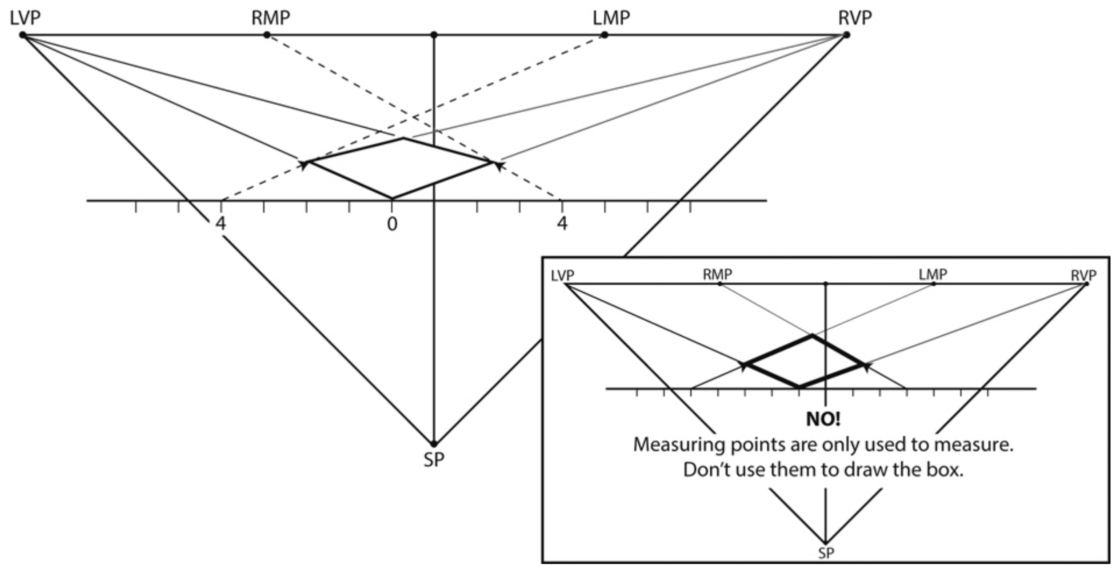


Figure 5.6 The back of the box connects to vanishing points. A common mistake is to use a measuring point where a vanishing point should be used.

Measuring Height

Measuring vertical dimensions in two-point perspective is no different than one-point perspective. Vertical lines touching the picture plane are actual size. Project the height to the desired location using a vanishing point or a reference point (Figure 5.7).

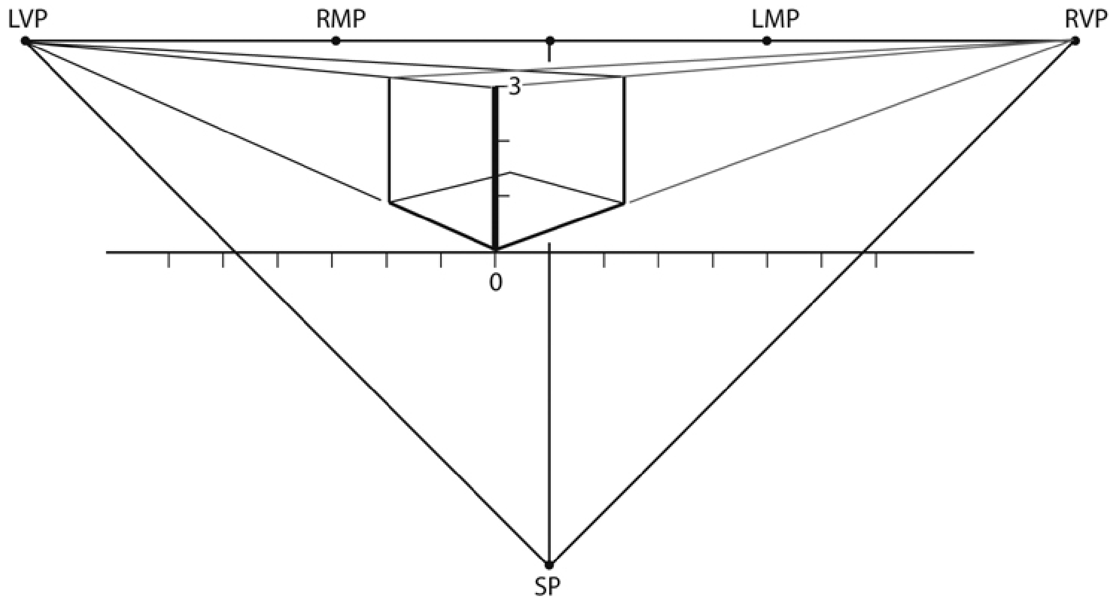


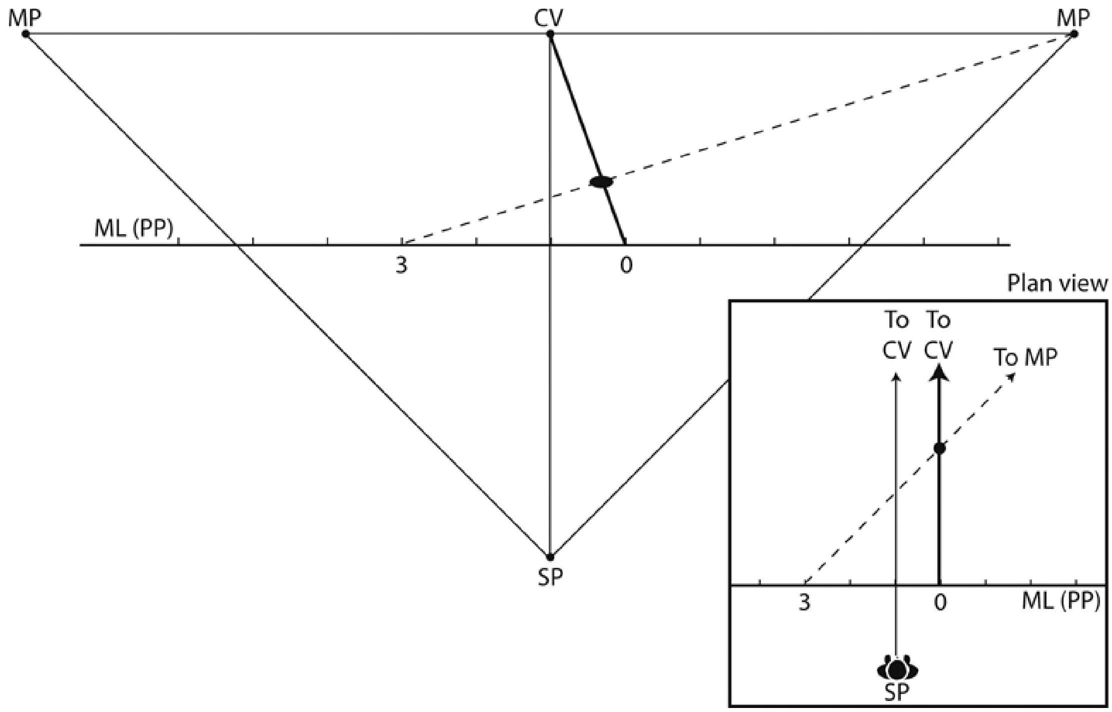
Figure 5.7 **Measure vertical dimensions at the picture plane**.

More Two-Point Measuring

In the previous example, the front of the box was touching the picture plane. This creates a convenient situation for measuring. The zero point was where the box contacts the measuring line. To measure depth, it was a case of simply counting to the left and right of zero. But what if the box does not touch the measuring line? How is this measured, and where is the zero point? Where does the counting begin? Before discussing the solutions, draw a sample square that does not touch the picture plane.

Location

Place the square 1 unit to the right of the center of vision and 3 units behind the picture plane. Use one-point perspective to find this location (Figure 5.8). Refer to Chapter 3, Figures 3.9–3.10 for additional information about measuring in one-point perspective.



[Figure 5.8](#) **The front corner of the box (represented by the dot) is 1 unit to the right of the center of vision and 3 units behind the measuring line.**

Measuring Behind the Picture Plane

There are two ways to measure shapes not touching the measuring line: 1) project the object to the measuring line, or 2) move the measuring line to the object.

Method 1: Project the Object to the Measuring Line

To measure an object, a point is needed to begin measurements—a zero point. To find the zero point, project the line being measured to the measuring line. How it is projected is critical; there is specific geometry to adhere to. The appropriate measuring point must be used to find the length of a foreshortened line. To find the zero point for lines drawn to the right or left vanishing point, use the right or left measuring point respectively. Use the right measuring point to measure lines connecting to the right vanishing point (Figure 5.9), and use the left measuring point to measure lines connecting to the left vanishing point (Figure 5.10).

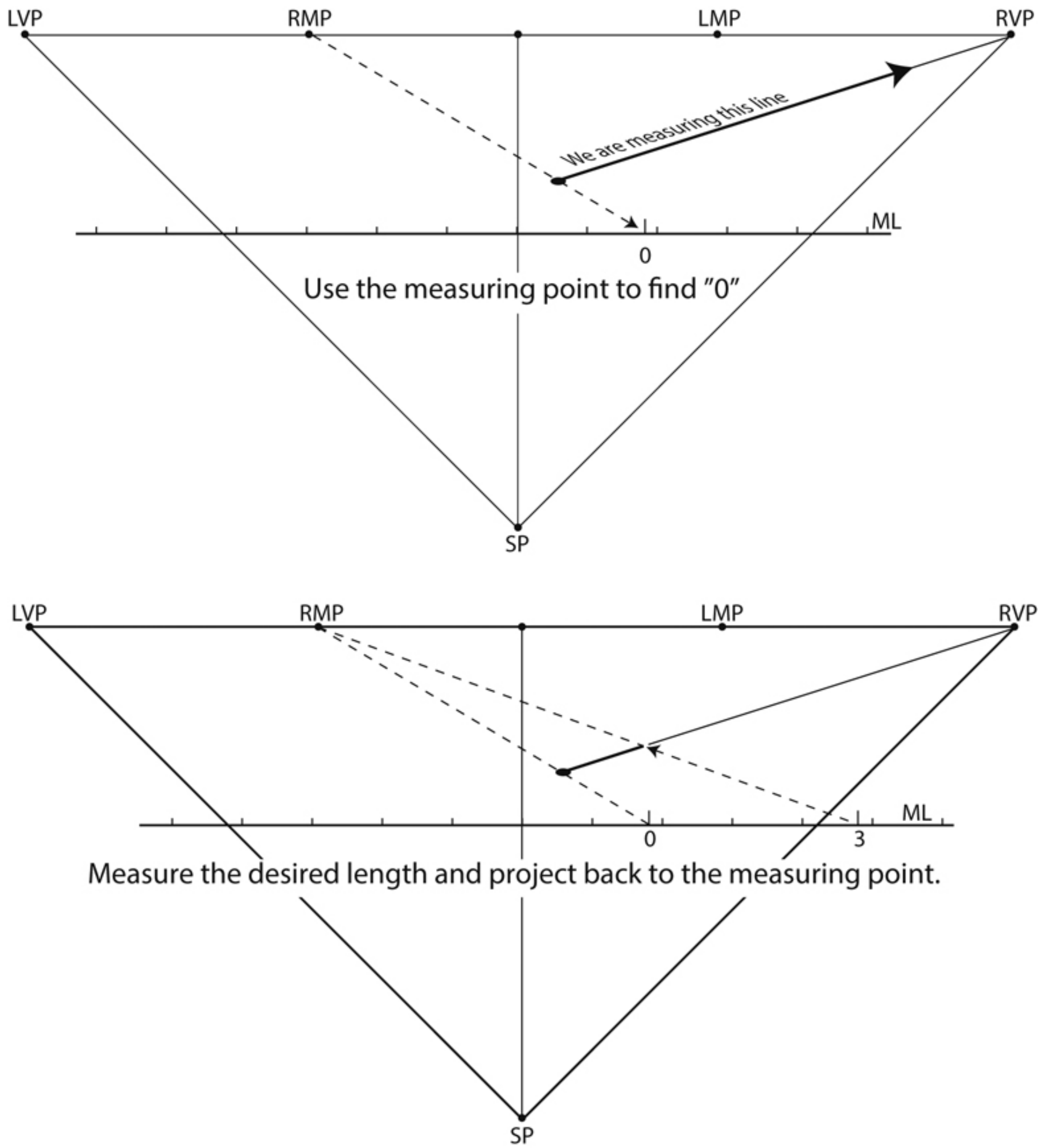


Figure 5.9 The right measuring point is used to measure lines connecting to the right vanishing point.

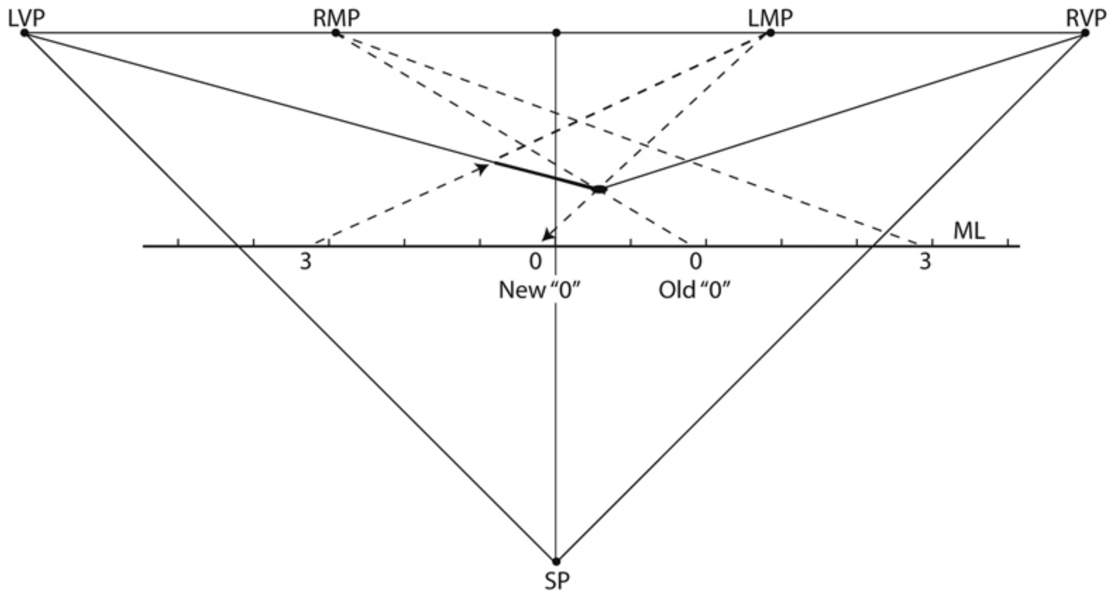


Figure 5.10 **Measure the left side by finding a new zero point. Measure the desired distance and project back to the left measuring point.**

Project the beginning of the line to the picture plane. This is the zero point. Count the desired distance along the measuring line, and then project back to the same measuring point.

Connect the lines to vanishing points to complete the square (Figure 5.11).

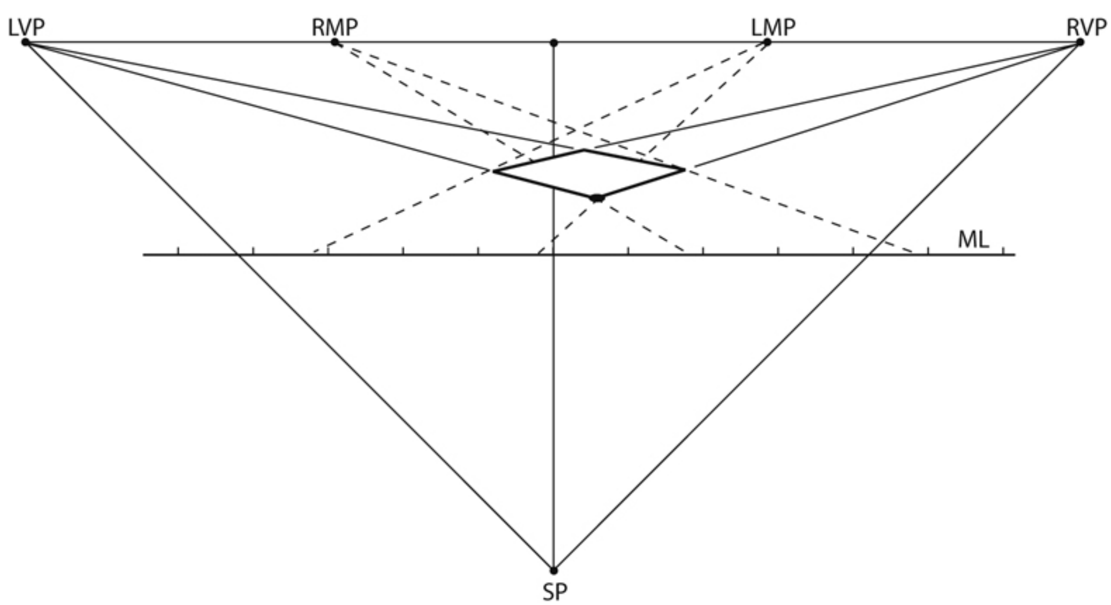


Figure 5.11 **Connect to vanishing points to complete the square.**

Method 2: Move the Measuring Line to the Object

An alternative method is to move the measuring line. Position it so that it touches the object being measured. The measuring line must be moved in perspective, using a reference point (Figure 5.12). Once the new measuring line is in place, follow the procedures outlined in Figures 5.4–5.6.

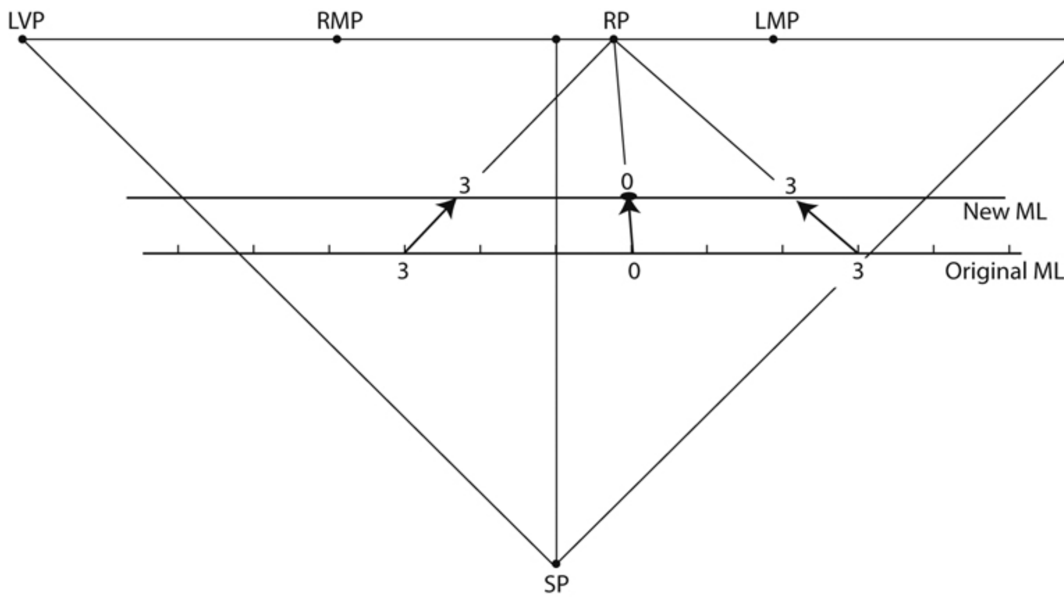


Figure 5.12 Use a reference point to move the measuring line.

Distant Objects

If the object being measured is located a great distance from the picture plane, using Method 1 can be inconvenient. In these situations, Method 1 would require a very long ruler. The second method, moving the measuring line back and creating smaller units closer to the object being measured, is the preferred method (Figures 5.13–5.15).

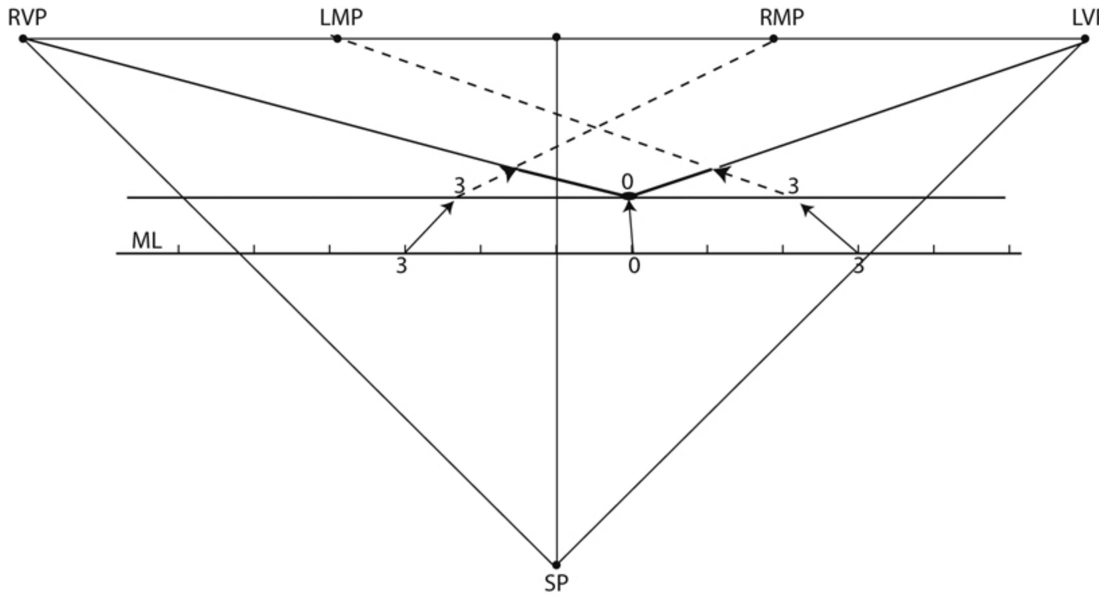


Figure 5.13 **With the measuring line relocated, measure the respective lines using the proper measuring points.**

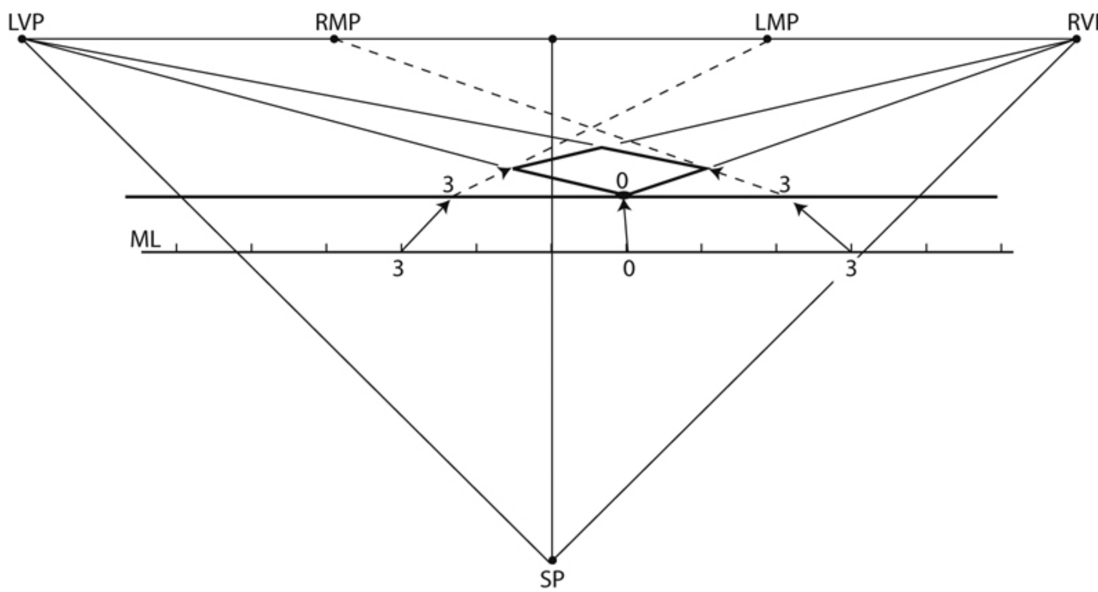
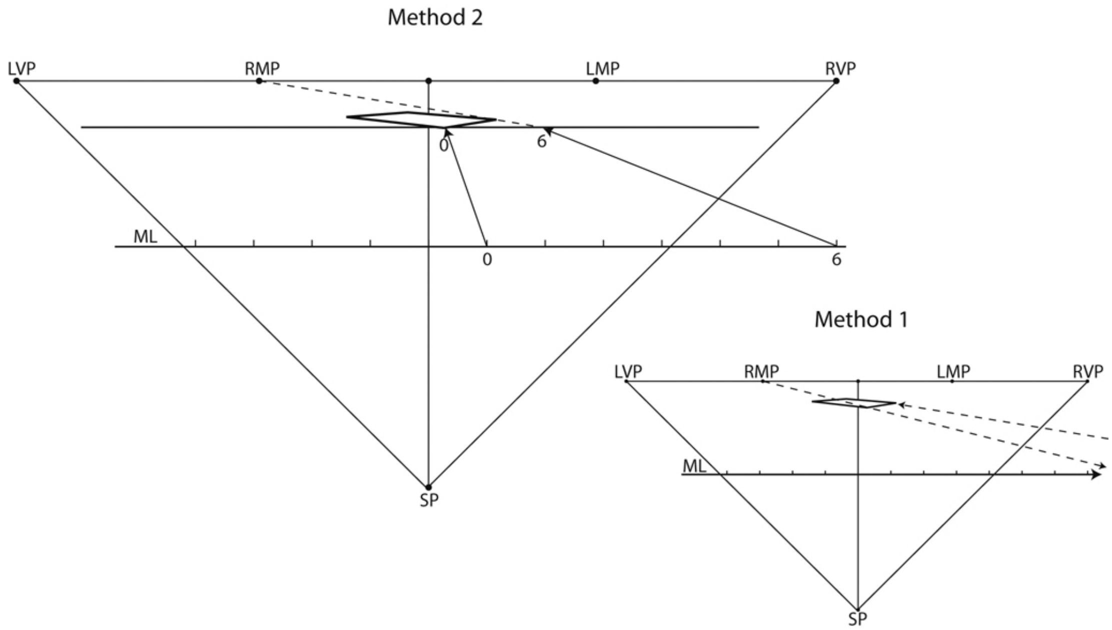


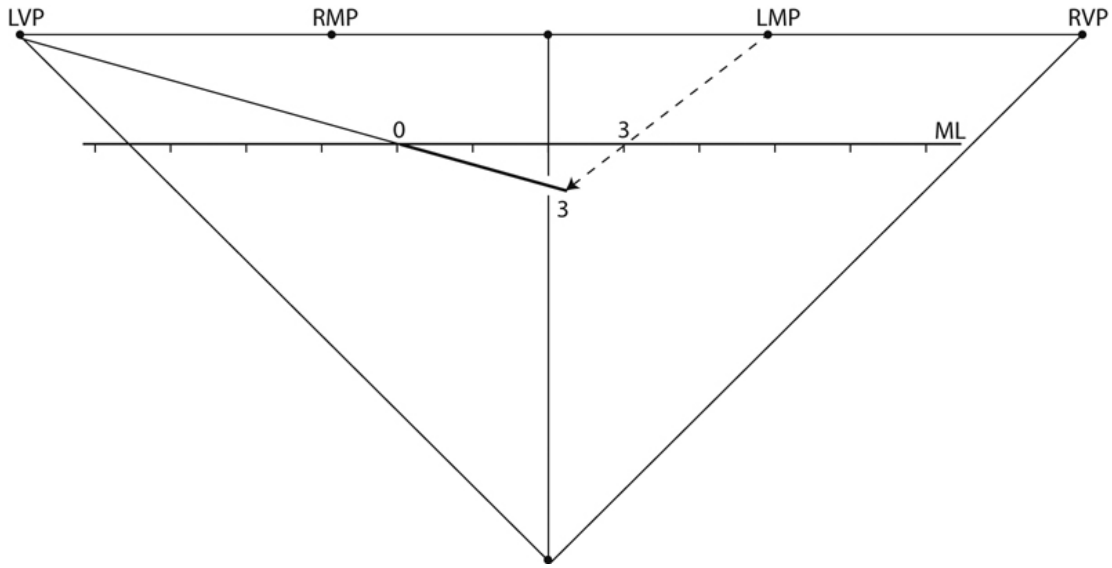
Figure 5.14 Connect lines to vanishing points to create the back of the square.



[Figure 5.15](#) **Moving the measuring line (Method 2) keeps measurements from exceeding the bounds of the paper (as compared to Method 1).**

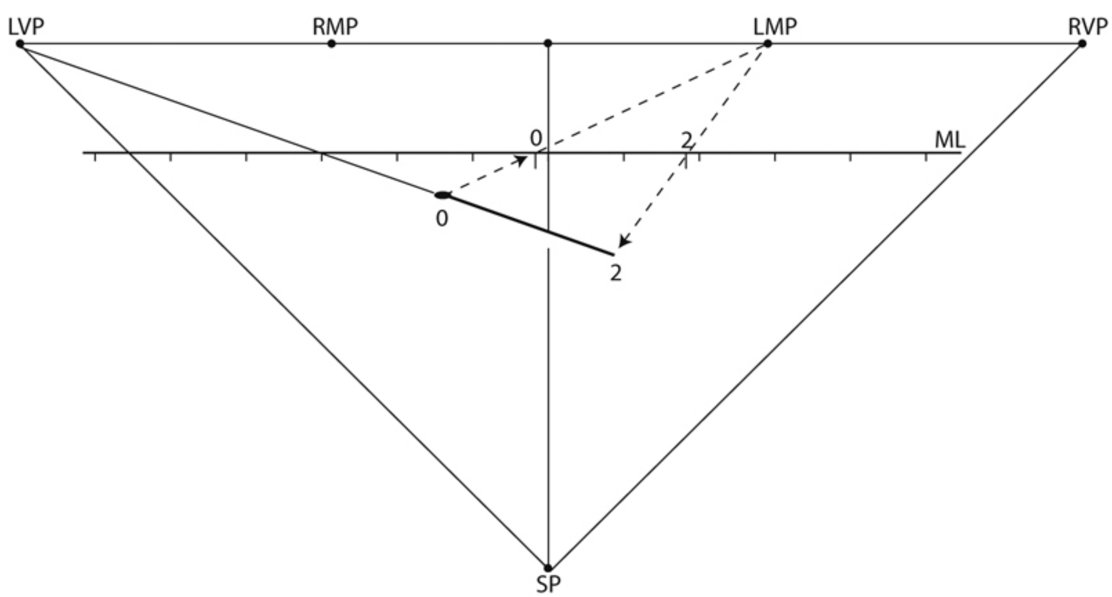
Measuring in Front of the Picture Plane

Measuring lines in front of the picture plane follows the same basic procedures as measuring lines behind the picture plane. The difference is that lines are projected forward from the measuring line instead of backward (Figure 5.16).



[Figure 5.16](#) **Measuring in front of the picture plane.**

If the line being measured does not touch the picture plane, use the measuring point to project the measurements forward ([Figure 5.17](#)).



[Figure 5.17](#) **Use the measuring point to project the line being measured to the measuring line.**

If a line straddles the picture plane it requires a combination of projecting measurements backward and forward ([Figure 5.18](#)).

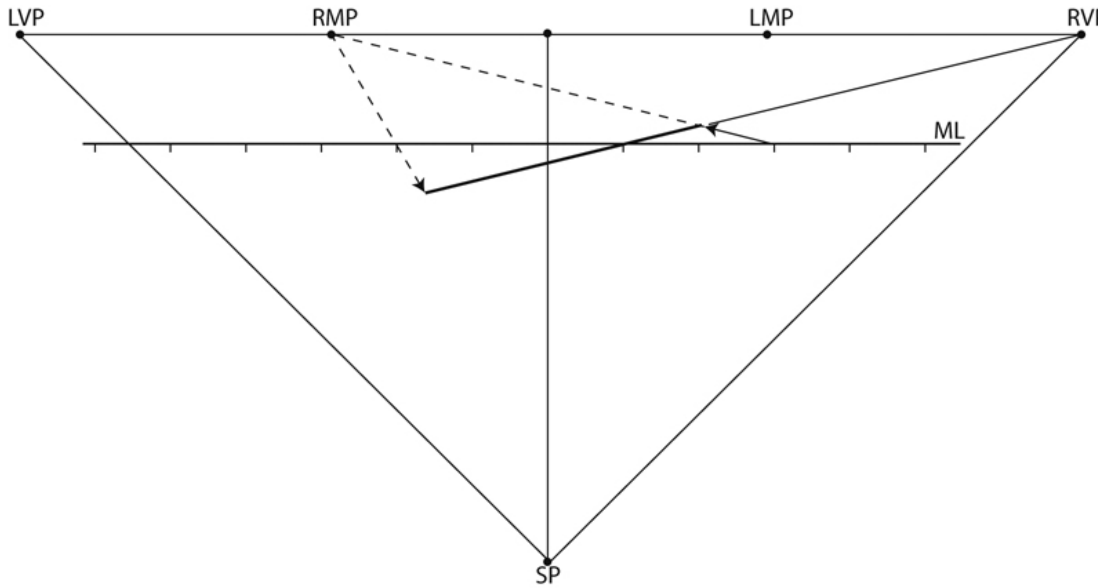
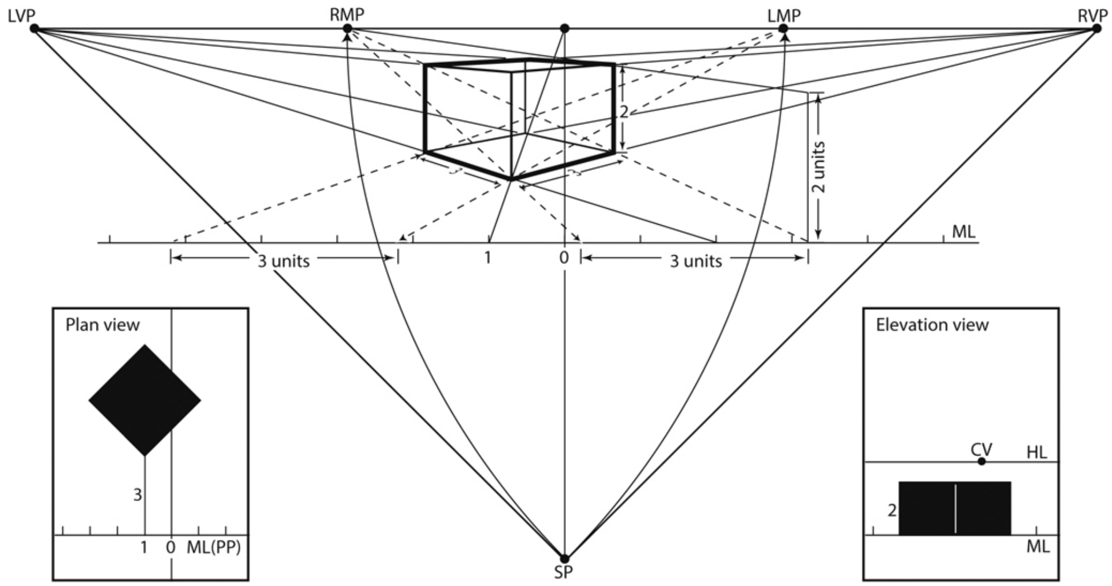


Figure 5.18 **This line is 3 units in front and 2 units behind the picture plane**.

Drawing A Two-Point Box

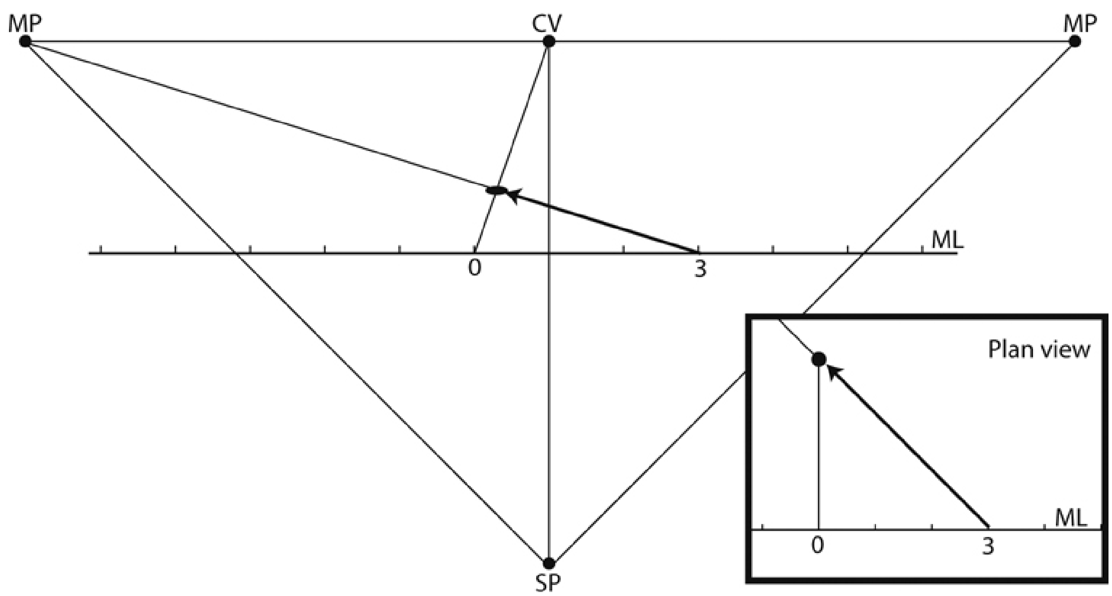
Now that the basics of two-point perspective have been covered, it is time to apply this information to a drawing. This example is a box, 2 units tall, 3 units wide, and 3 units deep. It is 1 unit to the left of the center of vision and 3 units behind the picture plane (Figure 5.19 shows how the finished layout will look). To approach this problem, do one step at a time.



[Figure 5.19](#) **The completed box with construction lines intact.**

Location

First, using one-point perspective, measure 1 unit to the left of the center of vision, then 3 units behind the picture plane ([Figure 5.20](#)).



[Figure 5.20](#) **The location of the box's front corner is 1 unit to the left and 3 units behind the measuring line.**

Left Side

First find the zero point. Using the left measuring point, project the front corner of the box (represented by the dot) to the measuring line (Figure 5.21). Measure 3 units to the left, and then project back to the left measuring point (Figure 5.22).

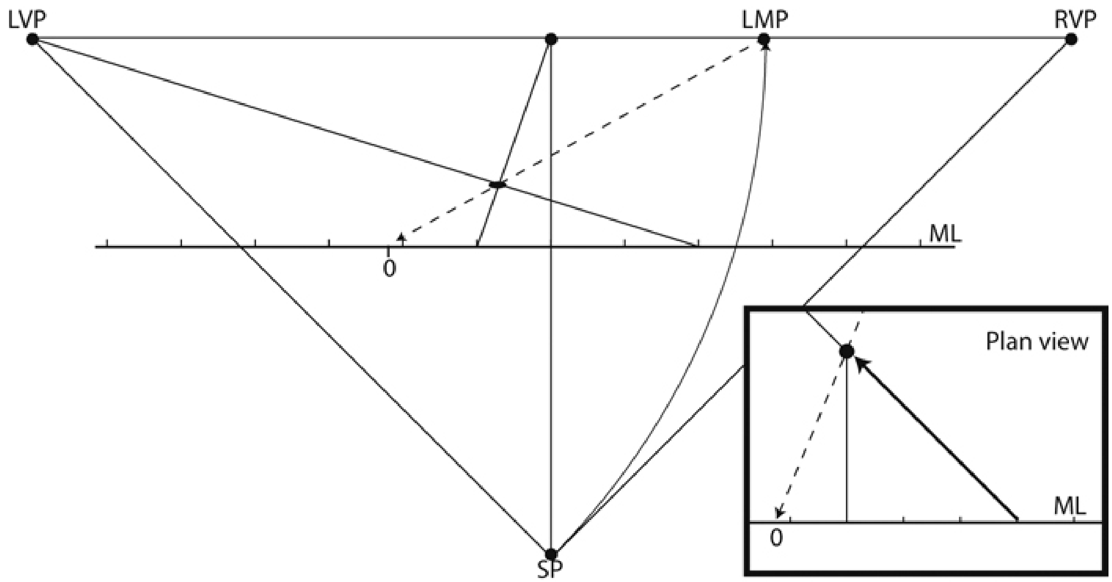


Figure 5.21 Finding the zero point.

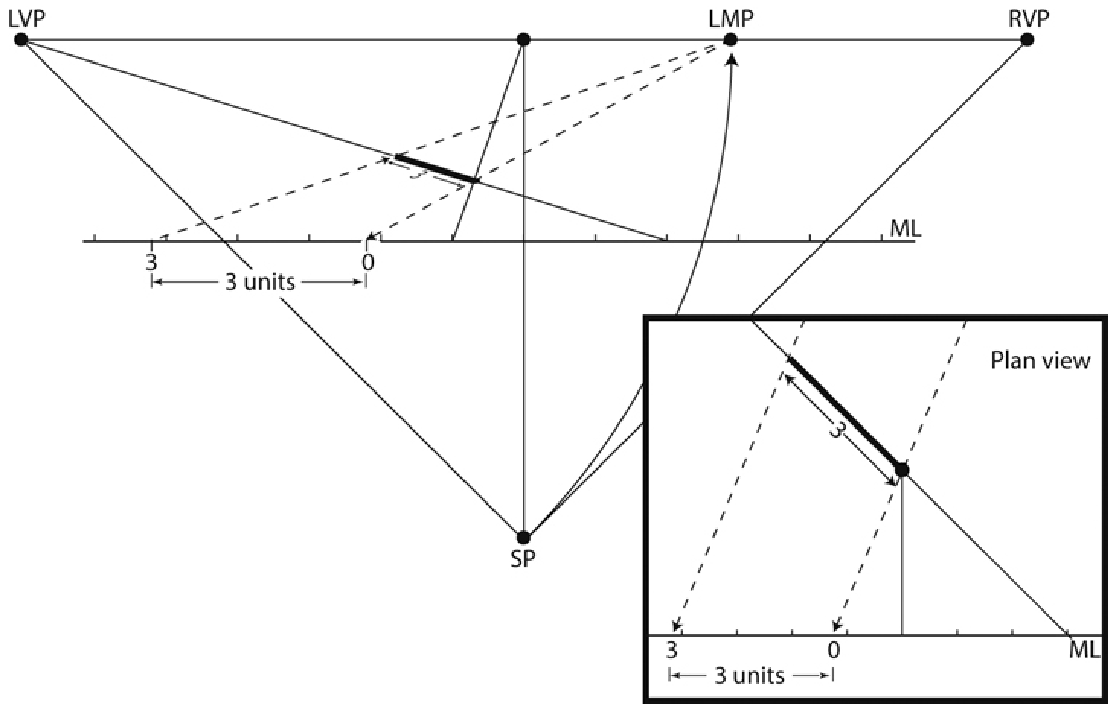
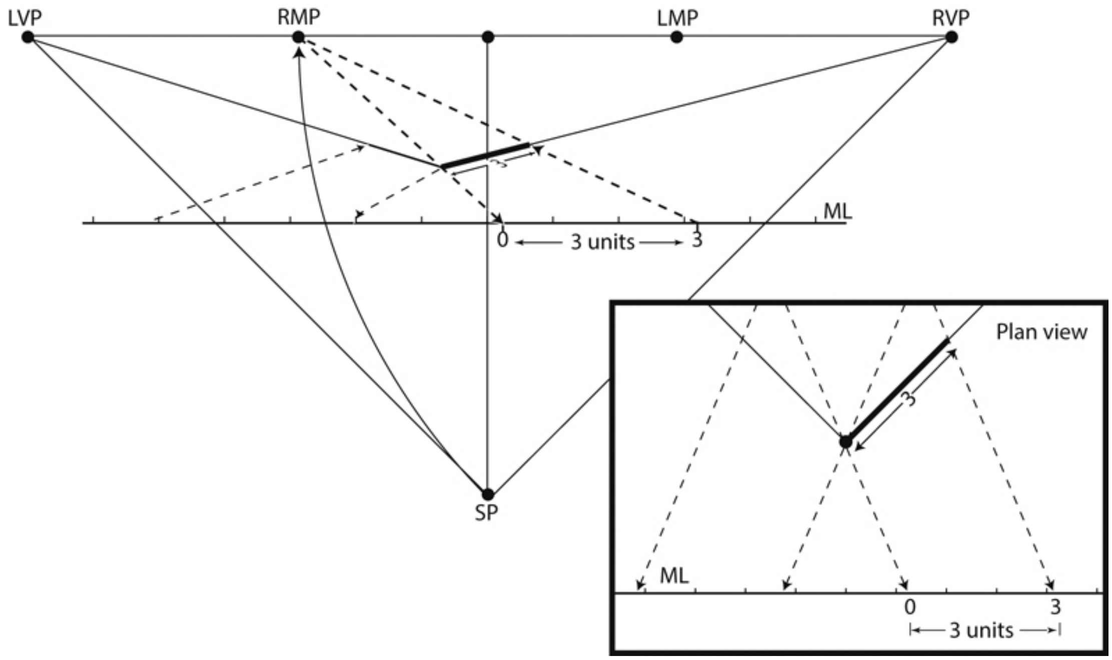


Figure 5.22 **Measuring the left side of the box.**

Right Side

Use the same procedure to measure the right side. But first, find a new zero using the right measuring point (Figure 5.23).



[Figure 5.23](#) **Measuring the right side of the box**.

Height

Height is not foreshortened. A reference point can be used to establish the height. A reference point can be any dot on the horizon line ([Figure 5.24](#) uses the right measuring point).

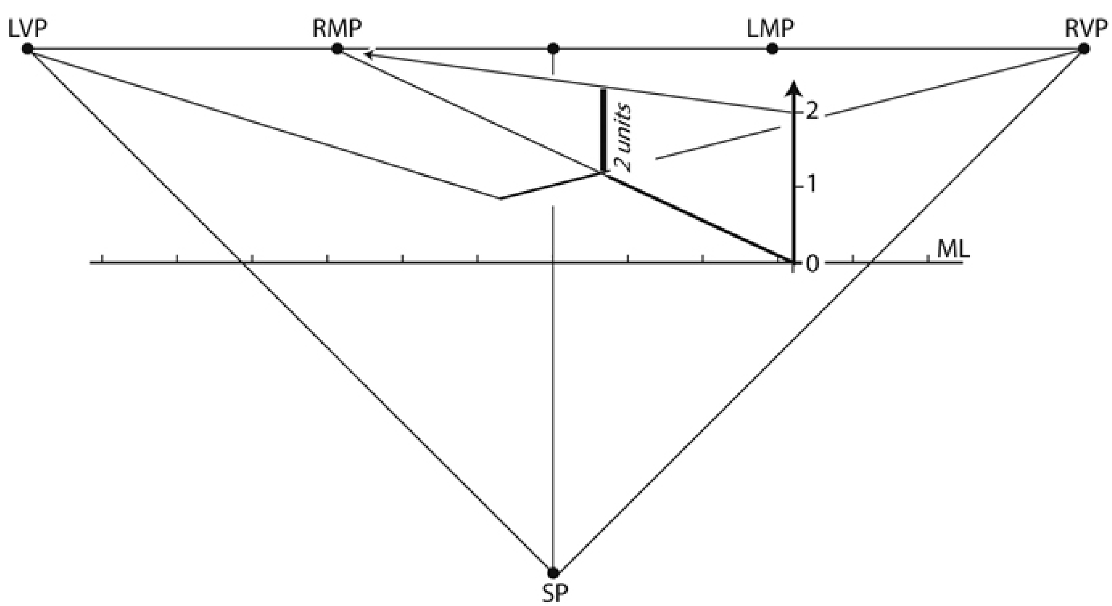


Figure 5.24 **Using the right measuring point as a reference point to measure height**.

Completing the Box

To complete the box, connect the horizontal lines to the vanishing points, and draw the vertical lines parallel with the picture plane (Figure 5.25).

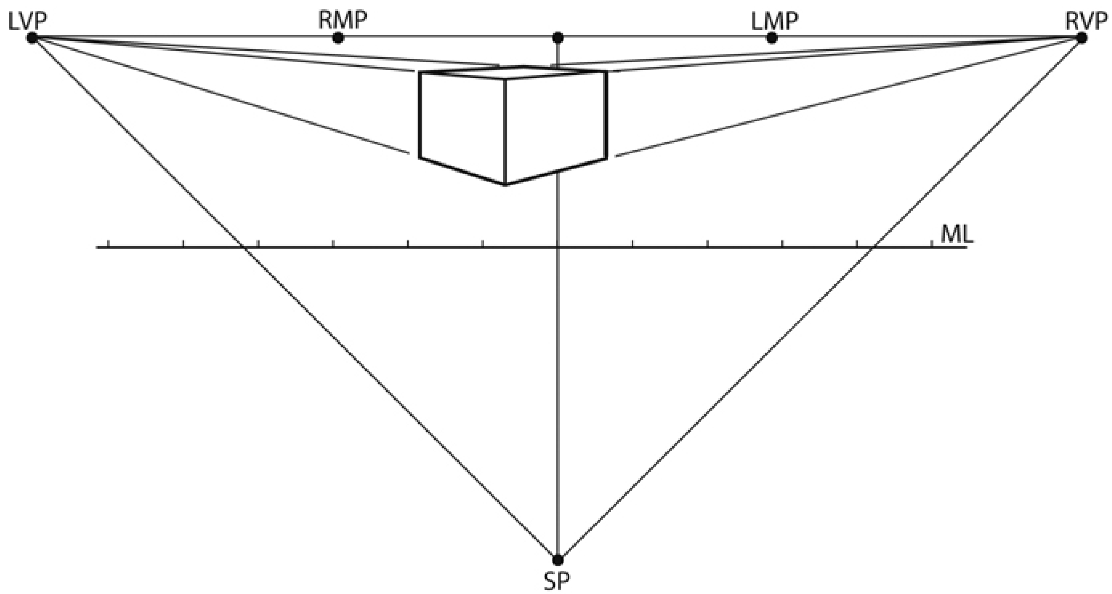


Figure 5.25 **Connect lines to vanishing points to complete the box**.

Two-Point Perspective Grid

As with one-point, a two-point perspective grid, once established, is an easy way to measure shapes and determine the proper direction of lines. But once confidence with perspective techniques has been achieved, grids become unnecessary. They take undue time to create and they make drawing objects at angles other than the grid angles awkward. Despite its drawbacks, the grid's simplicity is enticing, and there are situations where establishing a grid can be the best solution to a problem.

Diagram

To draw a grid, first construct the diagram (establish the HL, SP, VPs, MPs, and ML). Decide on the placement of the grid (the front corner is typically placed on the measuring line, aligned with the center of vision). Draw lines to the left and right vanishing points. Then, using the appropriate measuring point, divide these lines into equal increments. The number of increments made depends on the desired size of the grid (Figure 5.26).

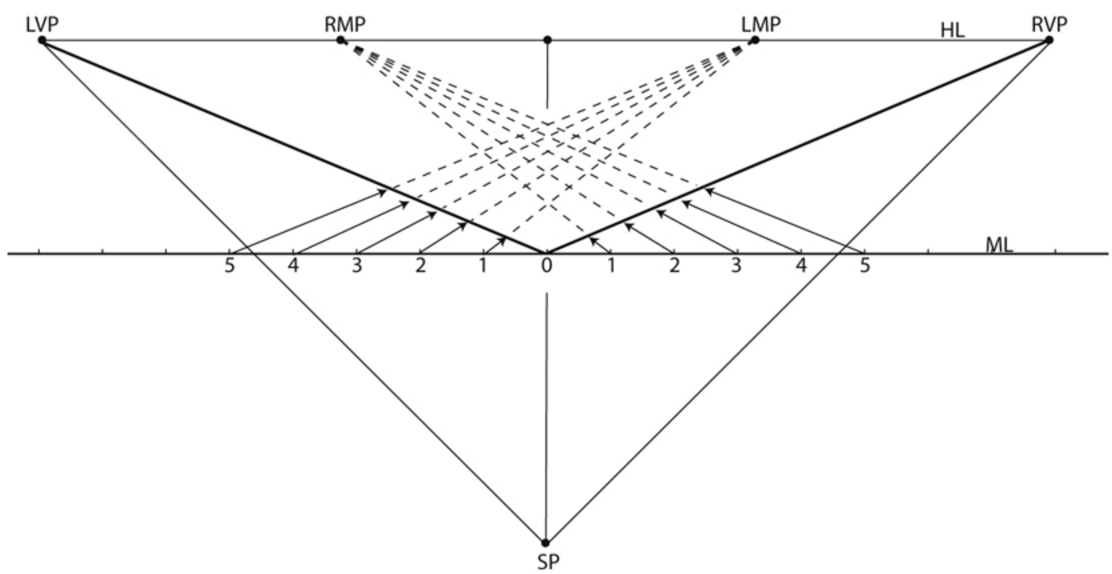
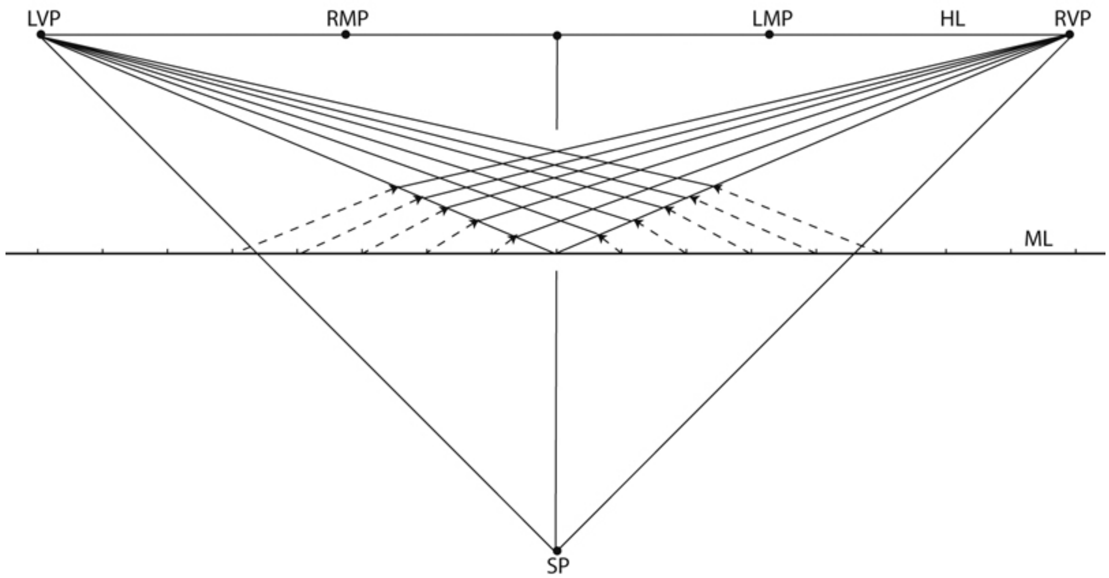


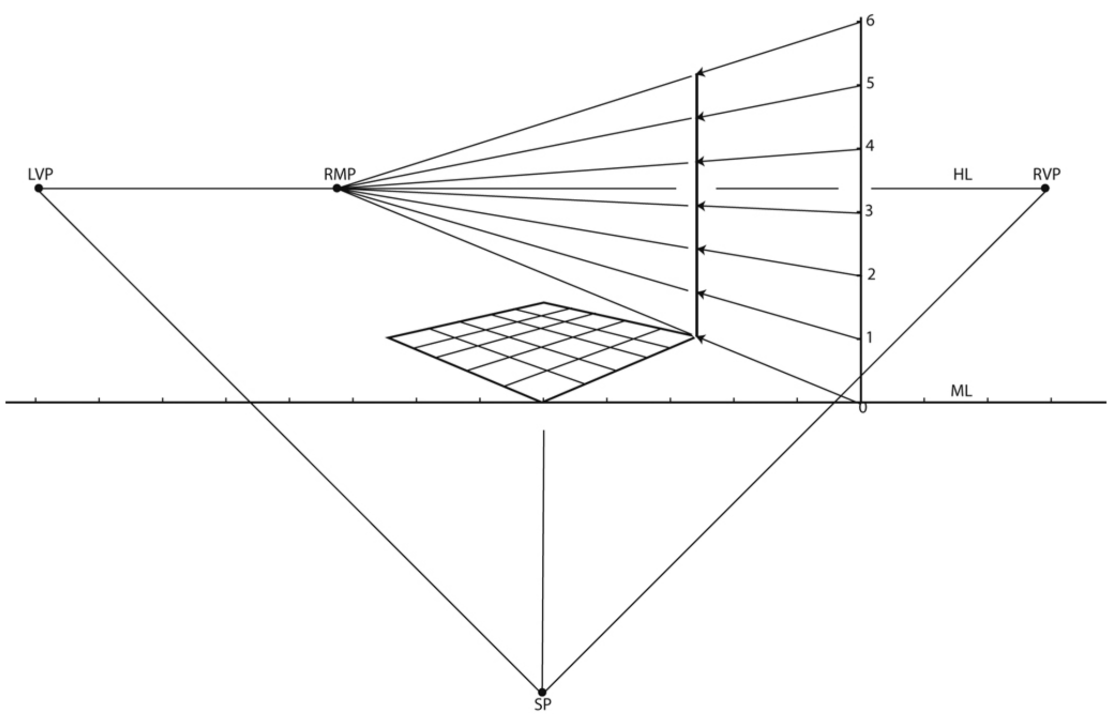
Figure 5.26 Measure evenly spaced segments along the lines connecting to the vanish points.

Horizontal Grid

After measuring the grid segments, connect these measurements to vanishing points (Figure 5.27).



[Figure 5.27](#) **Connect the measurements to vanishing points to create a grid.**



[Figure 5.28](#) **Create a vertical grid by transferring dimensions from the picture plane. The right measuring point is functioning as a reference point.**

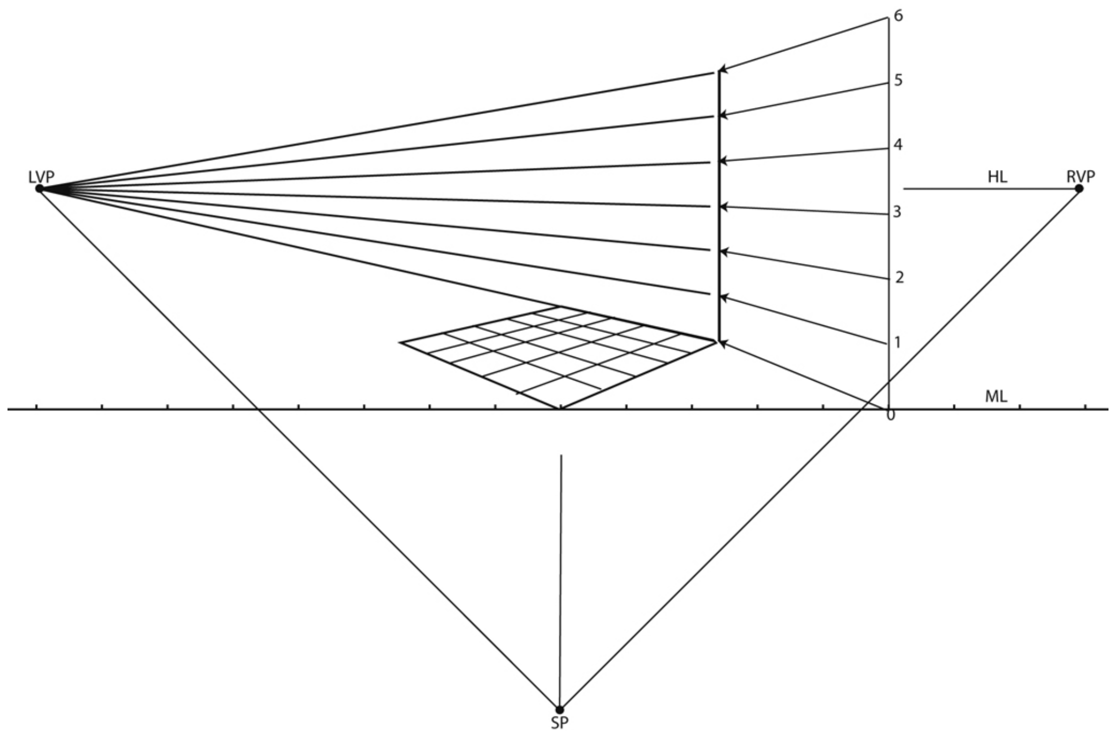


Figure 5.29 Connect vertical segments to the vanishing point.

Vertical Grid

Use a reference point to project the vertical dimensions from the picture plane (Figure 5.28). Project each segment to the vanishing point (Figure 5.29). Extend the horizontal grid lines vertically to finalize the grid (Figure 5.30). Create a grid on the left wall if needed.

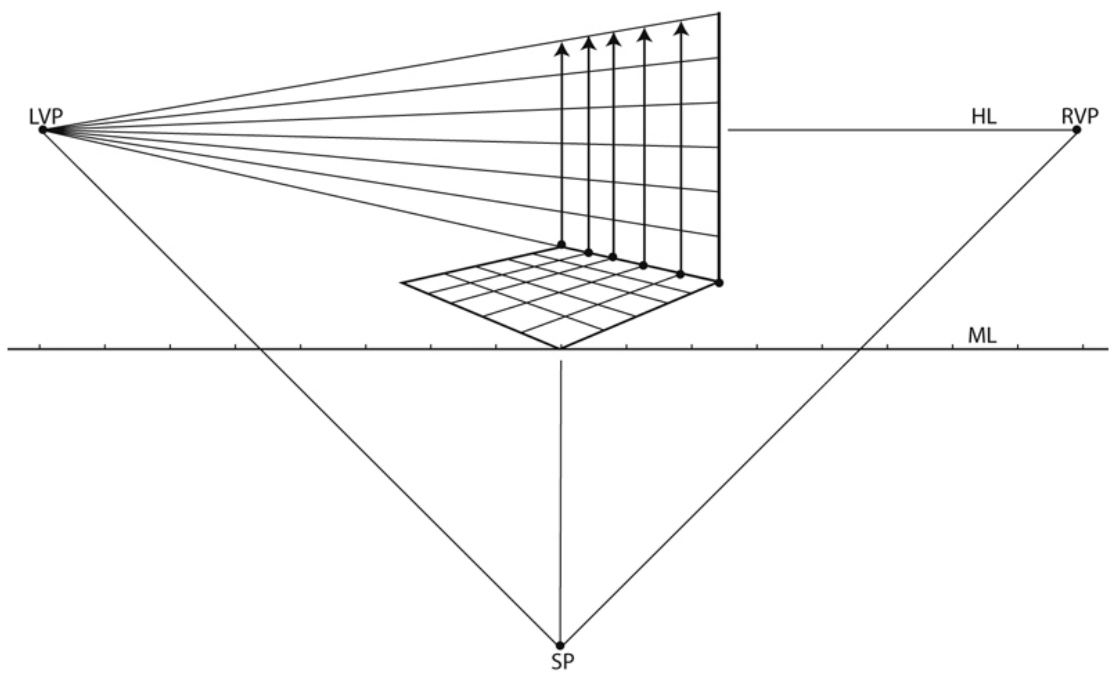
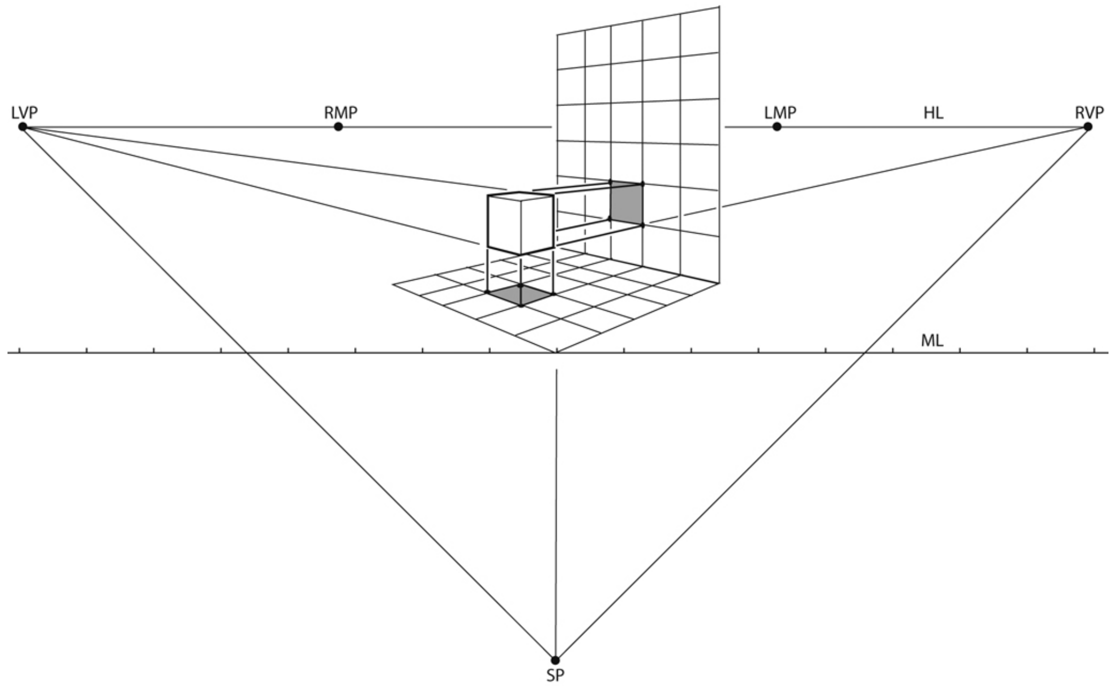


Figure 5.30 **Project vertical lines from the horizontal grid to complete the squares.**

Completing the Grid

Once the grid is complete, it is typically used as an underlay to guide drawing. Each square represents 1 unit. Using this as a guide, a shape of any size can be made by counting squares and conforming the lines to the grid.

Use the horizontal grid to determine the width, depth, and placement of the shape being drawn. Use the vertical grid to determine height (Figure 5.31).



[Figure 5.31](#) Use the grid to measure objects. This cube is 1 unit from the front left wall, 2 units from the front right wall, and 1 unit above the ground.

6
Measuring Point Geometry

Being well-versed in geometry is not required in order to draw in perspective. But understanding why these methods are used helps remove the mystery and confusion that surrounds the process. To understand why measuring points work, isosceles triangles need to be understood. Isosceles triangles have two sides (legs) that are equal in length. When creating a measuring point, an isosceles triangle is also created (Figure 6.1).

Connecting the vanishing point, station point, and measuring point forms a true isosceles triangle (Figure 6.1, right).

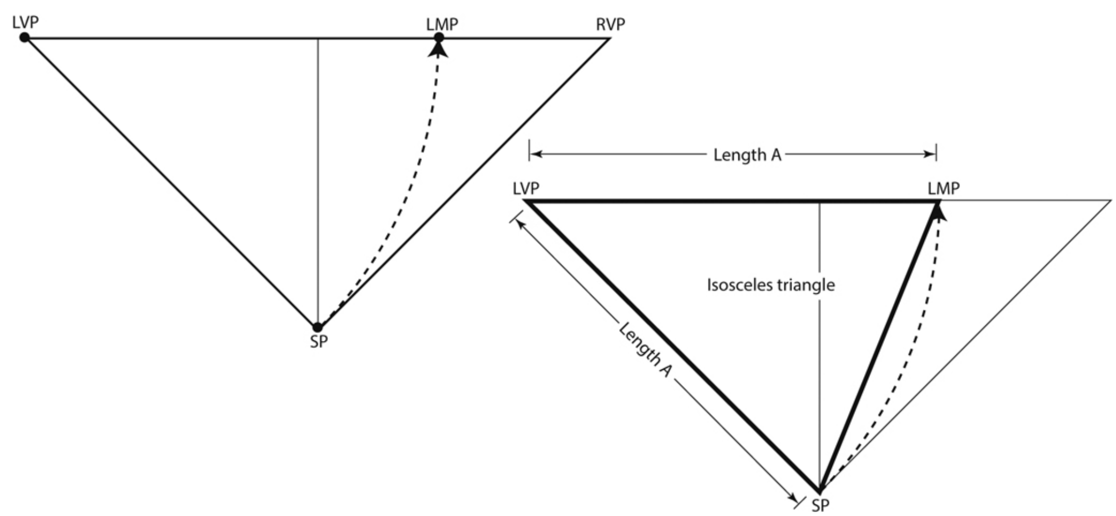


Figure 6.1 **The distance from the left vanishing point to the left measuring point is the same as the distance from the left vanishing point to the station point, an isosceles triangle.**

Lines drawn from the measuring point draw foreshortened isosceles triangles. The measuring line is always parallel with the picture plane; it is never foreshortened. When measuring, the length is transferred from the measuring line to a foreshortened line. The measuring line and the foreshortened line are the legs of the isosceles triangle. For the two legs of the triangle to be the same length, the angle created by the measuring point must be specific (Figure 6.2). It is therefore critical to use the proper measuring point. Otherwise, the shape drawn would not be an isosceles triangle; the two legs would not be the same, and the measurements would be inaccurate.

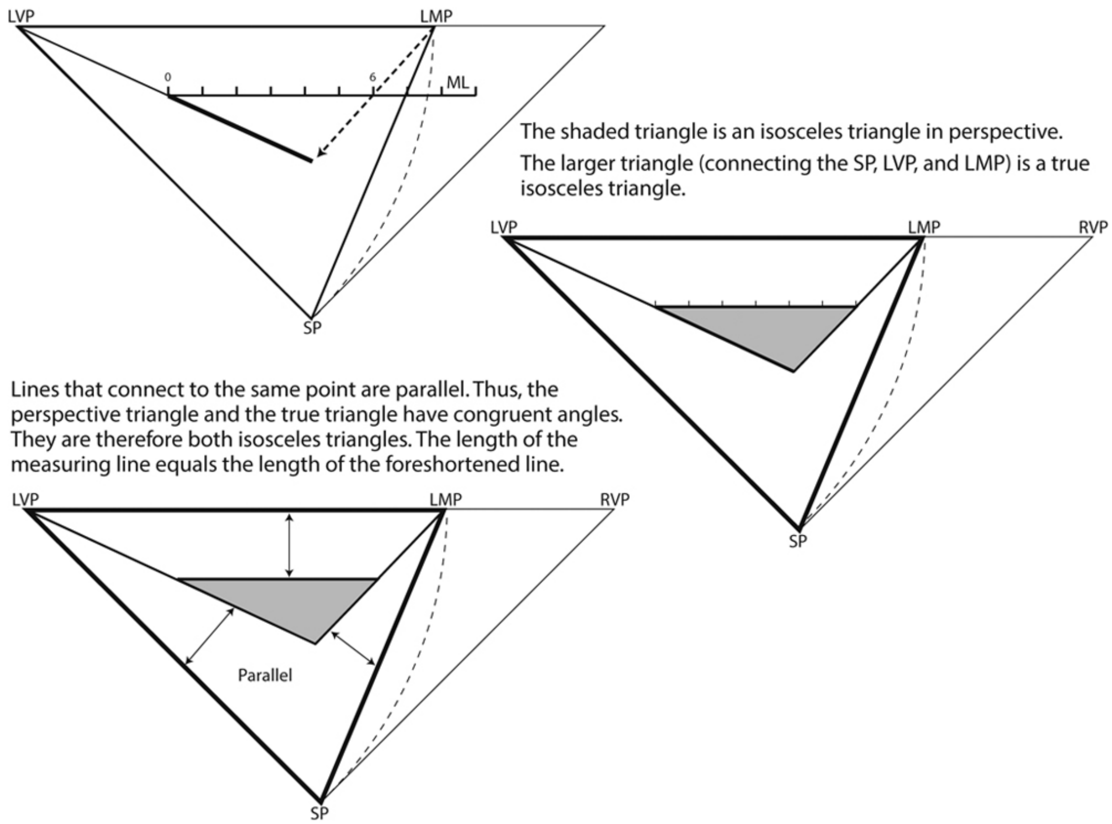


Figure 6.2 When using a measuring point, a foreshortened isosceles triangle is created.

7
Horizontal Angles

There is a one-to-one relationship between true angles at the station point and perspective angles from vanishing points. The station point serves as the axis (center point) for angles. Around the station point are angles totaling 360° (Figure 7.1). It is worth reviewing Chapter 5 as this content builds on that foundation.

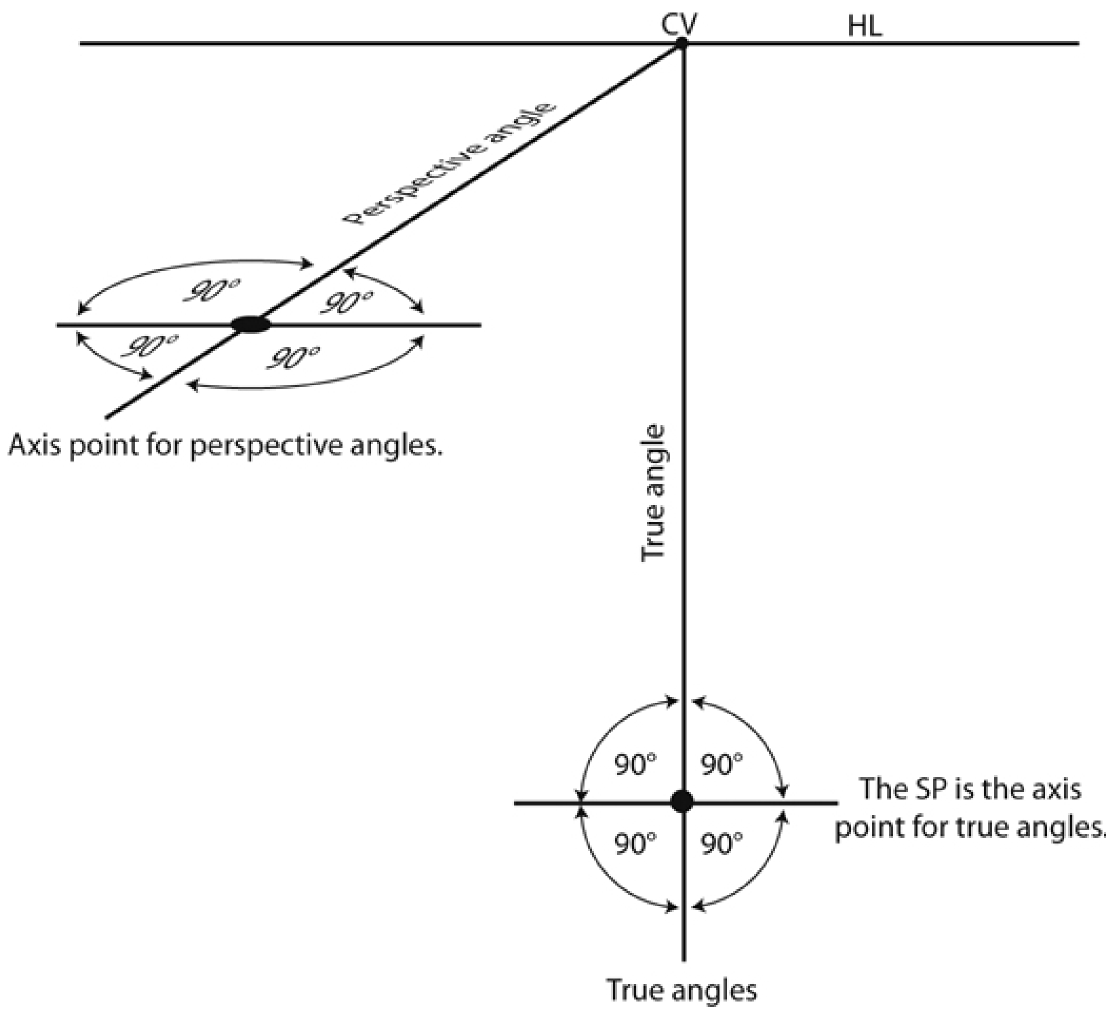


Figure 7.1 True 360° angles shown at the station point, and 360° angles shown in perspective to the upper left.

True angles at the station point mirror the perspective angles drawn from the vanishing points. An angle drawn from the station point, projected to the horizon line, creates a vanishing point that will draw that same angle in perspective. For example, to create a line 30° to the right of the center of vision, draw a true 30° angle at the station point. Project that angle to the horizon line. The resulting vanishing point will draw 30° angles in perspective (Figure 7.2). Keep in mind that there are 360° to consider. Here is another example: to create a line 130° to the right of the center of vision, draw that angle at the station point and project it to the horizon line. The resulting vanishing point draws that angle in perspective (Figure 7.3).

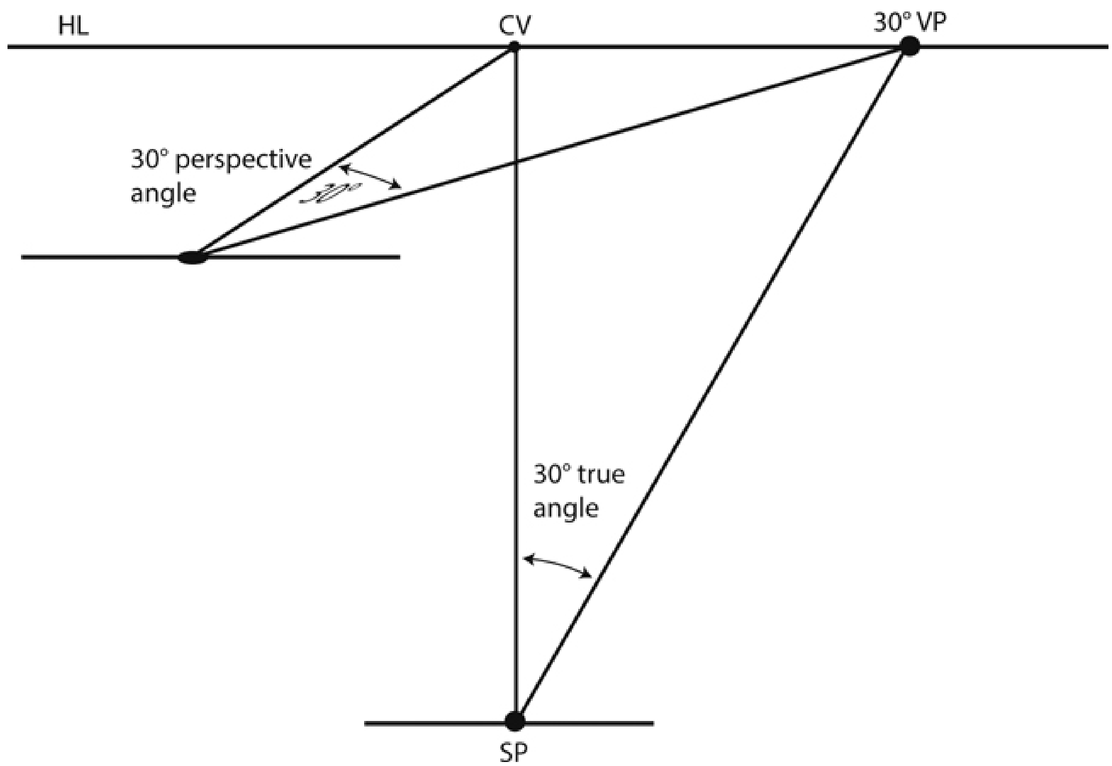


Figure 7.2 True angles projected from the station point create vanishing points that draw that same angle in perspective.

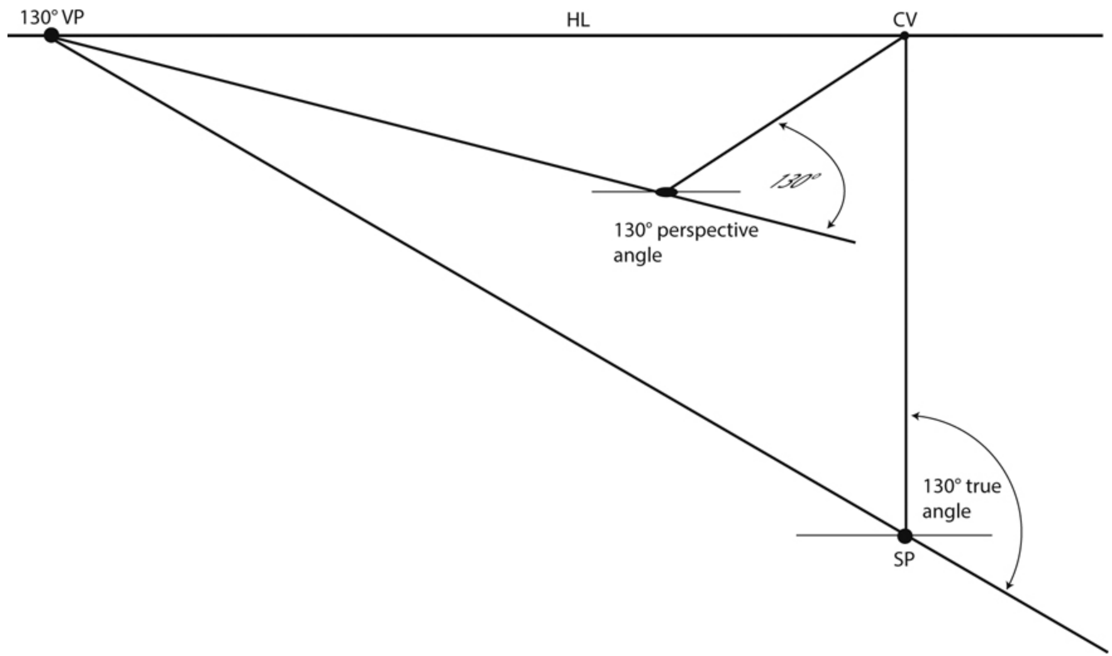
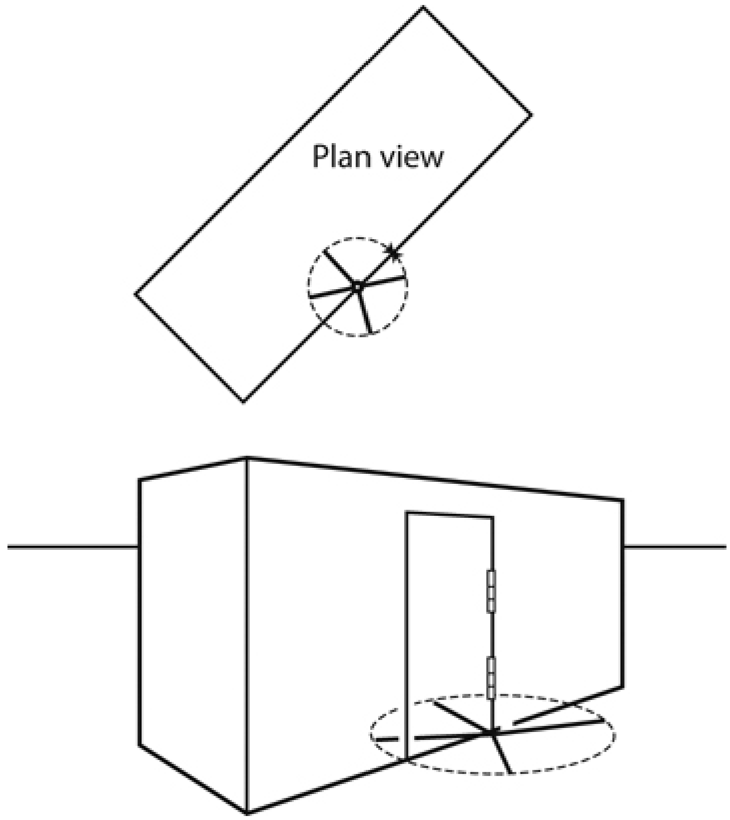


Figure 7.3 Angles can radiate from any direction around the station point, as there are 360°.

An Example of Horizontal Angles

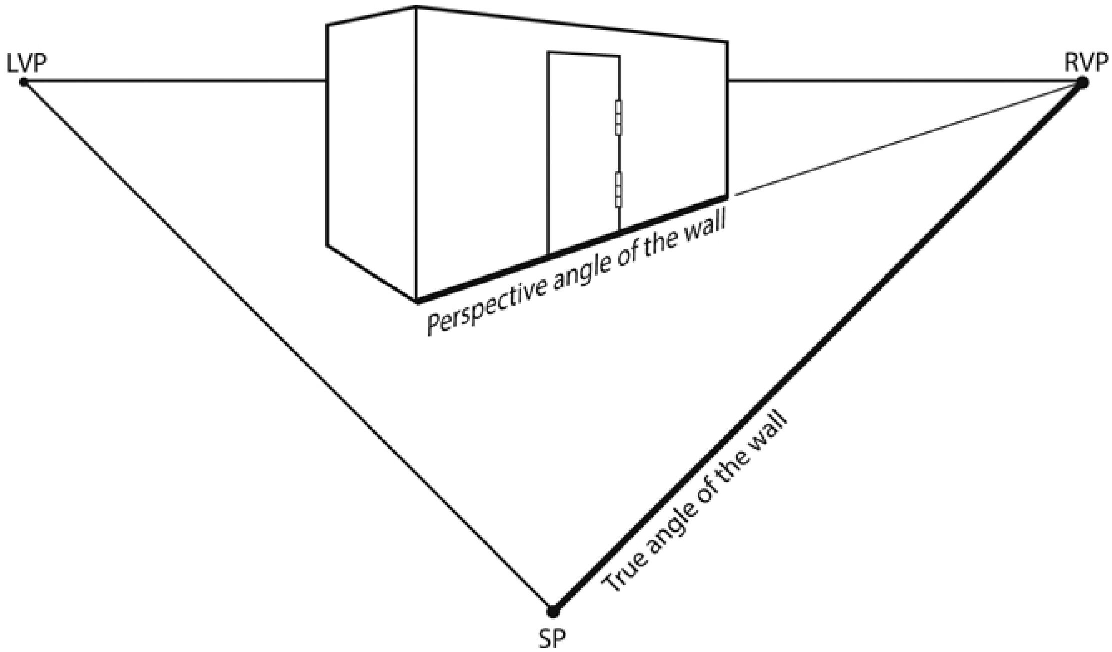


[Figure 7.4](#) **It is possible to open a door in or out, which makes it a convenient example to explore angles other than 90°.**

Most objects have right angles. But there are instances where angles other than 90° are called for. For an example, try applying this understanding of angles to an illustration. A door makes an excellent demonstration, as doors can swing out and swing in. There are a full 360° of possibilities (Figure 7.4). Of course, these general concepts apply to any situation that involves angles other than 90°.

The Wall's Angle

Place the door on the right, opening out 60° from the wall. The first step is to determine the angle of the wall. The wall is in perspective, so, to find the true angle of the wall, look to the station point. Angles at the station point are true angles. The line connecting the station point to the right vanishing point indicates the true angle of the right wall (Figure 7.5).



[Figure 7.5](#) **True angles of any foreshortened line can be found at the station point.**

The Door's Angle

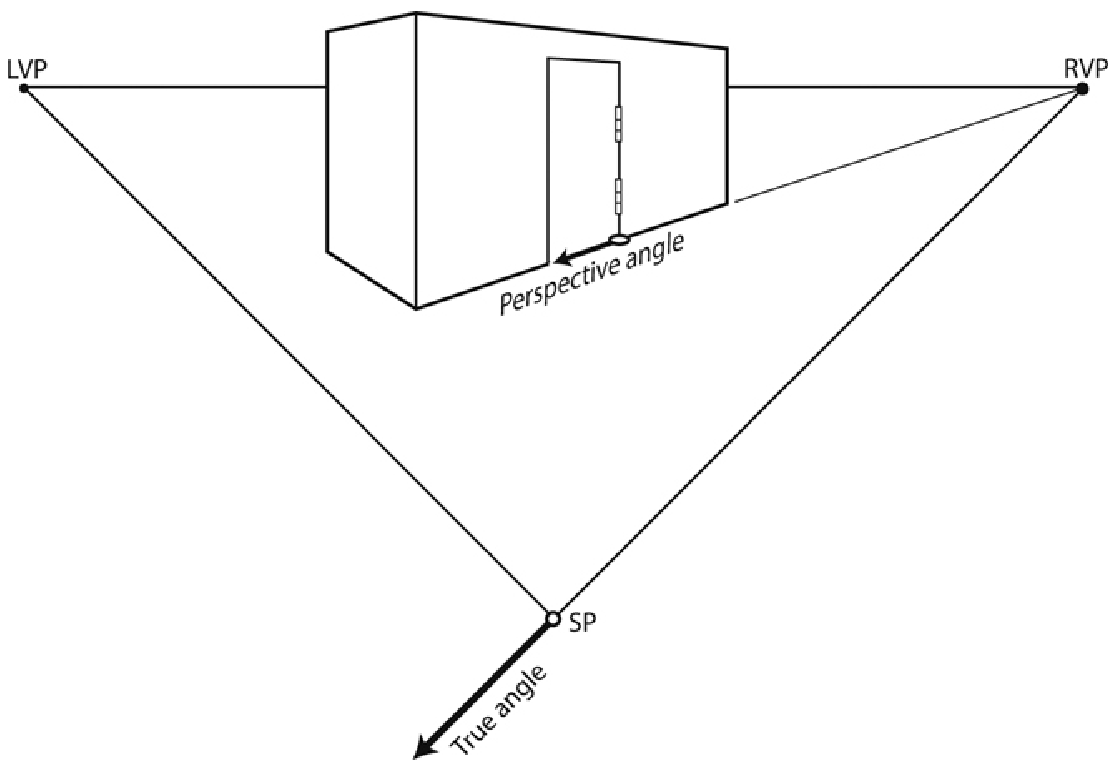


Figure 7.6 **Angles at the station point reflect the true angles of perspective lines.**

The station point is the axis point for true angles. The door's hinge is the axis point for perspective angles. Since the door's threshold projects forward, in front of the hinge, create that same angle at the station point (Figure 7.6).

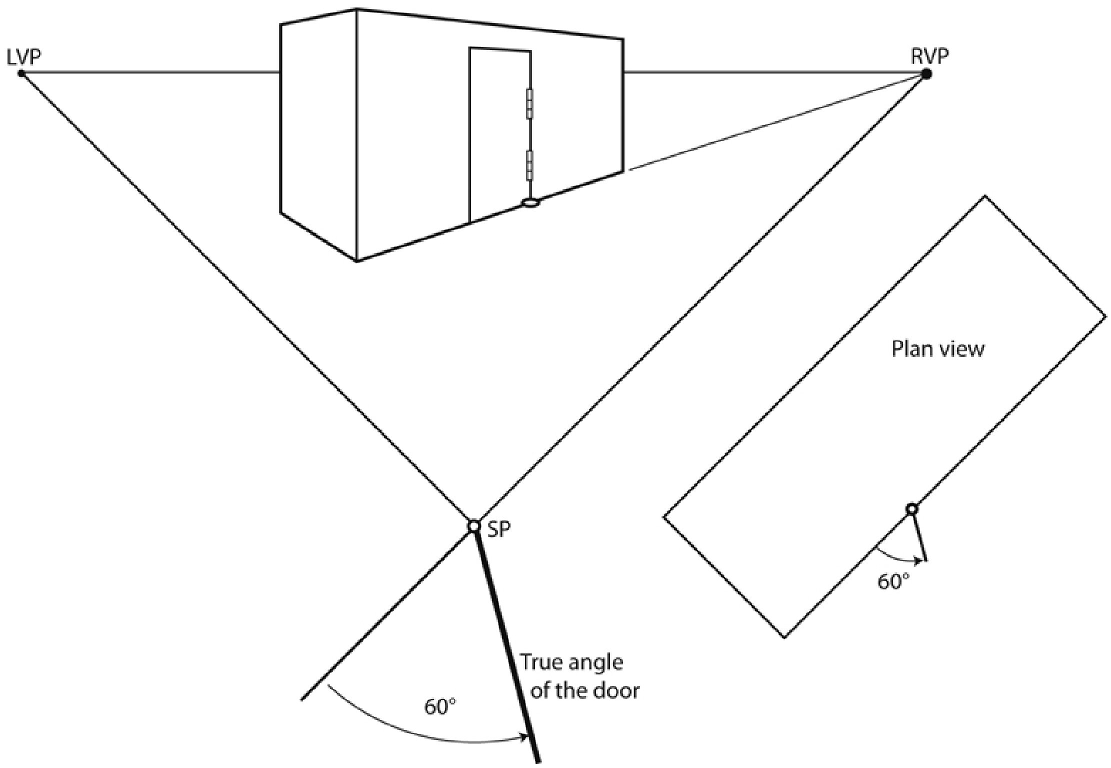


Figure 7.7 **Draw the true angle of the door at the station point. The angles at the station point are the same as those in a plan view.**

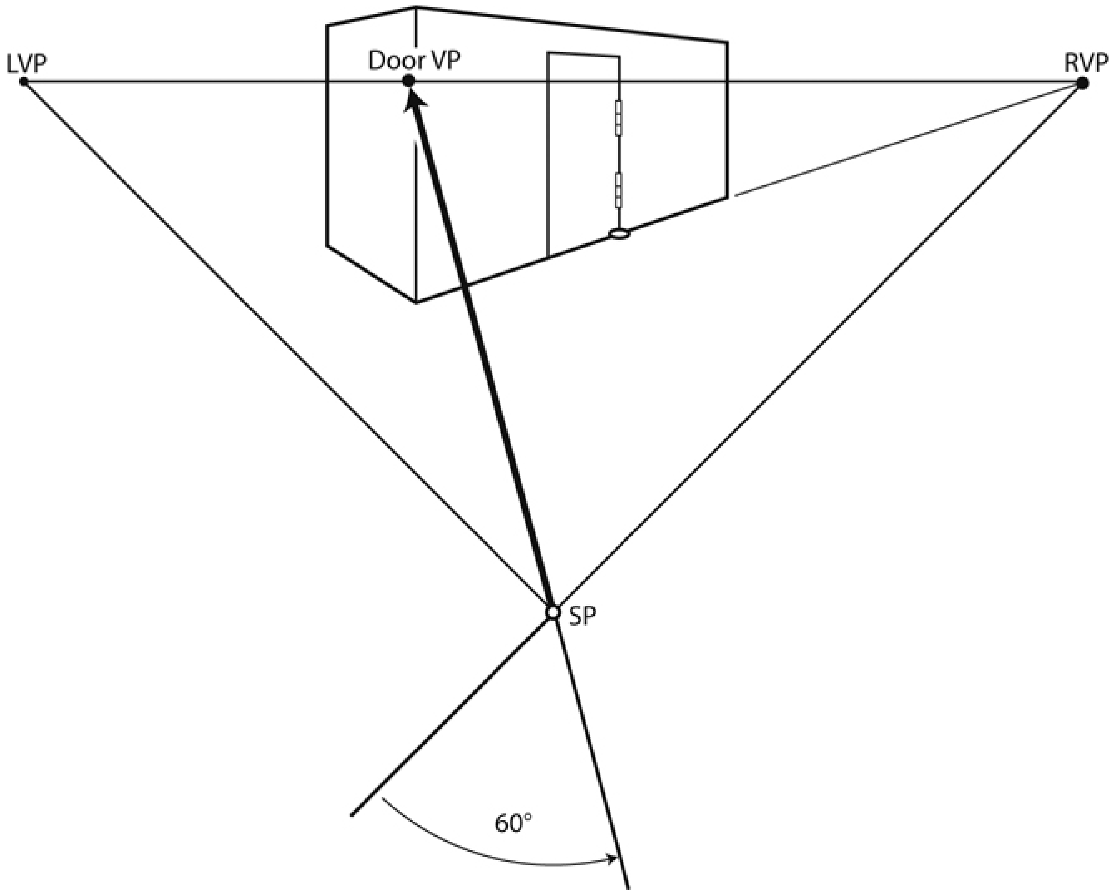


Figure 7.8 **Project the true angle of the door to the horizon line, creating a vanishing point that draws that same angle in perspective.**

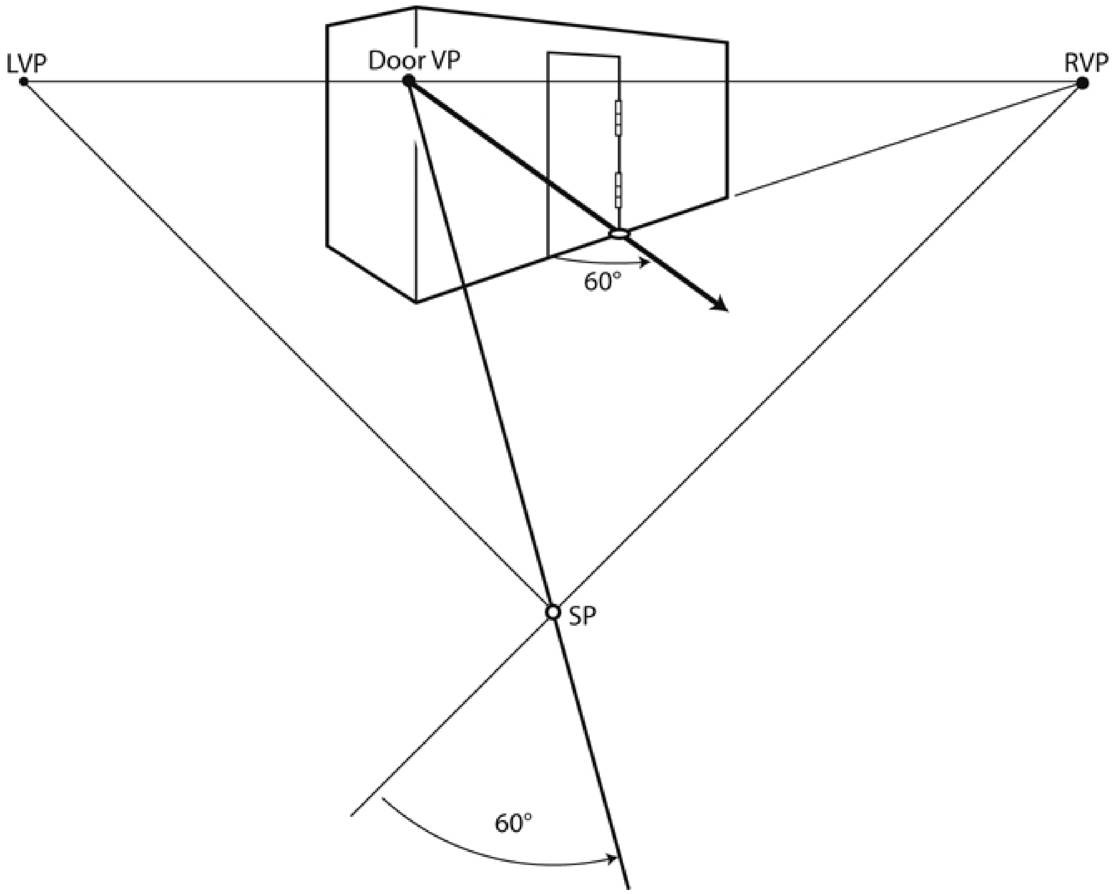


Figure 7.9 Lines drawn from the door vanishing point are 60° from lines drawn to the right vanishing point.

This door opens outward 60°, so draw that angle at the station point (Figure 7.7). Project the 60° angle to the horizon line (Figure 7.8). This vanishing point draws angles that are 60° from lines drawn to the right vanishing point (Figure 7.9).

The Threshold's Length

The door must close properly, so it must be the same length as the threshold. The next step is to measure the door. This is done with the right measuring point (Figure 7.10).

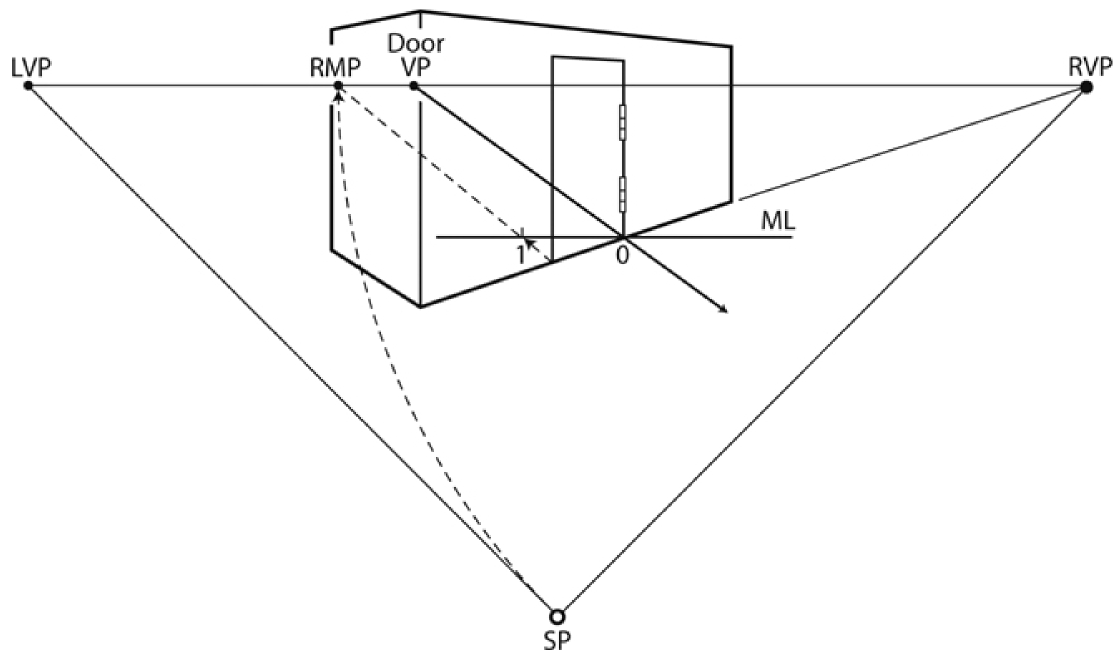


Figure 7.10 Use the right measuring point to measure the door's threshold.

The Door's Length

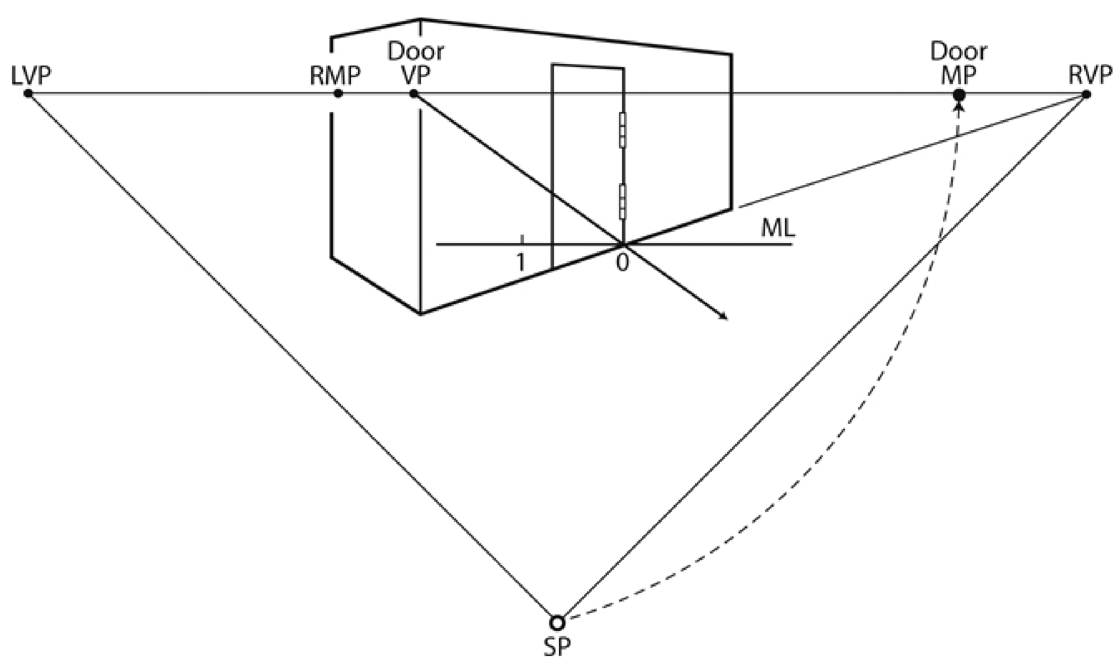


Figure 7.11 Create a door measuring point.

After determining the length of the threshold, measure the door. Since every vanishing point has its own measuring point, a new measuring point for the door needs to be created (Figure 7.11). Measure the door using the door measuring point (Figure 7.12). Complete the door by connecting lines to vanishing points (Figure 7.13).

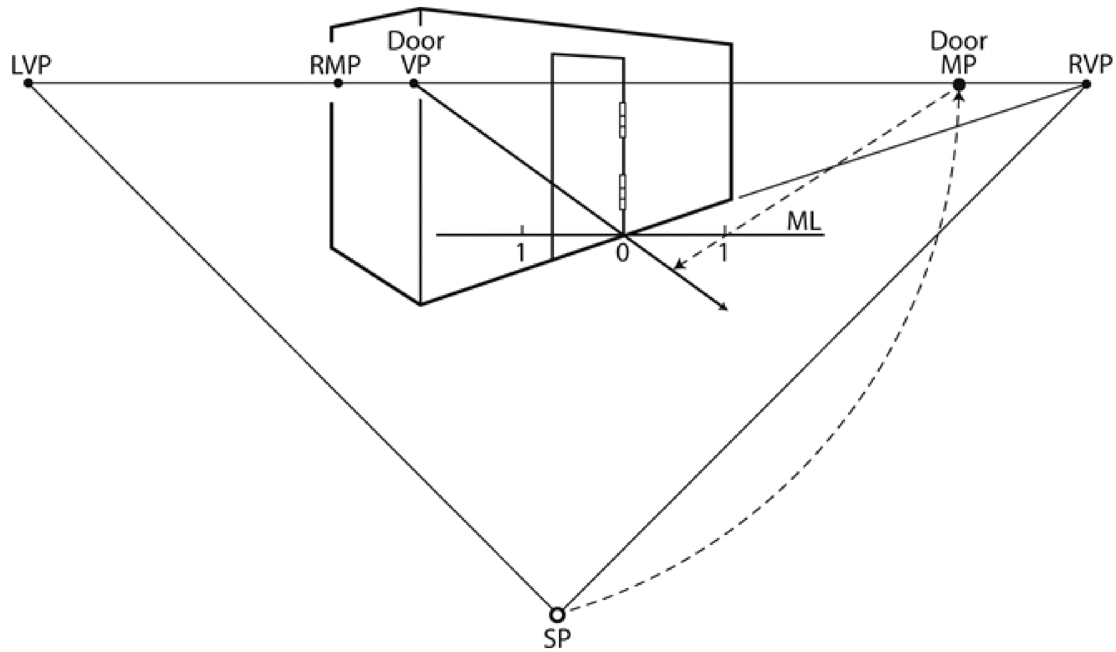


Figure 7.12 Use the right measuring point to measure

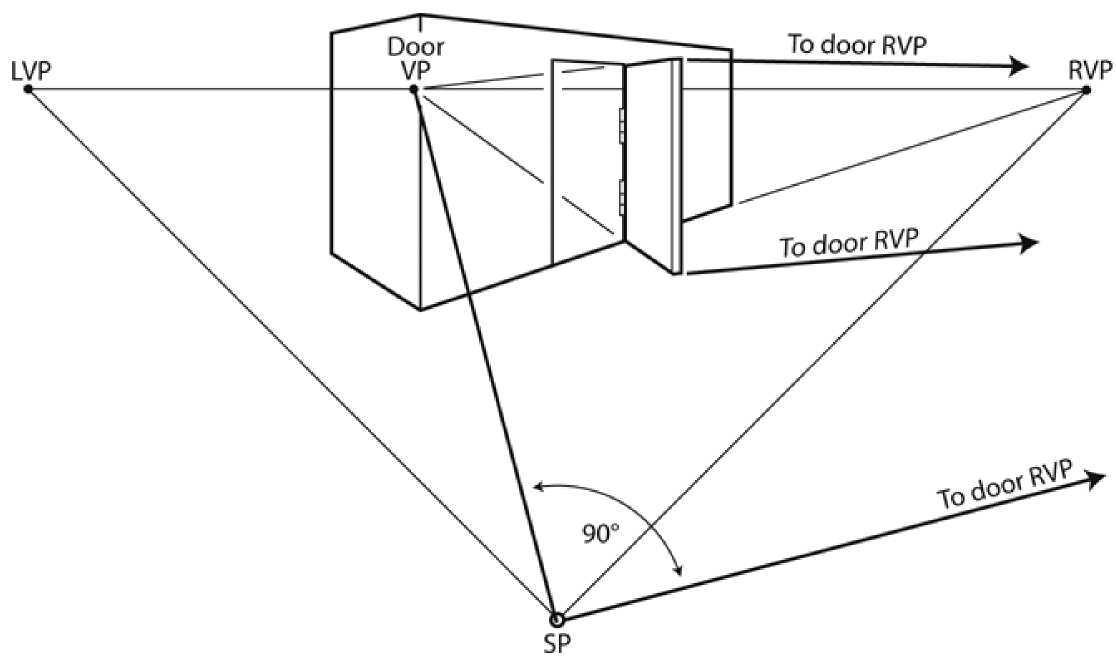


Figure 7.13 Completed door. To draw the thickness, project a 90° angle at the station point.

8
Ellipses, Spheres, Spiral Forms, and Random Curves

Ellipses

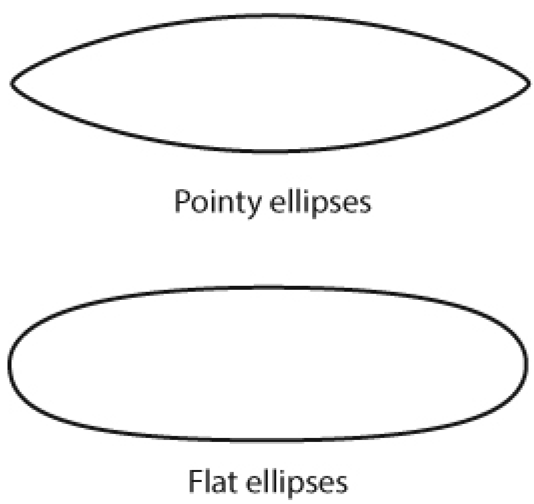


Figure 8.1 **Two of the most common mistakes when drawing ellipses**.

Ellipses are circles in perspective. Two common mistakes are drawing pointy ellipses and flat ellipses (Figure 8.1). A sure way to correct ellipses is to plot them in perspective. There are many ways to plot an ellipse. Each involves finding points along the circle and connecting the dots. This method is standard for drawing any curved object. The more points that are plotted, the more accurate the curve. The techniques used to plot ellipses are not especially complicated, but drawing smooth, beautiful ellipses involves more than knowing how to plot them. It involves a level of skill and finesse; it requires practice.

There are many methods to plot an ellipse, more than discussed in this book, but they all accomplish the same task—they all draw circles in perspective. The following pages illustrate some of the best methods, beginning with a four-point ellipse.

Four-Point Ellipse

Typically, four points do not give enough information to draw an accurate ellipse. But, when used for small ellipses, they are usually adequate, as well as simple and fast. Start by drawing a square. A circle touches a square at the center of each side—at four points. Locate these four points by finding the center of the square (draw an X through the corners), then project outward from the center point. Connect the four points with a smooth curve (Figure 8.2). Four points are adequate for small ellipses. For larger ellipses, it is desirable to have more points. The more points there are, the more accurate the ellipse.

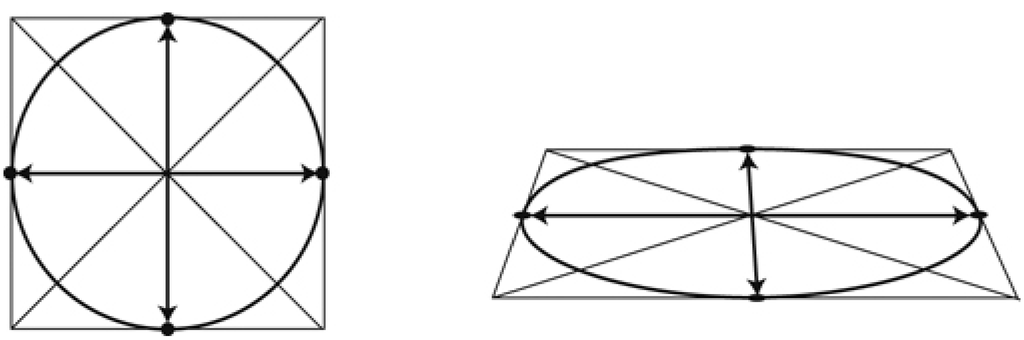


Figure 8.2 **Drawing a four-point ellipse**.

Ellipses are especially susceptible to distortion. As ellipses stray from the center of vision they tend to look tilted (Figure 8.3, left). This is an unavoidable byproduct of perspective—objects drawn beyond the cone of vision will look stretched. Ellipses seem to magnify this problem. These distorted ellipses can be seen in many historical paintings. It is a frustrating predicament. The ellipses are plotted correctly, but look askew. This dilemma can be fixed by "cheating" the ellipses: their shape is modified so they appear visually correct. This requires a bit of artistic surgery and eyeballing. Modify the shape so the left side of the ellipse appears symmetrical with the right side (Figure 8.3, right). If the left and right side of an ellipse are symmetrical, the ellipse will look flat. Sometimes it is best to use a little artistic license, correcting the ellipse to compensate for distortion.

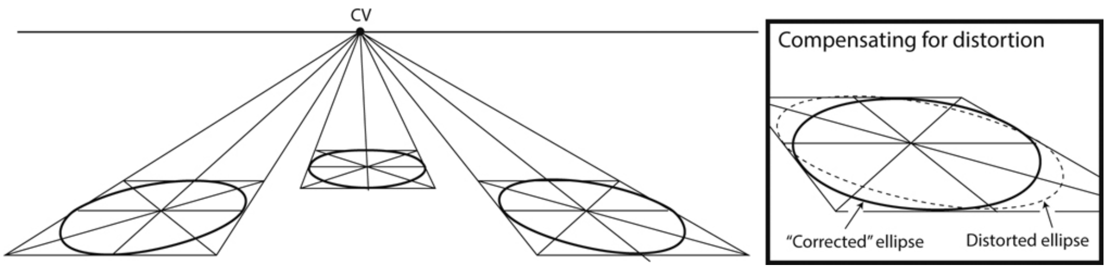


Figure 8.3 Ellipses look increasingly distorted as they approach the cone of vision's border.

Eight-Point Plotted Ellipse

The eight-point ellipse is most commonly used. Having eight points, each being 45° apart, gives an accurate guide to draw most ellipses. There are many ways to draw an eight-point ellipse. One of the oldest methods used to find these additional four points begins by drawing a true circle. The four points are then transferred from the true circle to the perspective ellipse (Figure 8.4).

Shortcut

This method works well but can be long-winded as there are quite a few unnecessary lines. Instead of drawing the entire true circle, draw a quarter circle. Once one point is plotted, that point can be transferred to the other three locations (Figure 8.5).

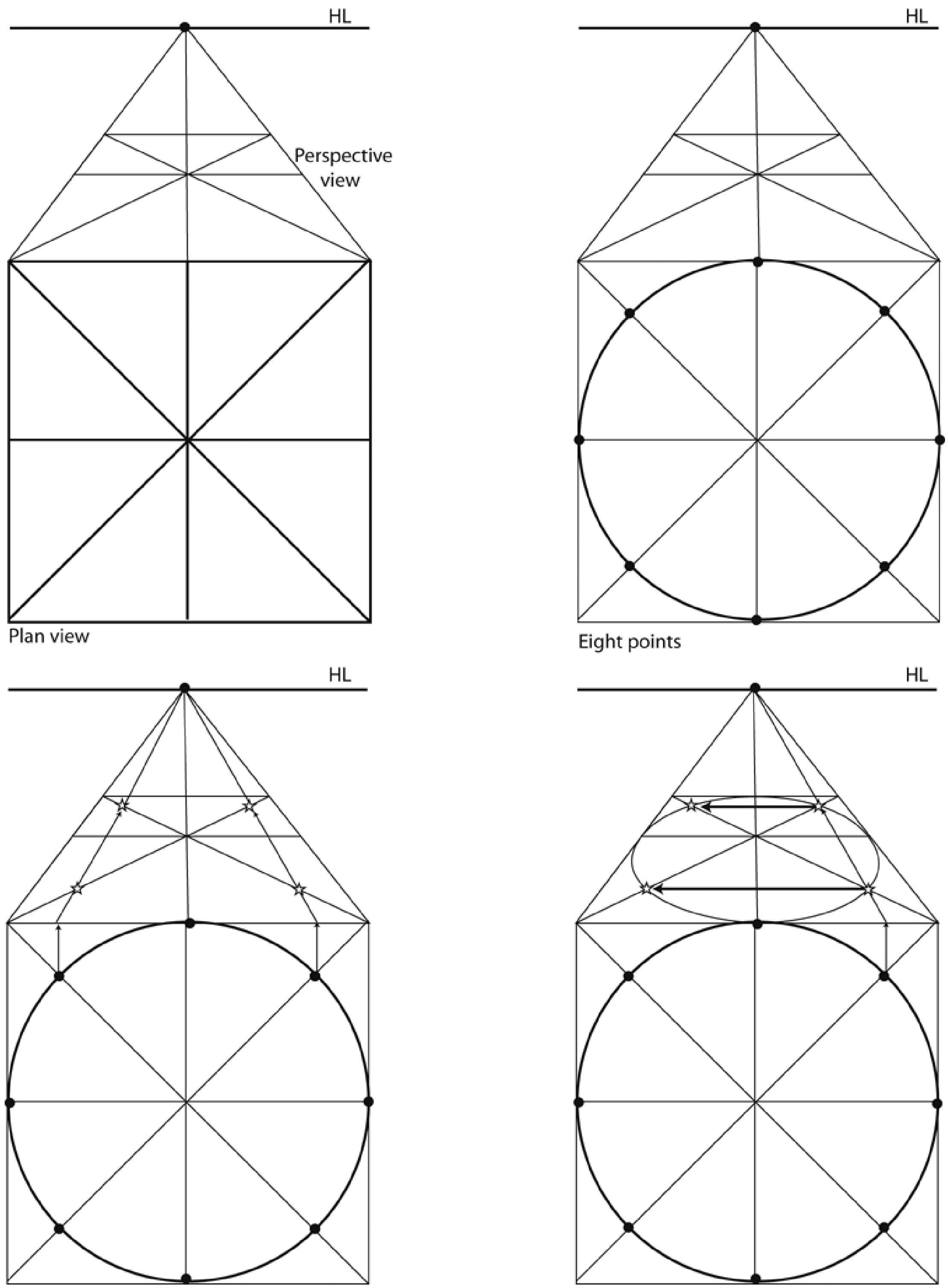


Figure 8.4 How to draw an eight-point ellipse in perspective.

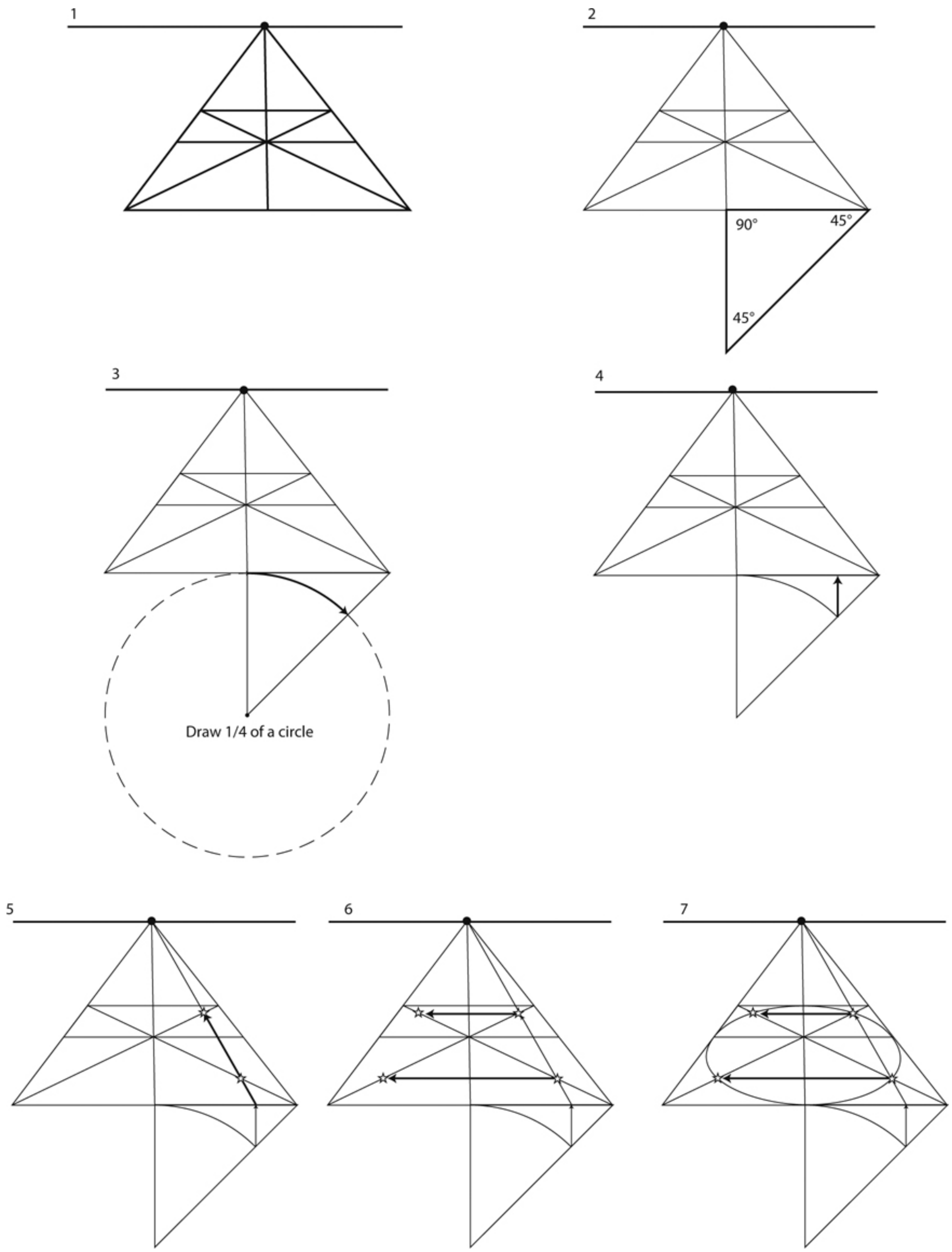
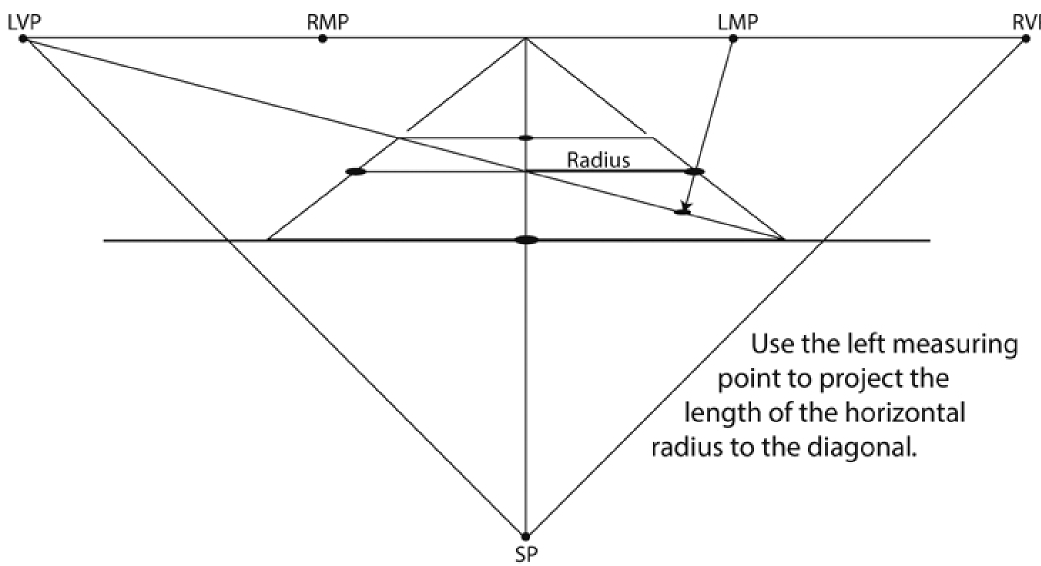
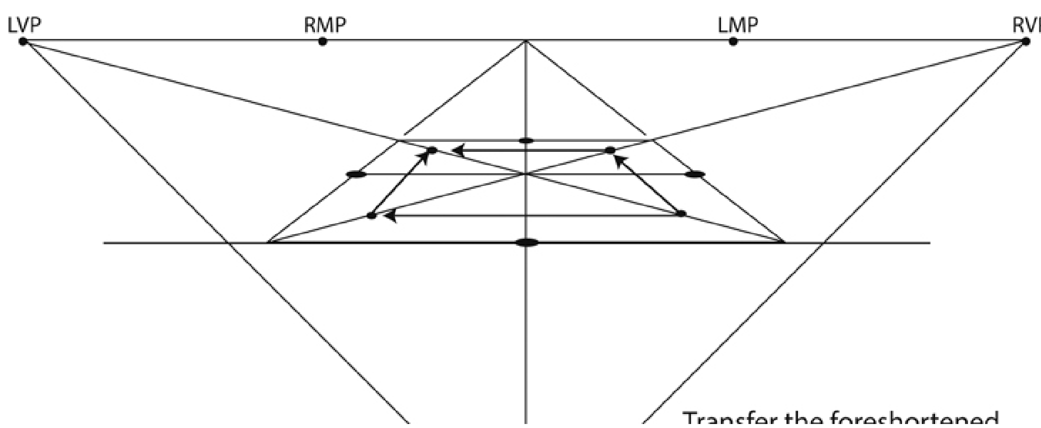


Figure 8.5 **This is a shortcut to drawing an eight-point ellipse, saving time and space.**

Eight-Point Measured Ellipse

This is the most streamlined— and arguably the simplest— method to plot an ellipse. Here's how it works. Four points of the ellipse touch the square. The other four points are located on the diagonal lines. The diagonal lines have a vanishing point; they are foreshortened. The horizontal line representing the **radius** of the ellipse is parallel with the picture plane, so it is not foreshortened. Use this horizontal line as a measuring line. Using the appropriate measuring point, transfer the length of the un-foreshortened radius to the diagonal line ([Figure 8.6](#)). This method works well for horizontal ellipses. It also works for vertical ellipses, but is a bit more complicated. It involves using auxiliary vanishing points (AUX. VPs) and auxiliary measuring points (AUX. MPs). This technique is best confined to horizontal ellipses.





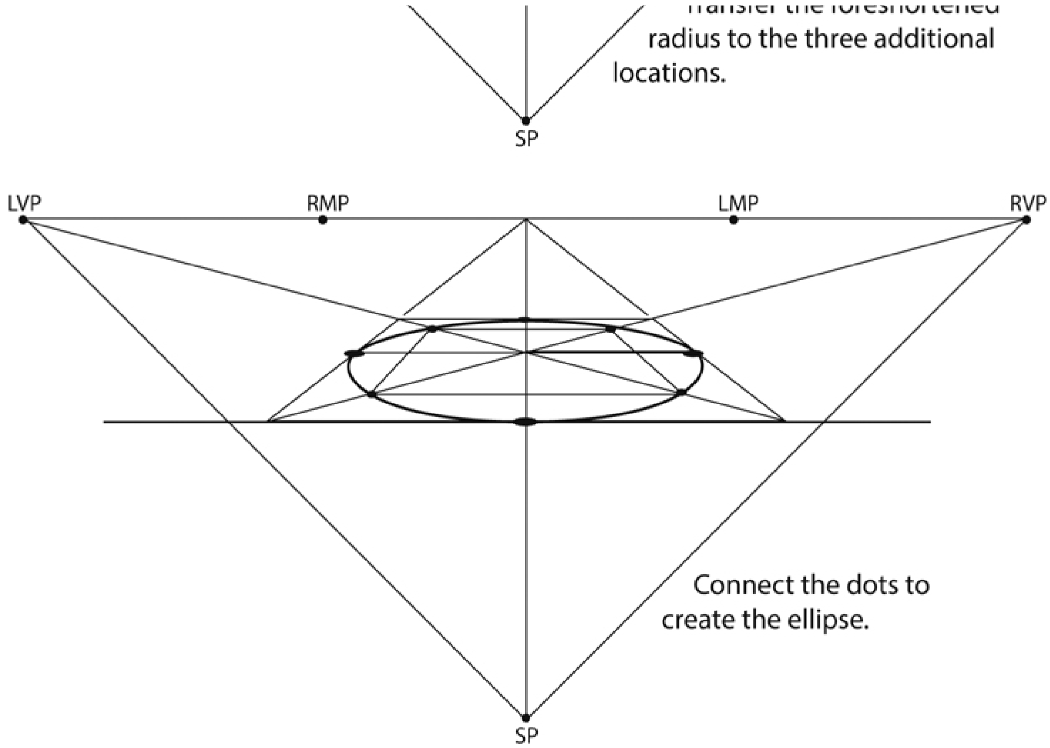


Figure 8.6 **Using a measuring point to create an eight-point ellipse**.

Eight-Point Projected Ellipse

This method is based more on coincidence than on geometry; there is no real logic to the procedure. Follow the steps, and an eight-point ellipse can be drawn. It is quick and relatively simple, but this method is not 100 percent accurate—it is extremely close, but not mathematically exact. It creates a radius along the diagonal, slightly longer than it should be (about 2 percent longer). This variation would be unacceptable to a mechanical engineer— but it is insignificant for a hand-drawn ellipse (Figure 8.7).

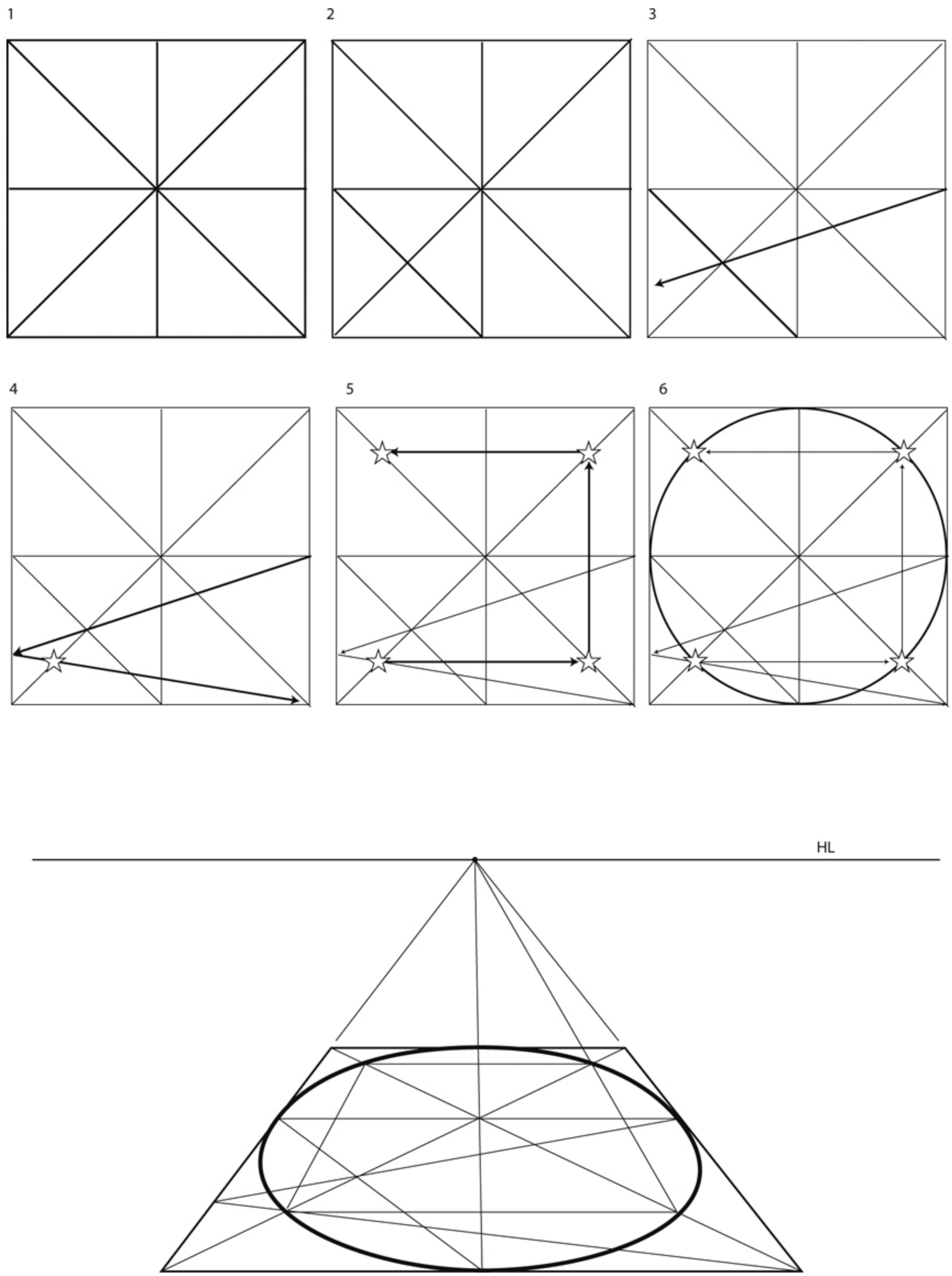


Figure 8.7 **This eight-point ellipse technique is simple and compact.**

Twelve-Point Ellipse

The more points that are plotted, the more accurate the ellipse. There are two methods to draw a twelve-point ellipse. In the first method, the ellipse is not mathematically perfect as before, but the deviation is insignificant. This method is best used for larger ellipses. Follow the steps to find the twelve points (Figure 8.8).

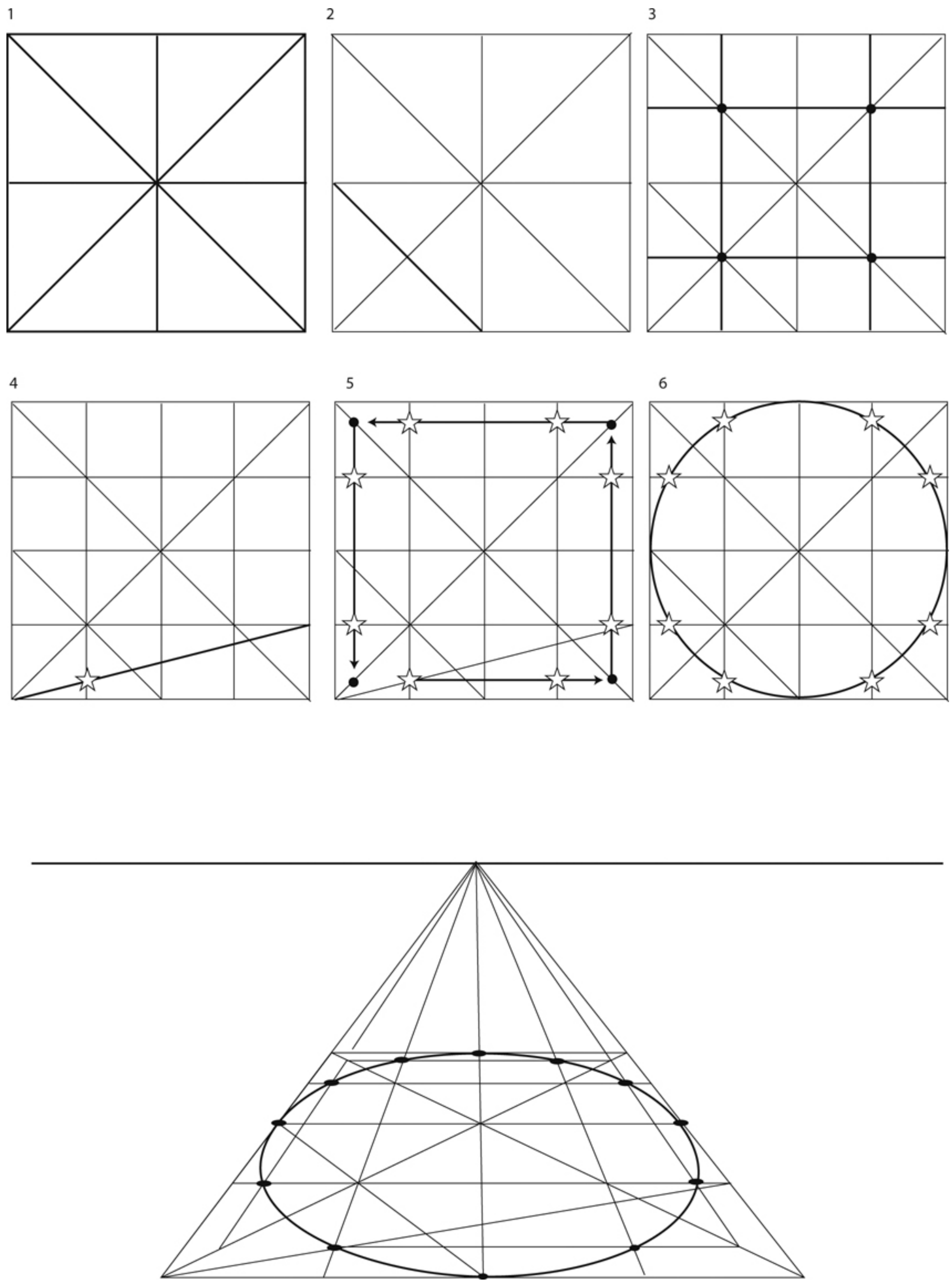


Figure 8.8 **A twelve-point ellipse works well for large circles.**

The points in the second version are a little closer to each other than in the previous twelve-point diagram (Figure 8.9).

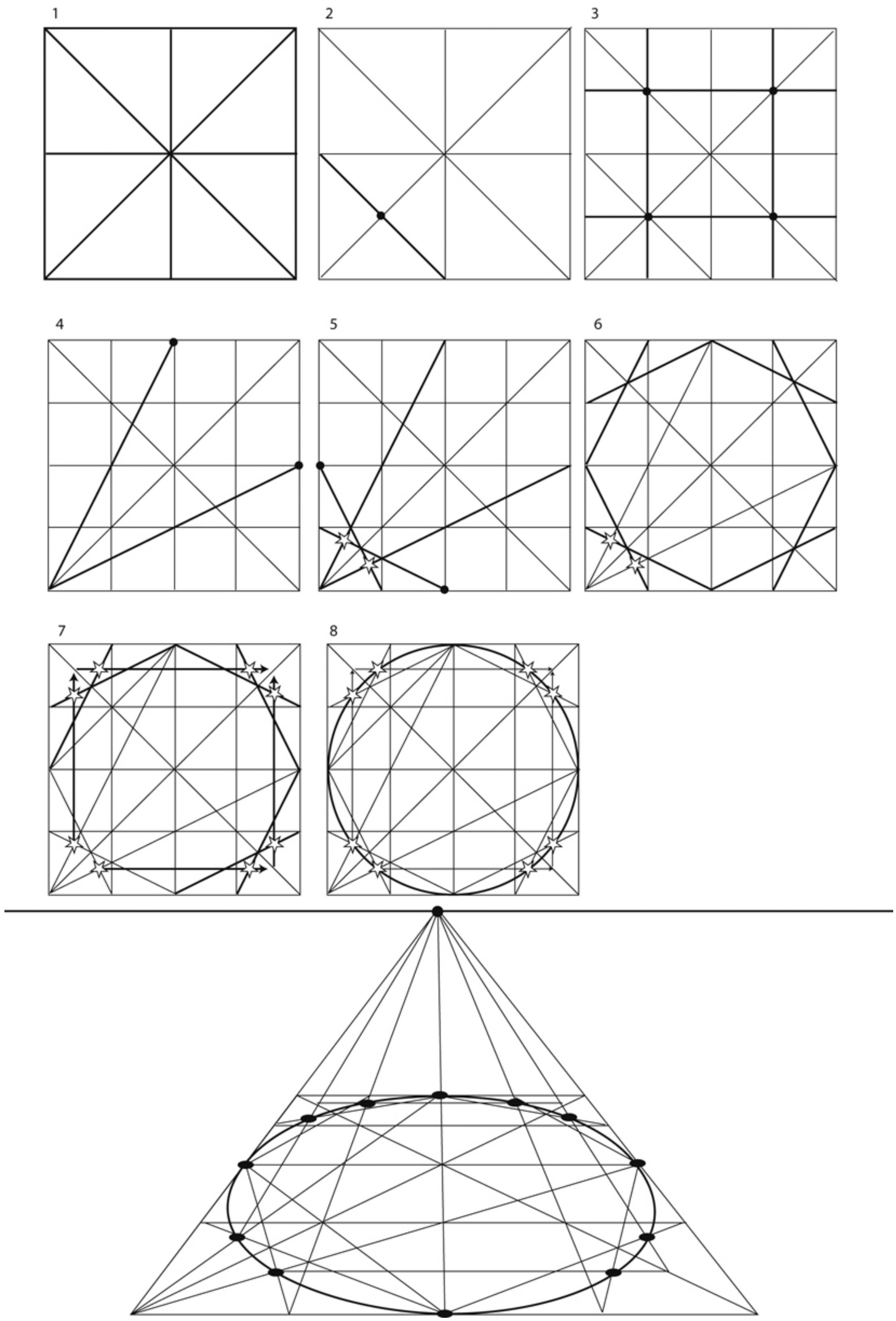


Figure 8.9 This is an alternative twelve-point ellipse method.

Twenty-Four-Point Ellipse

Combining the eight-point with one of the two twelve-point methods creates an ellipse with twenty-four points. This is a lot of points to draw, but if drawing a very big ellipse, a greater number of points is more desirable (Figure 8.10).

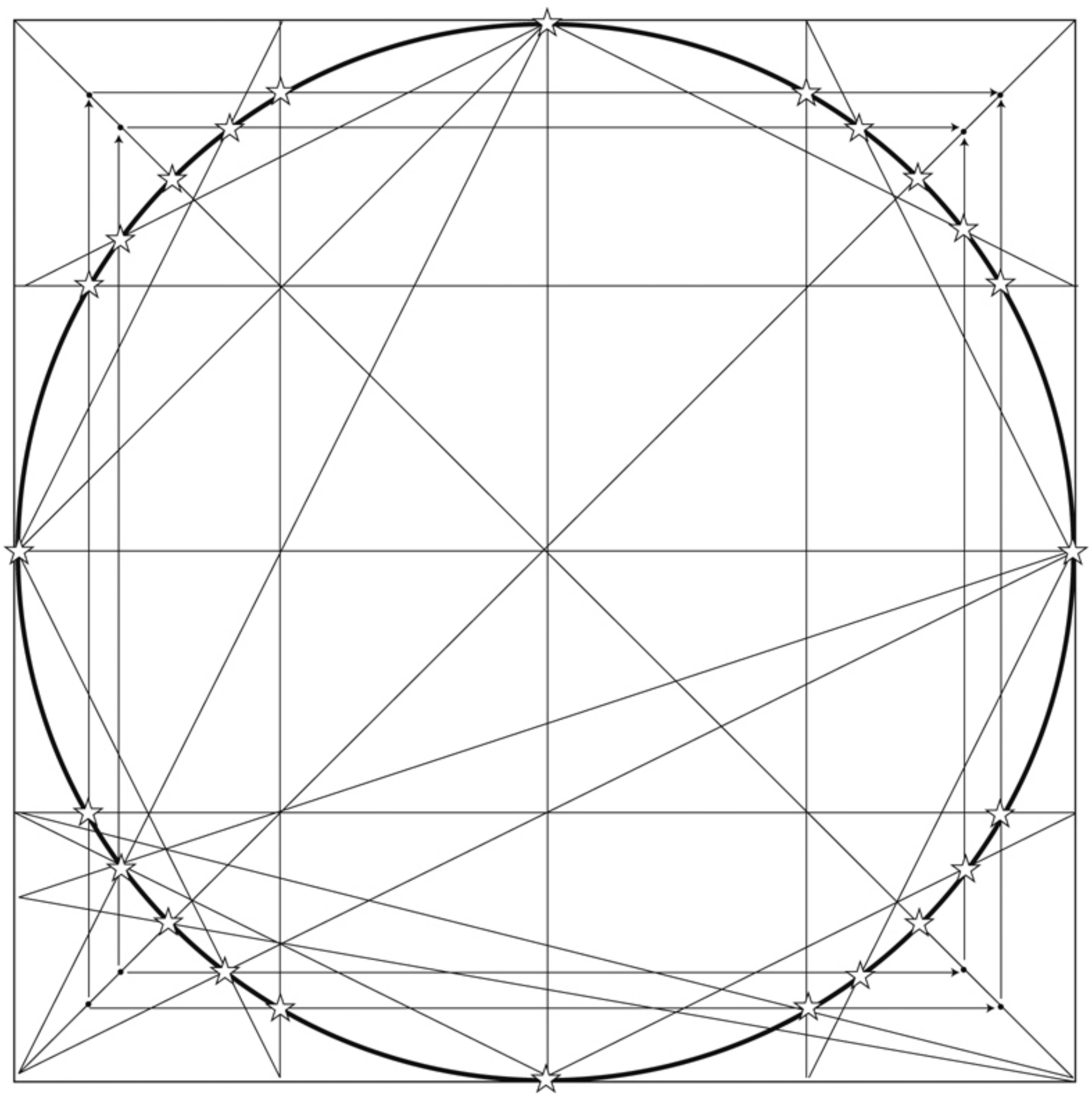


Figure 8.10 Twenty-four-points are used for very large ellipses.

Ellipse Guides

Ellipse guides may seem the obvious solution to drawing an ellipse. They are convenient, fast, and create perfectly smooth ellipses, but they have one major problem—they are not in perspective. An ellipse guide creates an oval, not an ellipse. An oval is symmetrical vertically *and* horizontally; the center is not a perspective center. A true perspective ellipse is not symmetrical; the front half of the circle is closer to the viewer than the back half. Therefore, the shape of the front is different from the shape of the back (Figure 8.11). Ellipse guides work well for small ellipses. But if the ellipse is large and is to have a feeling of depth, a perspective ellipse is needed—an ellipse that is plotted. However, despite their drawbacks, ellipse guides can be used successfully in a wide range of situations.

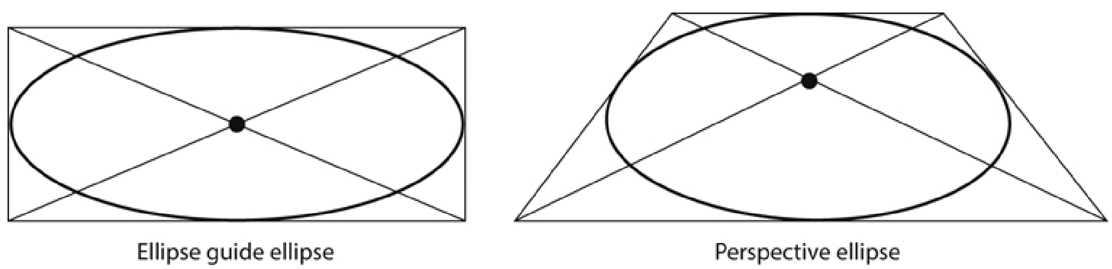
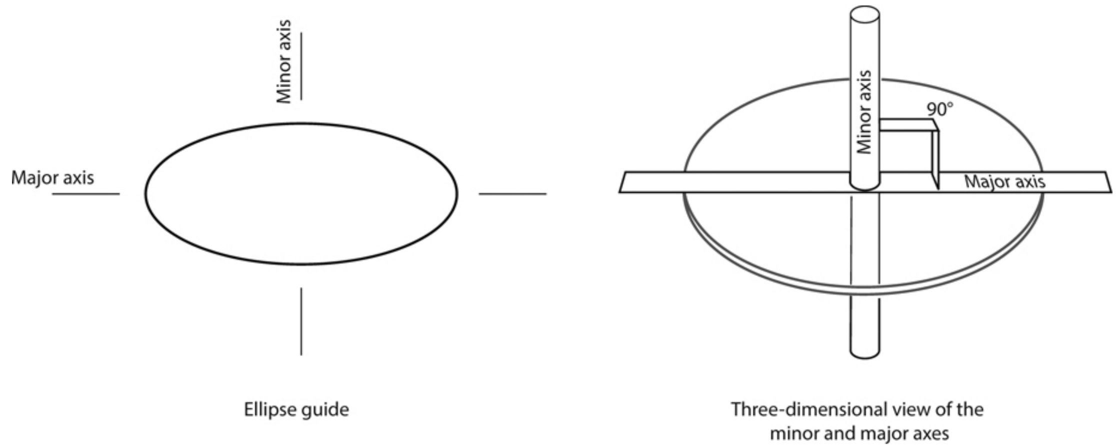


Figure 8.11 **An ellipse guide ellipse compared to a perspective ellipse**.

Ellipse guides come in sets ranging from 10° to 80°, in 5° increments. To decide which degree to use, first draw a square in perspective. The proportions of the square will determine which guide to use. Find the guide that fits best. The ellipse needs to touch the center of the square's sides. A word of caution: the ellipse will not align properly with perspective points. Remember, the ellipse guide is not in perspective, but the square is in perspective. The points on the square will not match the ellipse guide, nor will the center of the ellipse align with the perspective center of the square.



[Figure 8.12](#) **The major and minor axes are printed on the ellipse guide (left). A three-dimensional view shows the minor axis oriented 90° from the ellipse's surface (right).**

The proper orientation of an ellipse guide is critical. This is where mistakes are often made. If the ellipse guide is not oriented correctly, the ellipse will look tilted or angled. Using ellipse guides can be anti-intuitive; there are rules to their use, and they often go against instincts. To understand how to orient an ellipse guide, the **major** and **minor axes** need to be understood. The minor axis is the short side of the ellipse, and the major axis is the long side. The minor axis is the primary concern. It should be thought of three-dimensionally, going *through* the circle. It is like an axle on a tire: it intersects the center of the ellipse at a right angle (Figure 8.12).

Ellipse Orientation

The correct orientation of an ellipse is determined by the direction of the minor axis. Horizontal ellipses are oriented differently than vertical ellipses, and one-point ellipses are oriented differently than two-point ellipses.

Horizontal Ellipses

The minor axis is oriented vertically for all horizontal ellipses. It does not matter where the ellipse is placed, or if the ellipse is drawn in one- or two-point perspective. If the ellipse is parallel with the ground plane, then the minor axis is oriented vertically, parallel with the picture plane and perpendicular to the ground (Figure 8.13).

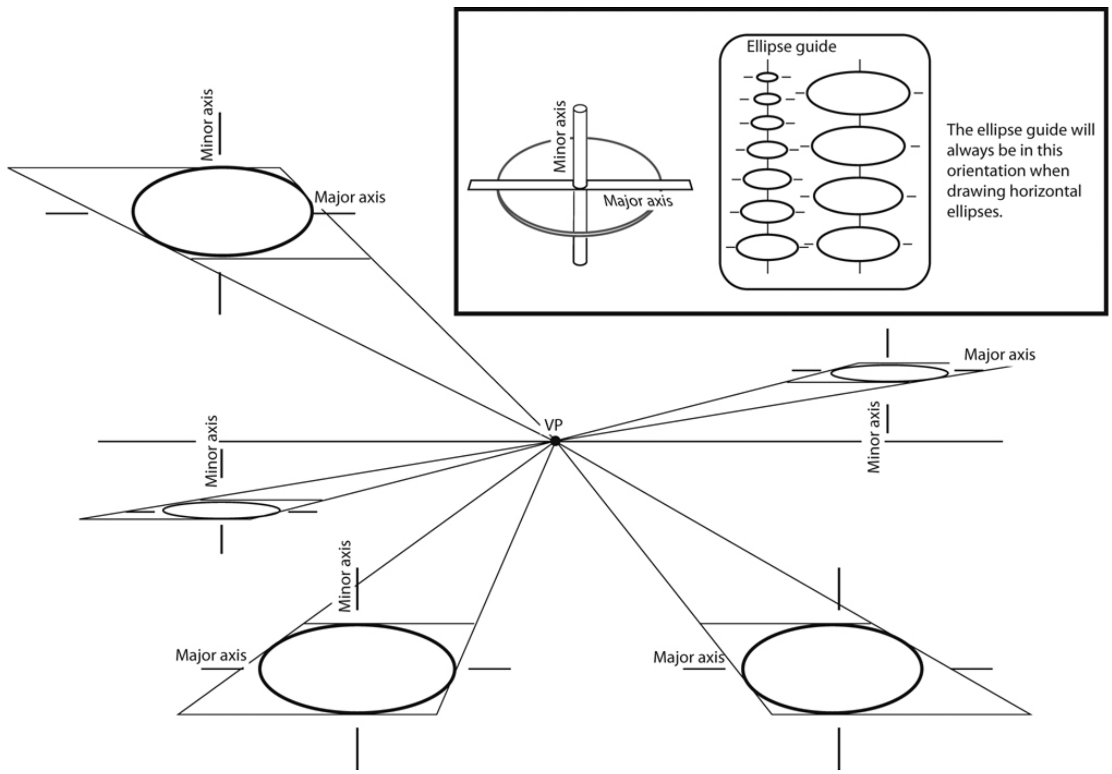
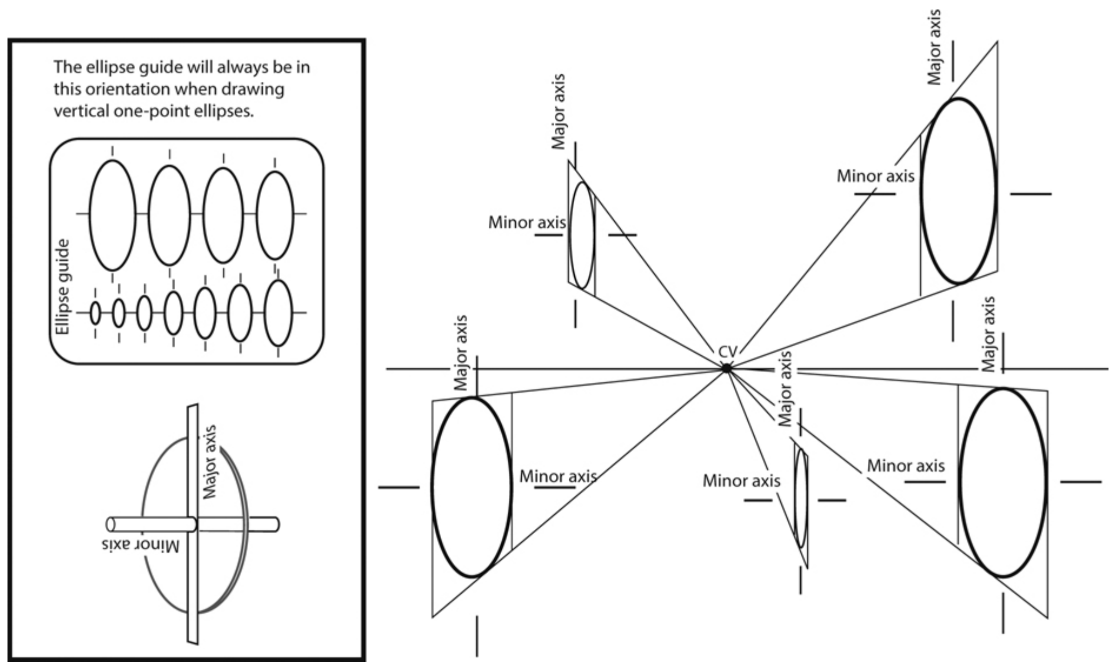


Figure 8.13 **When drawing horizontal ellipses, the minor axis is vertical, perpendicular to the ground plane**.

One-Point Vertical Ellipses

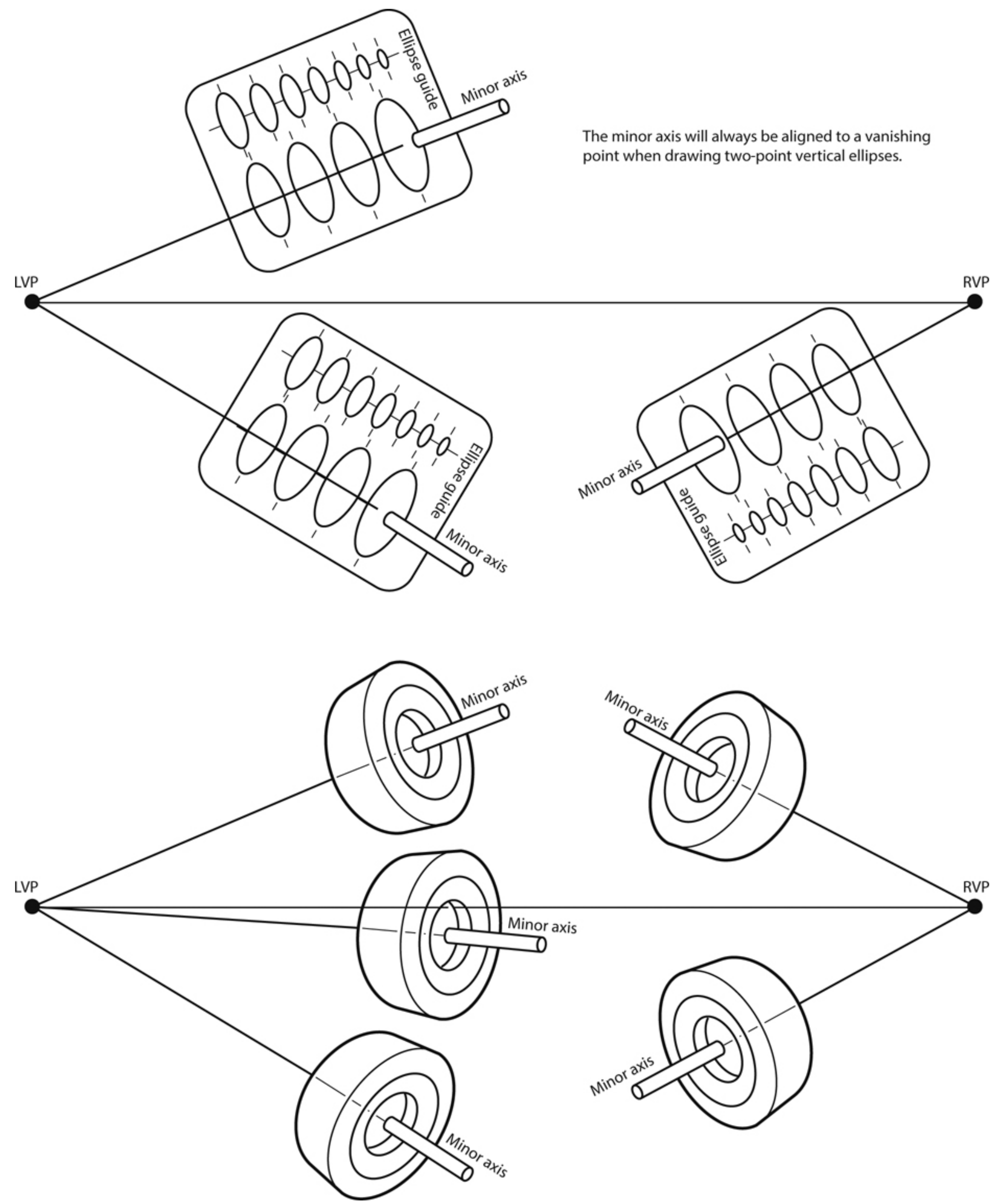
The minor axis is oriented horizontally for all one-point perspective vertical ellipses. It does not matter where the ellipse is placed. If the ellipse is vertical and in one-point perspective, the minor axis is oriented horizontally, perpendicular with the picture plane and parallel with the ground (Figure 8.14).



[Figure 8.14](#) **When drawing vertical one-point ellipses, the minor axis is horizontal, parallel with the ground plane**.

Two-Point Vertical Ellipses

The minor axis connects to a vanishing point for all two-point perspective vertical ellipses. But there are two vanishing points—which one does the minor axis connect to? It is helpful to think of the minor axis as an axle on a tire. The minor axis—like an axle—goes *through* the ellipse. The minor axis is a three-dimensional form. It is 90° from the surface of the ellipse ([Figure 8.15](#)).



[Figure 8.15](#) When drawing vertical two-point ellipses, the minor axis connects to a vanishing point. Think of the ellipse as a tire, and the minor axis as an axle.

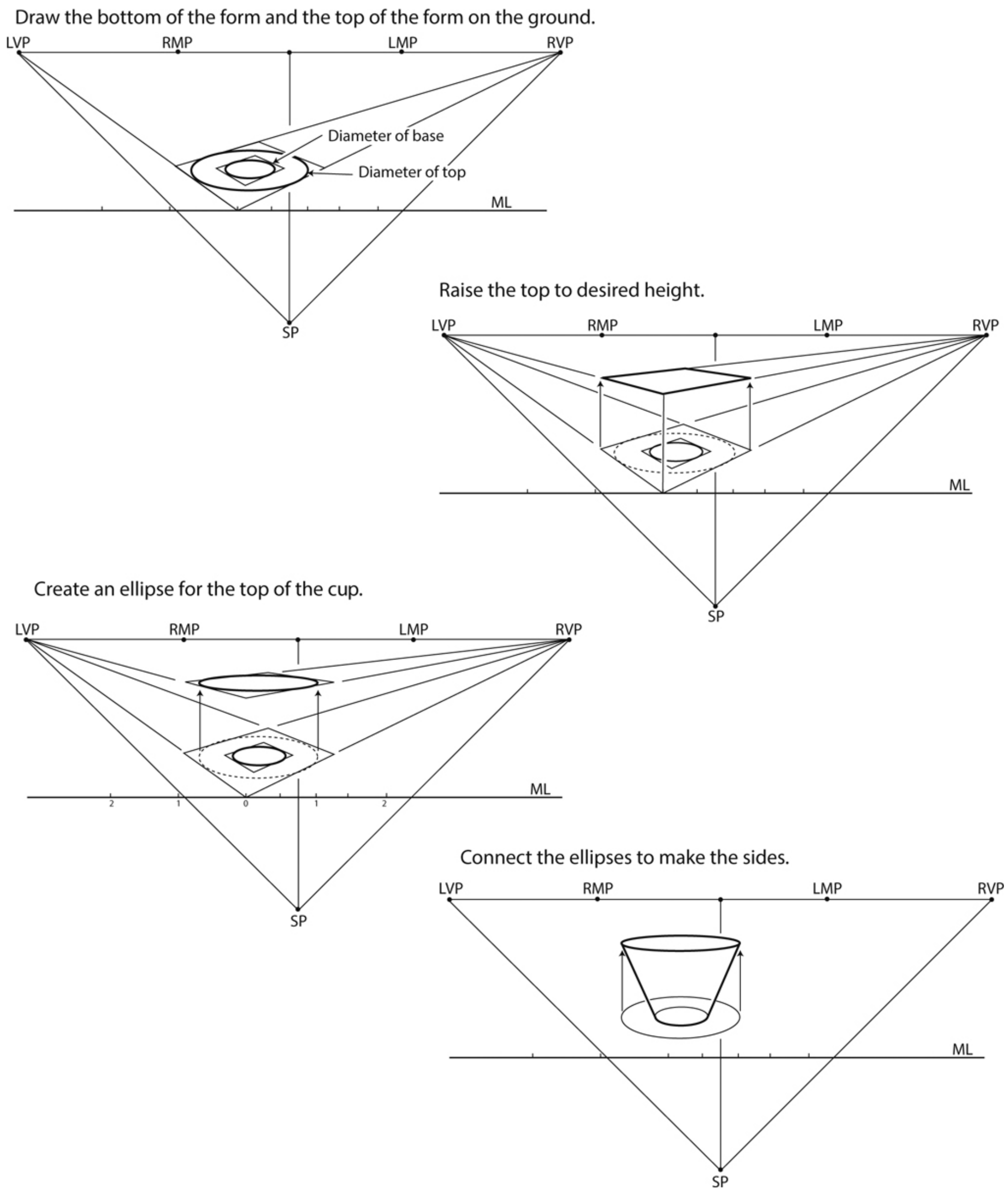


Figure 8.16 To draw tapered cylindrical forms, measure the diameter on the ground plane, then project the ellipse to the desired height and connect the ellipses.

Tapered Forms: Cups, Bottles, and the Like

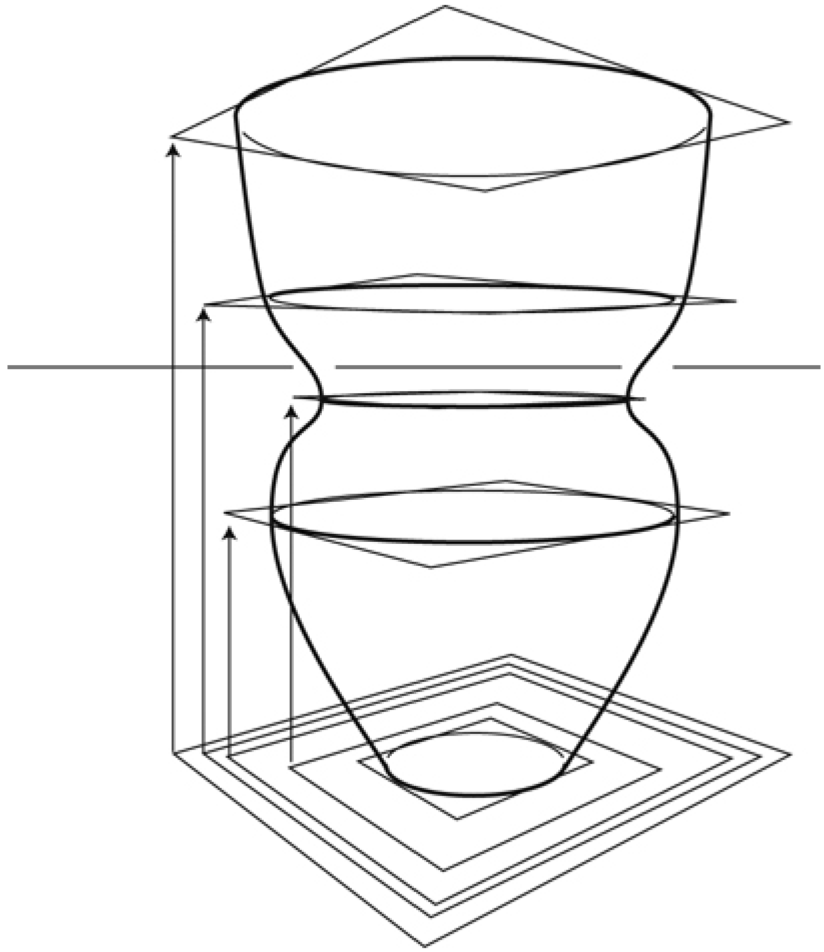


Figure 8.17 **Create squares on the ground plane, raise the squares to the desired height, draw ellipses, and then follow the contour to create a curved cylindrical shape.**

To draw cylindrical forms with various **diameters**, first draw squares of the appropriate size on the ground plane. Raise each square to the desired height. Then draw an ellipse in each square. For example, a simple tapered cup will have a smaller diameter ellipse on the ground and a larger diameter ellipse above. Draw both on the ground (Figure 8.16, top). Then project the top of the cup to the desired height. Connect the two ellipses to create the cup (Figure 8.16, bottom).

For complex forms of varying diameters, make more ellipses. The ellipses serve as key **cross-sections** and guide the contour of the form— the more ellipses, the more accurate the shape (Figure 8.17).

Spheres

Drawing a sphere in perspective is more complicated than one might think. A compass can be used to draw a circle, and considered finished. But if a specific size or placement for the sphere is desired, then a **cube** must be drawn first. The sphere fits into the cube touching the center of all six sides. The cube defines where the sphere touches the ground. The cube also defines the diameter of the sphere.

Distortion

Before discussing how to fit the sphere into the cube, some issues concerning the shape of the sphere will be explored. This may be surprising, but, in perspective, spheres are not necessarily round. In fact, perspective spheres are seldom round due to distortion. As an object moves away from the center of vision, it becomes distorted. This is unavoidable. A plotted sphere would be perfectly round at the center of vision. As the sphere moves farther away from the focus point, the distortion increases.

A look at conic sections can further explain this phenomenon. A conic section is the intersection of a plane and a cone. The silhouette of a sphere is circular. When that round shape is projected to the eye, the visual pyramid is conical. A cone intersecting a flat plane at an oblique angle creates an ellipse (Figure 8.18). Thus, a sphere plotted in perspective is not round unless its center is aligned with the focal point. A sphere plotted in perspective is an ellipse.

Any circular object not aligned with the center of vision is drawn elliptical by the rules of perspective. This elliptical shape will appear correct (circular) if seen from the position it was plotted. If the person looking at the drawing places their eye at the location of the station point, the ellipse will appear circular. But, if a viewer looks at the drawing from a place other than where the image was plotted, the perspective sphere will appear elliptical. It is difficult to control the position from which a viewer will look at the artwork. Ideally, spheres should look circular, not elliptical. The bottom line: use a compass to make spheres. When drawing spheres, it is better for them to look correct than to be correct.

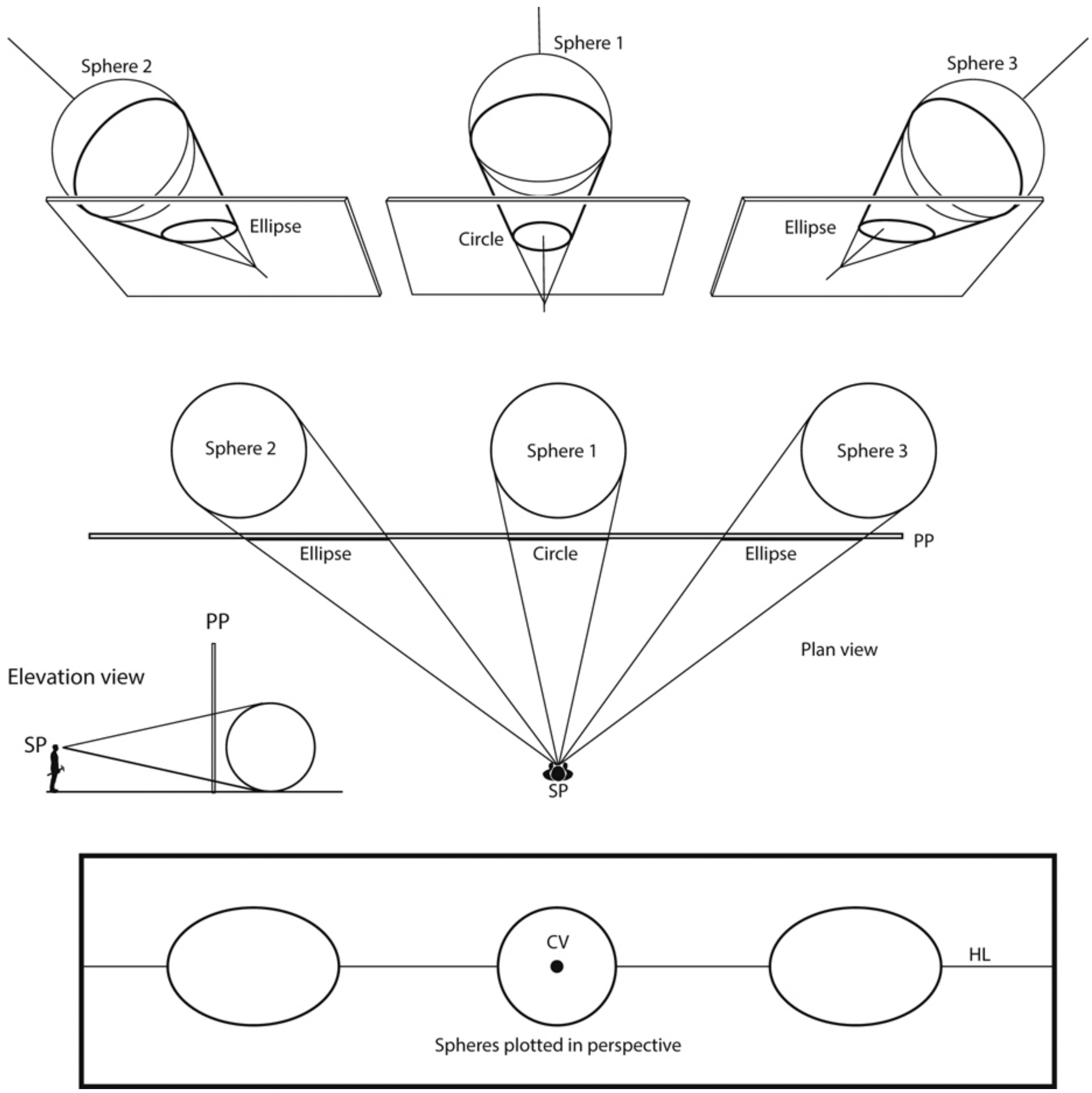
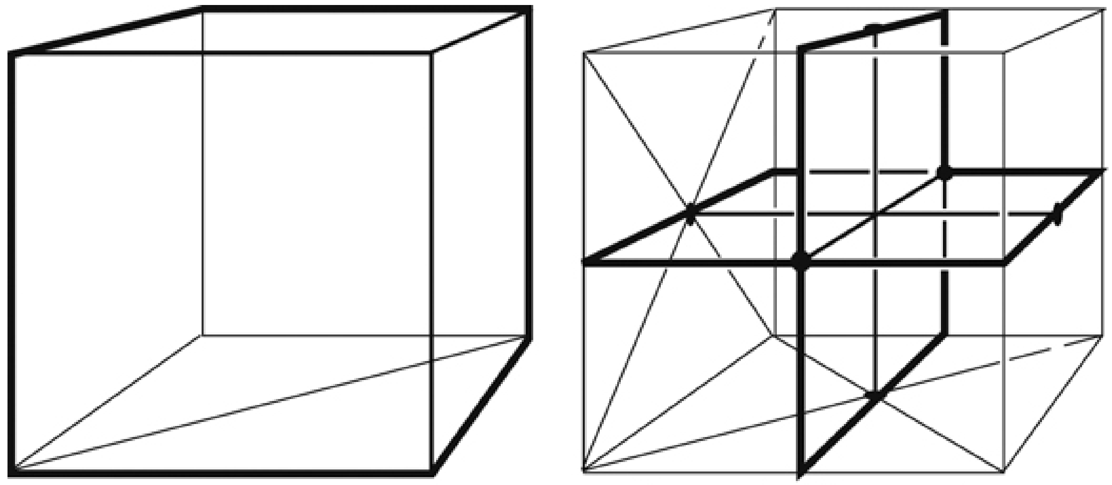


Figure 8.18 **When an object is seen obliquely, distortion is inevitable.**

Drawing a Cube

When using a compass to create a sphere, the sphere still needs to be a specific size and in a specific location. First, draw a cube and bisect it vertically and horizontally creating six touch points (Figure 8.19).



[Figure 8.19](#) **Locating the six touch points of the sphere**.

Draw an ellipse in the cross-sections ([Figure 8.20](#), left). The edges of the two ellipses indicate the diameter of the sphere ([Figure 8.20](#), right). These points are in perspective and will not align perfectly with a compass circle. So, this is where some "cheating" is required—use artistic license and draw the sphere with a compass.

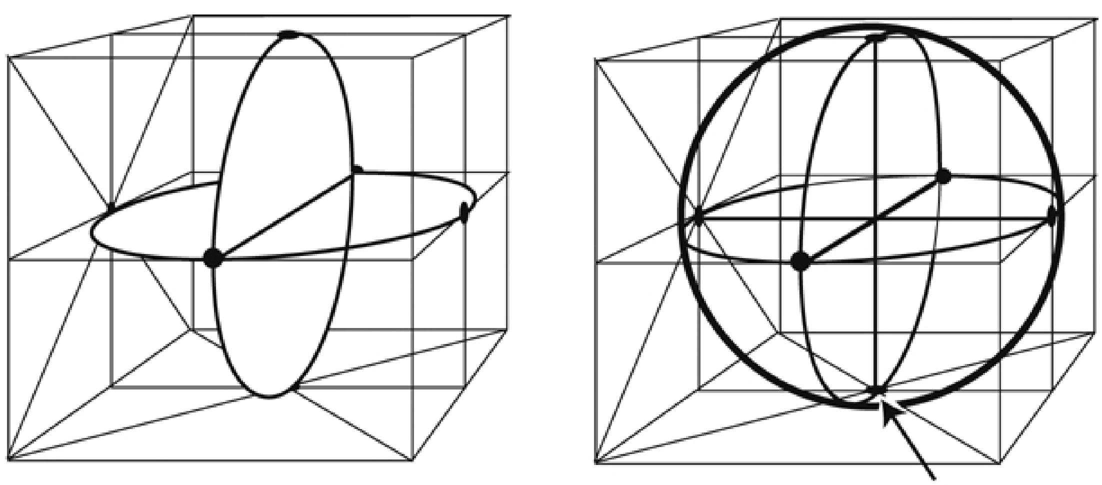


Figure 8.20 **When drawing a perspective sphere, the horizontal and vertical cross-sections determine the diameter of the circle.**

Spiral Forms

Draw all sinuous lines by plotting points along the curve. A spiral is a stretched circle. It moves around as it moves up. Spiral forms are based on a series of stacked ellipses. A point is plotted along each ellipse. Closer ellipses create a tighter spiral. First, draw an ellipse representing the diameter of the spiral. Then decide on the spacing of each ellipse (Figure 8.21).

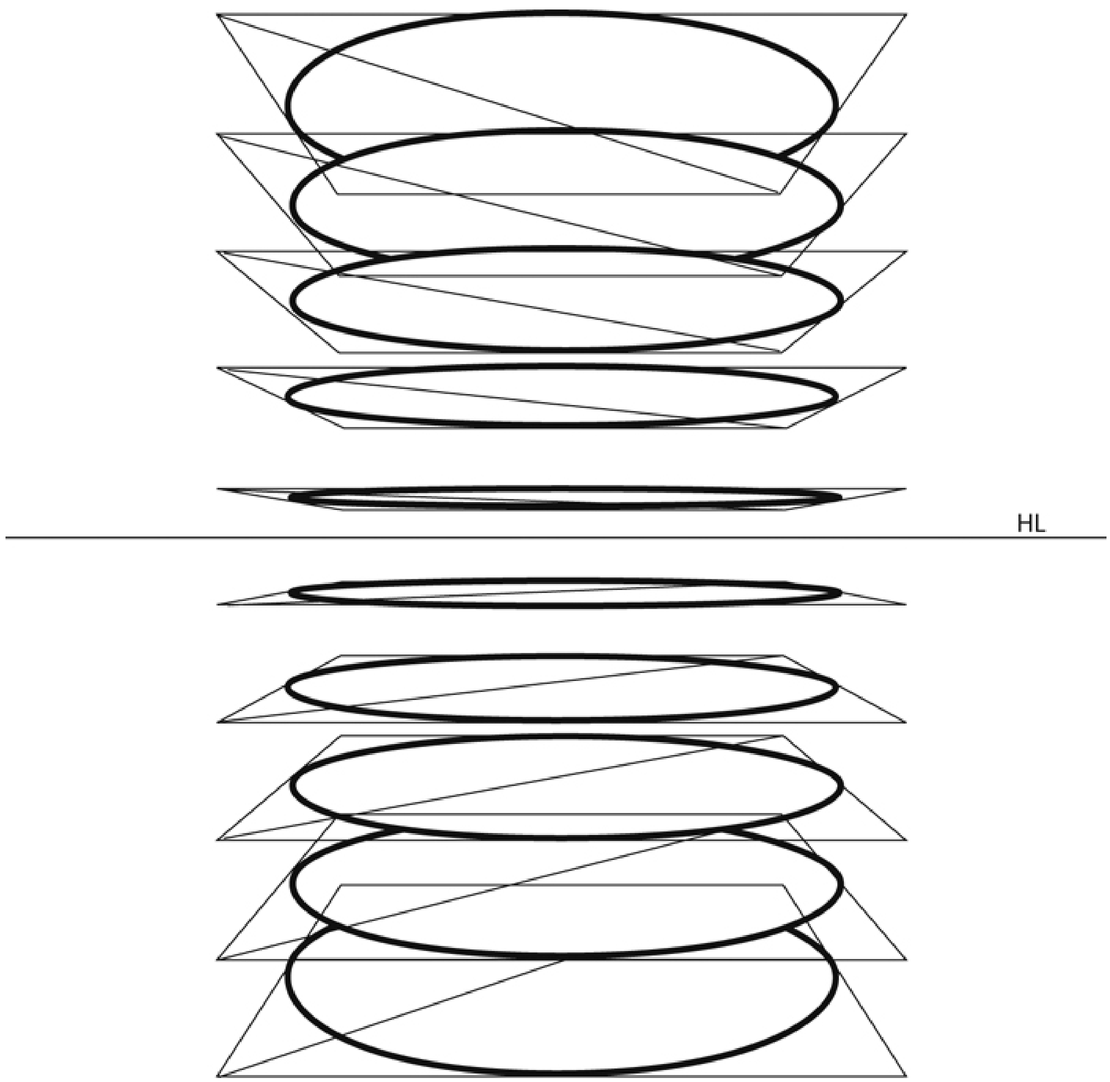


Figure 8.21 **This stack of ellipses looks tight, but the resulting spiral will be surprisingly stretched. The ellipses need to be very close to achieve a tight spiral.**

Each point on an eight-point ellipse represents one-eighth of a coil. A coil is one complete turn of a spiral. It takes nine points (nine ellipses) to create a complete coil.

Start at the bottom ellipse and pick a point to begin the spiral. Then, on the ellipse above, move one point (one-eighth of a turn) counter-clockwise (or clockwise depending on the rotation of the spiral). Repeat this process, moving up one ellipse, and over one-eighth a turn. As the point moves up, it also moves around the circle (Figure 8.22). Connect the dots, resulting in a spiral (Figures 8.23–8.24). The method is not difficult, it is just time-consuming. It is also dense with lines. All spiral forms (springs, barbershop poles, candy canes, spiral staircases, etc.) are done using this basic technique.

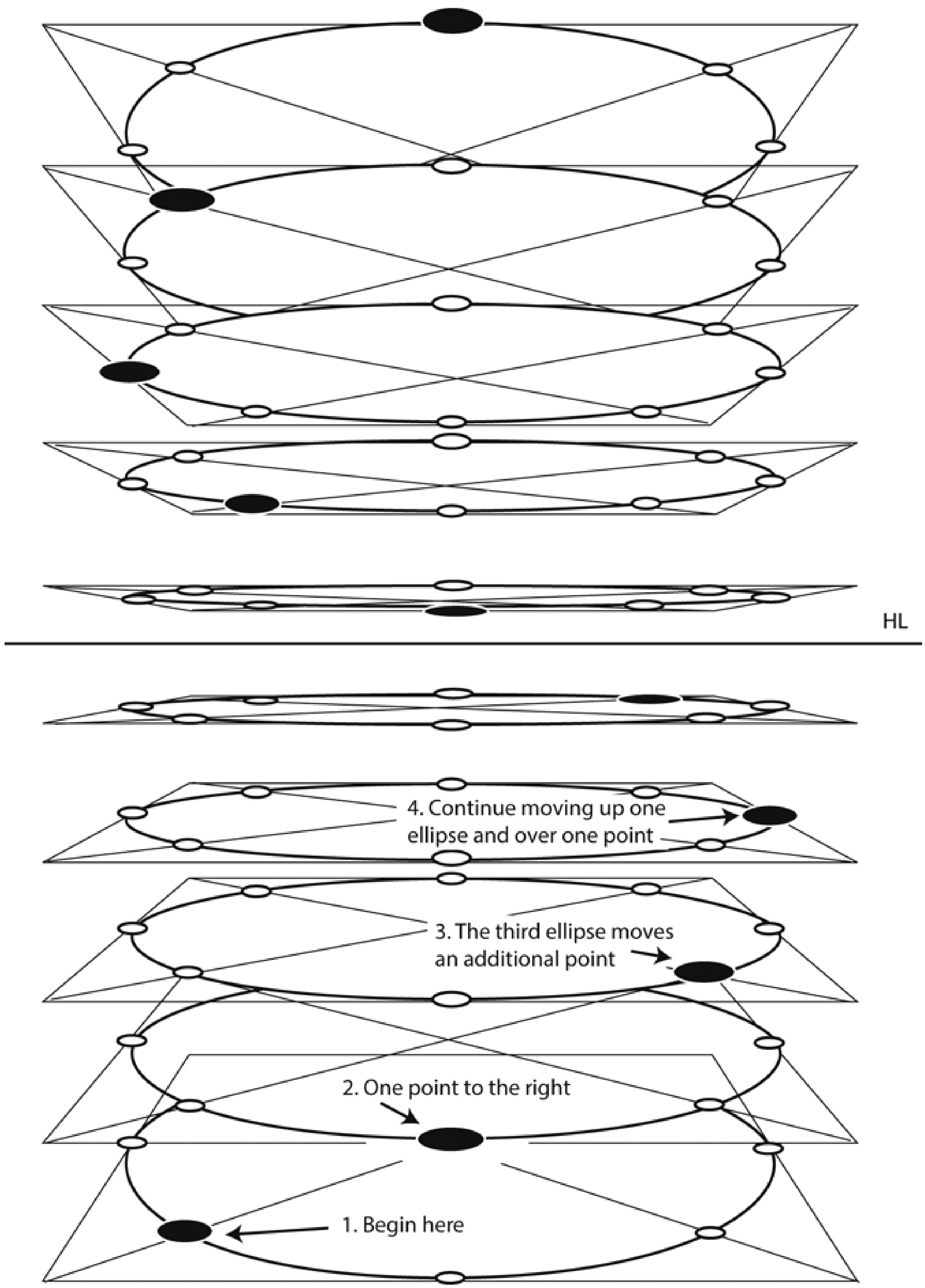
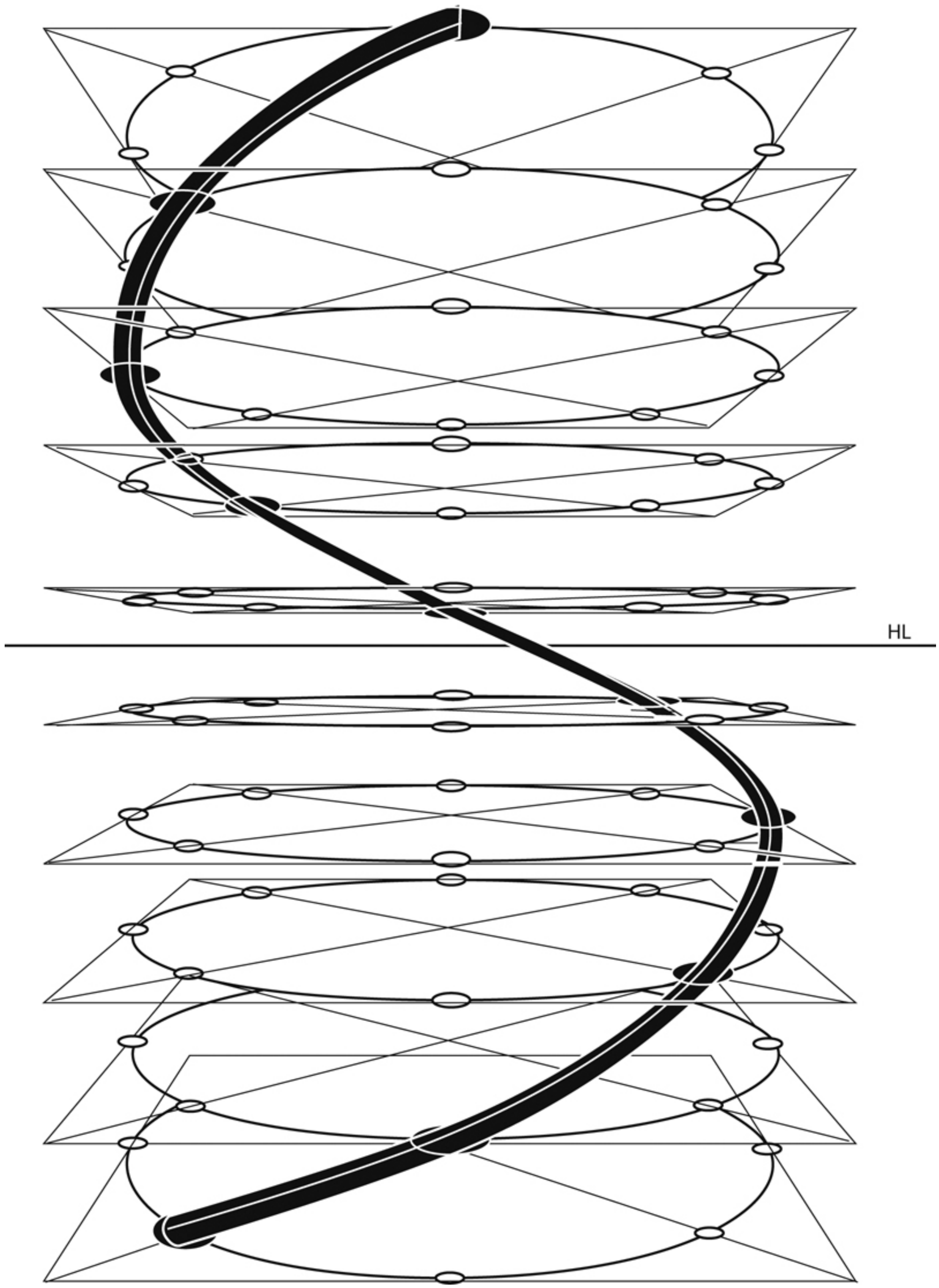


Figure 8.22 The solid dots represent points along the spiral. Each dot moves up one level and counter clockwise one-eighth of a turn. Using an eight-point ellipse, nine ellipses will make one complete coil.



[Figure 8.23](#) **Connect the dots to create a spiral. Add thickness to the spiral to create a spring.**

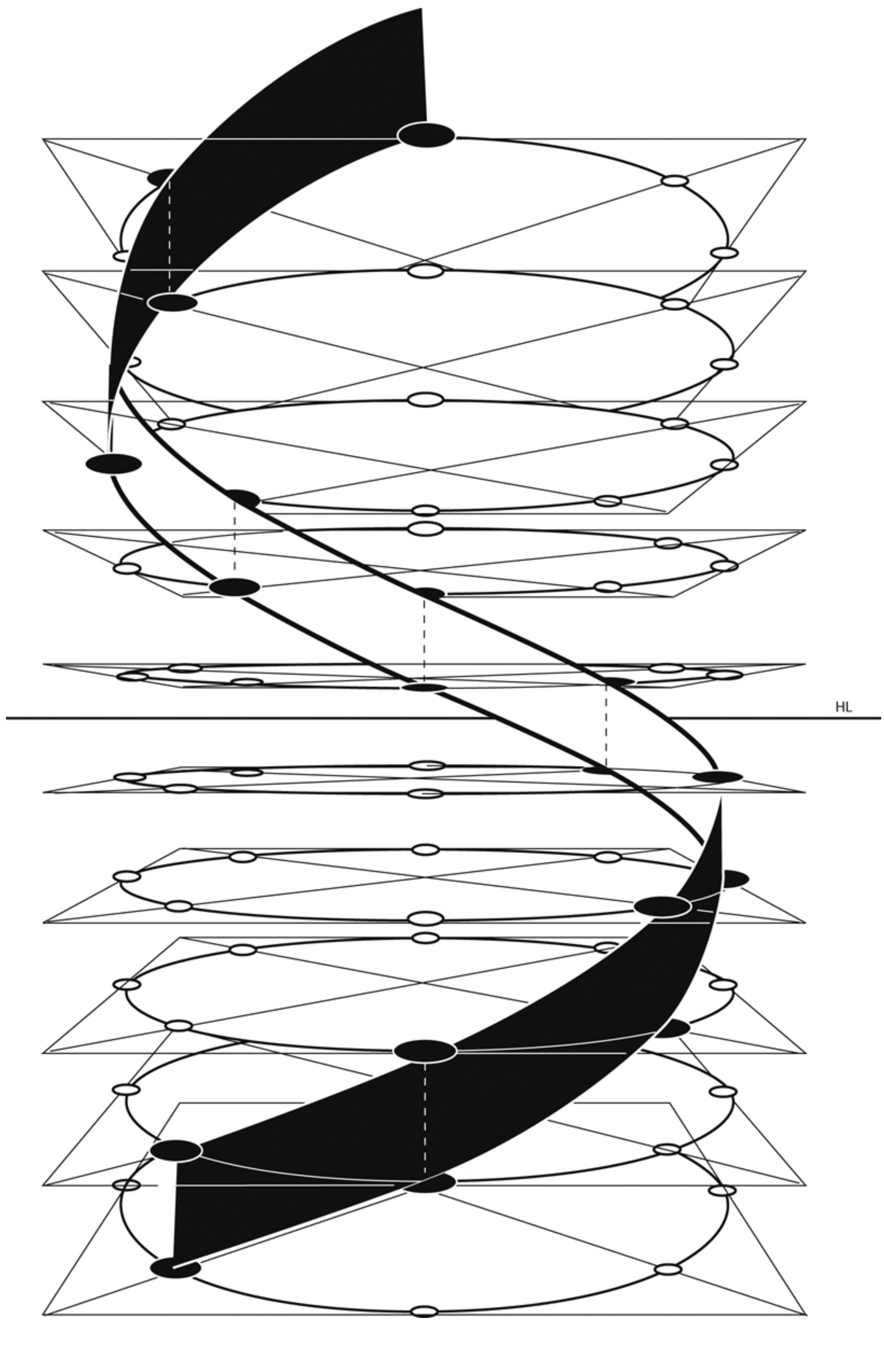

HL

Figure 8.24 **Create another spiral above the first to make a ribbon, candy cane, or barbershop pole.**

Spiral Staircase

Drawing a spiral staircase follows the same principles as any spiral form. A spiral staircase is a series of triangles moving up and around a circle. First, decide on the size of the steps. Divide the ellipse like a slice of pie. Each slice represents one step. Draw one step. Then draw another, one level above the first. The third step is one level above the second, and so on. Each step moves to the left and up one level (spiral staircases always go up clockwise). The spiral staircase will turn and rise as each step is built (Figure 8.25). Continue adding steps. Draw handrails by following the same guidelines (Figure 8.26).

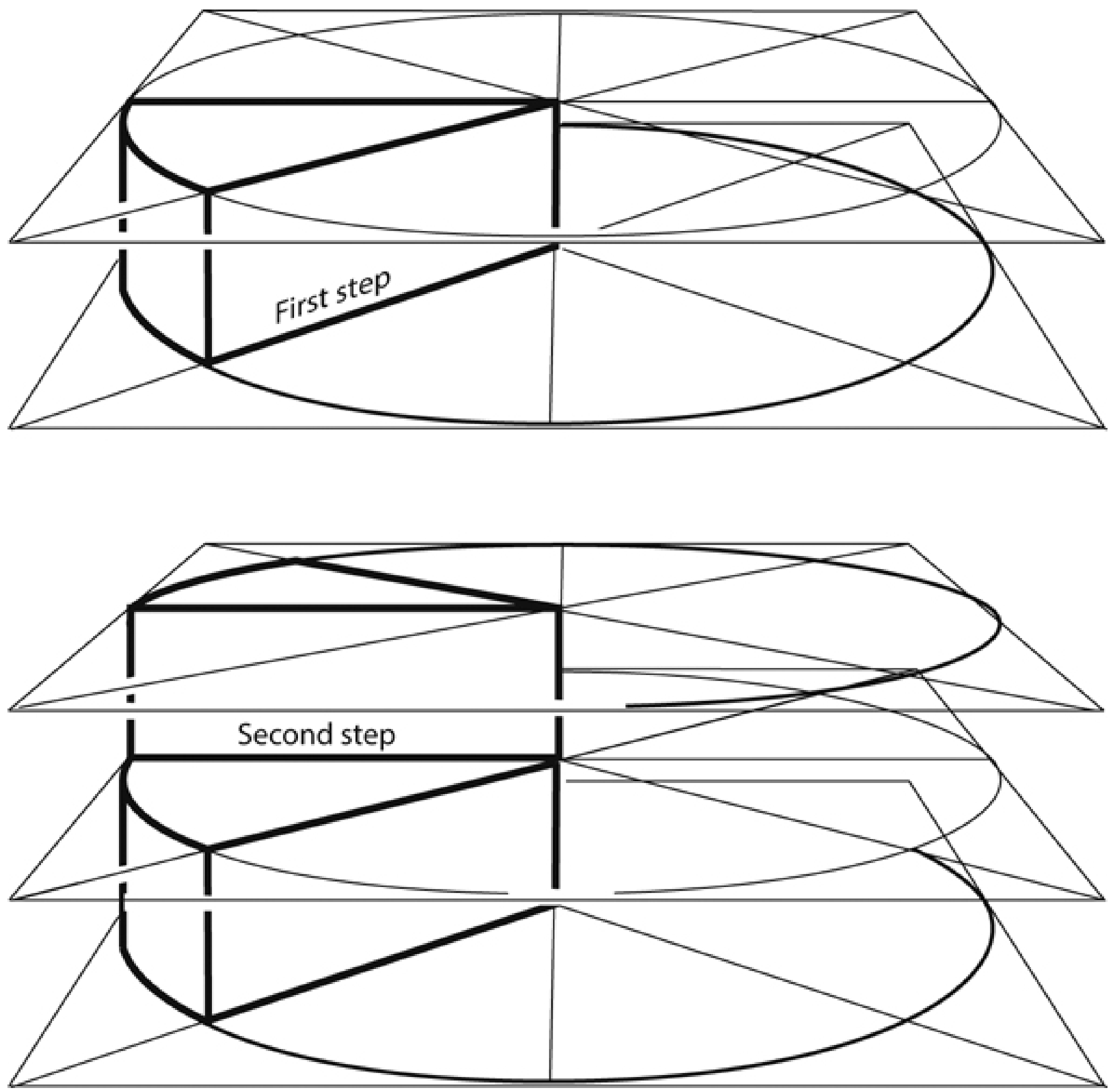
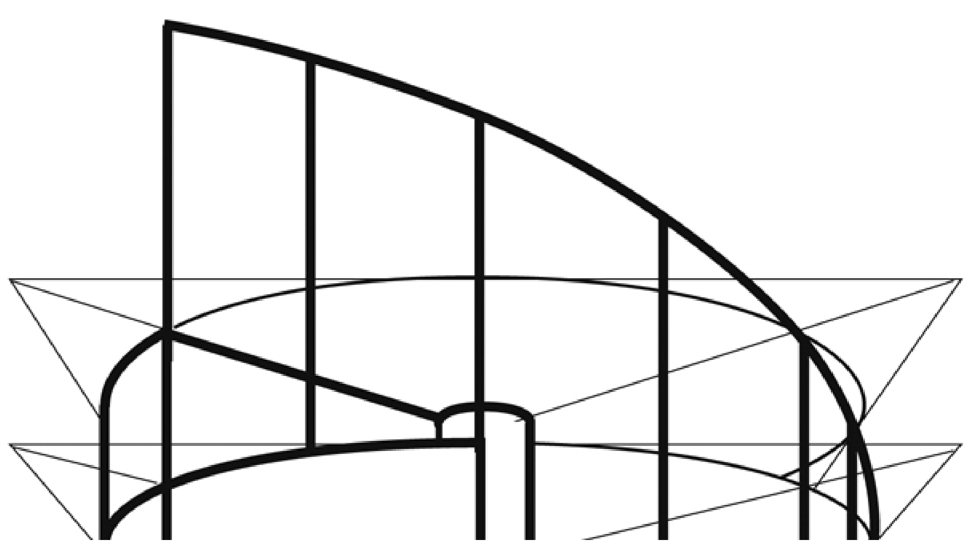


Figure 8.25 Build a spiral staircase one step at a time.



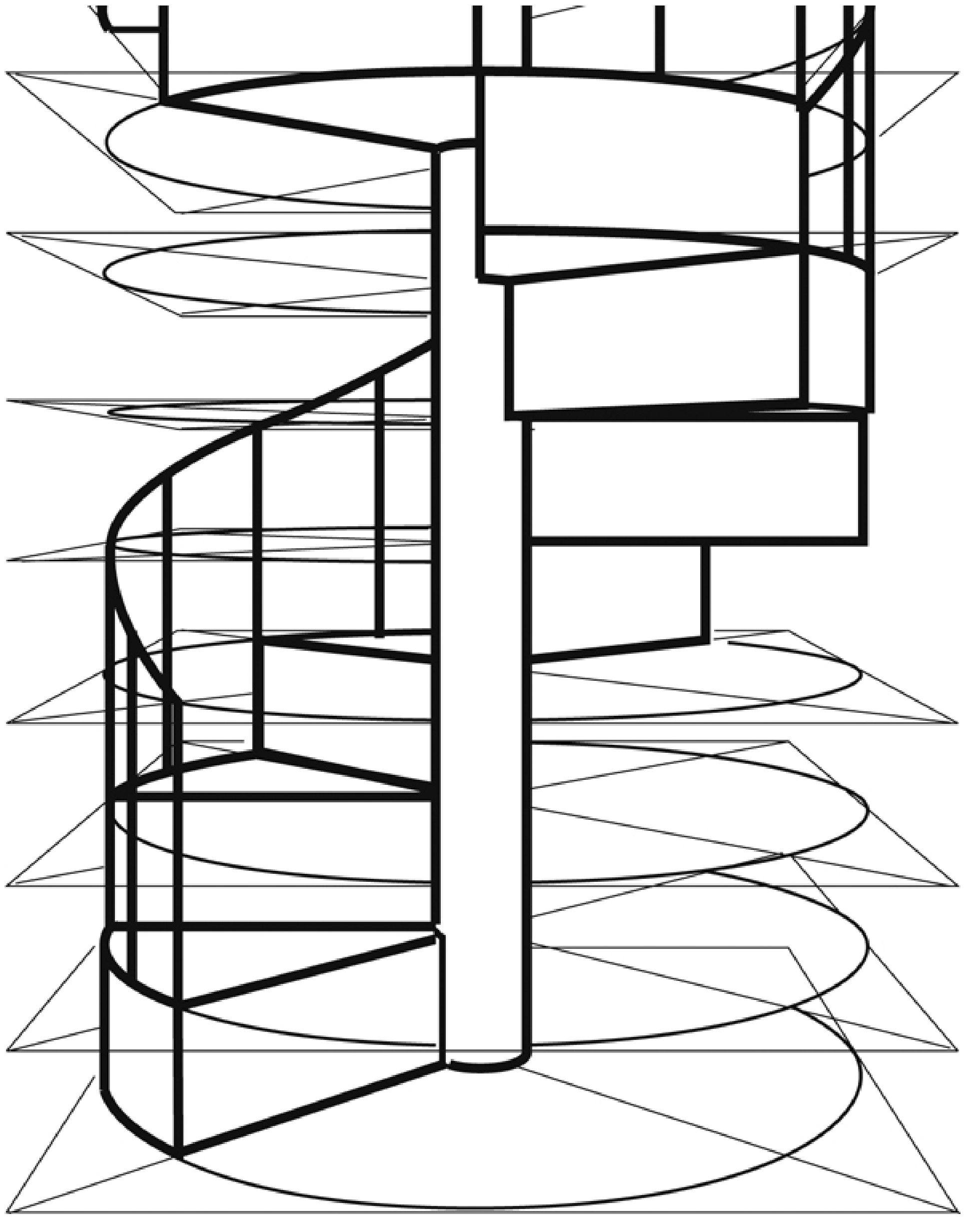


Figure 8.26 **Add a handrail to finish the staircase.**

Random Curves

The solution to drawing sinuous undulating lines where one shape flows seamlessly into another is not complicated. It is more tedious than elegant. Drawing curved lines involves plotting points along the curve and then connecting the dots. The more points that are plotted, the more accurate the curve. First, draw the serpentine shape in a plan or elevation view, a view without perspective. Measure key points along the curve. Then transfer those dimensions to a perspective view (Figure 8.27).

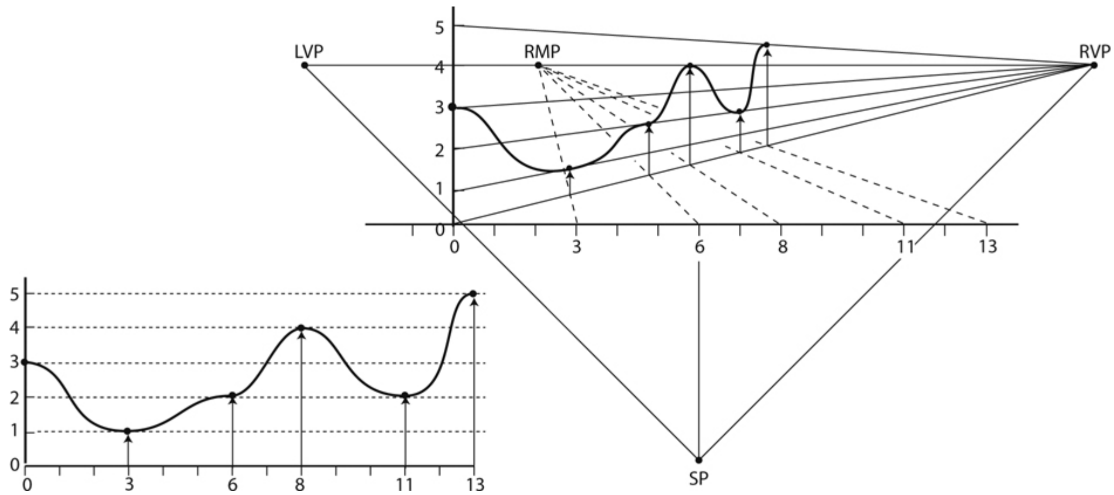


Figure 8.27 This curved line was first drawn in an elevation view (lower left). Key points along the curve were measured. Those points were then plotted in perspective. Connect the points to create a perspective curve (upper right).

9
Inclined Planes

Inclines continued to challenge artists long after the foundations of perspective had been established. When artists discovered that foreshortened lines connect to vanishing points *on* the horizon, it was a difficult concept to shed. Artists who attempted to draw inclines using points on the horizon line were met with curious results. The shapes looked tapered (Figure 9.1). This outcome made it obvious that something was amiss. Eventually it was reasoned that inclines connect to points *above* or *below* the horizon. Vanishing points on the horizon line draw lines parallel with the ground plane. Vanishing points above or below the horizon line draw lines at an angle to the ground plane.

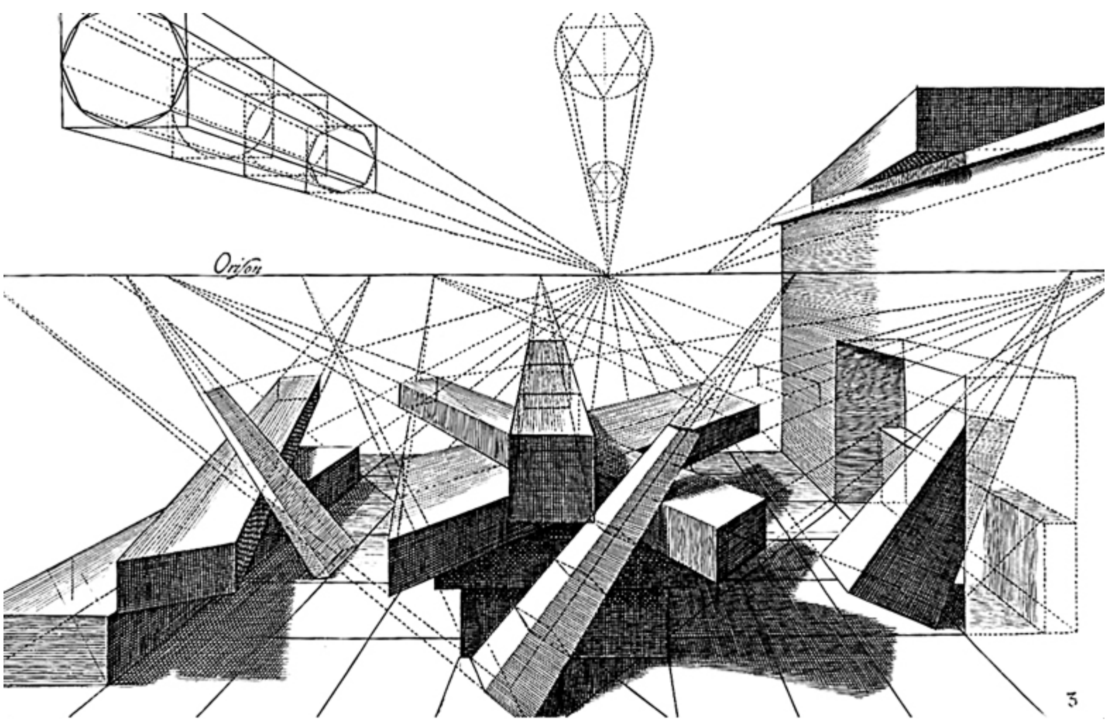
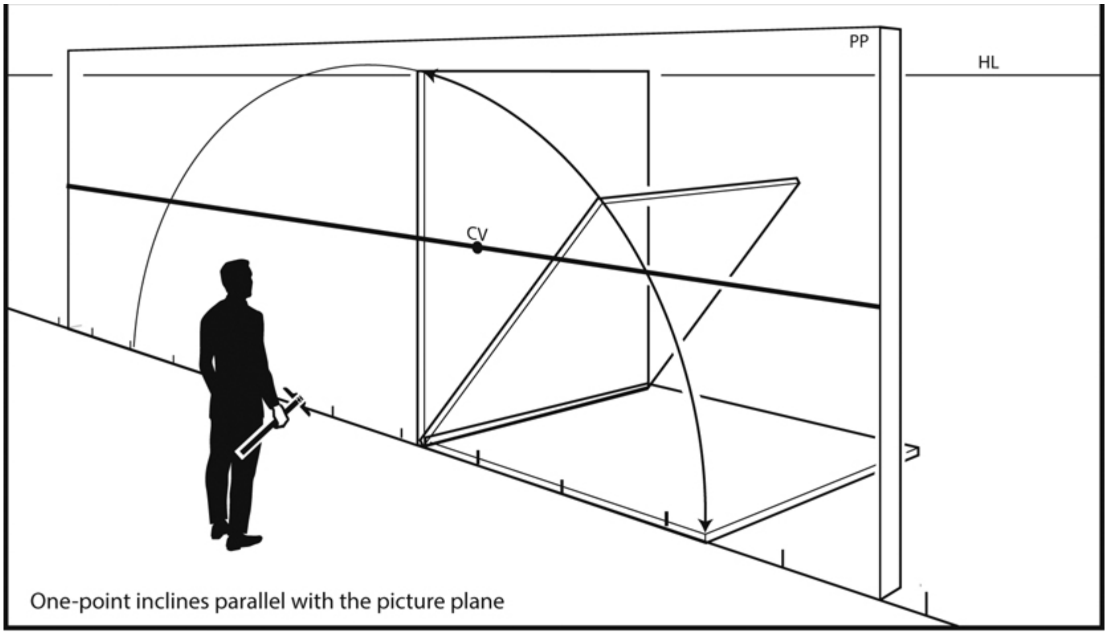
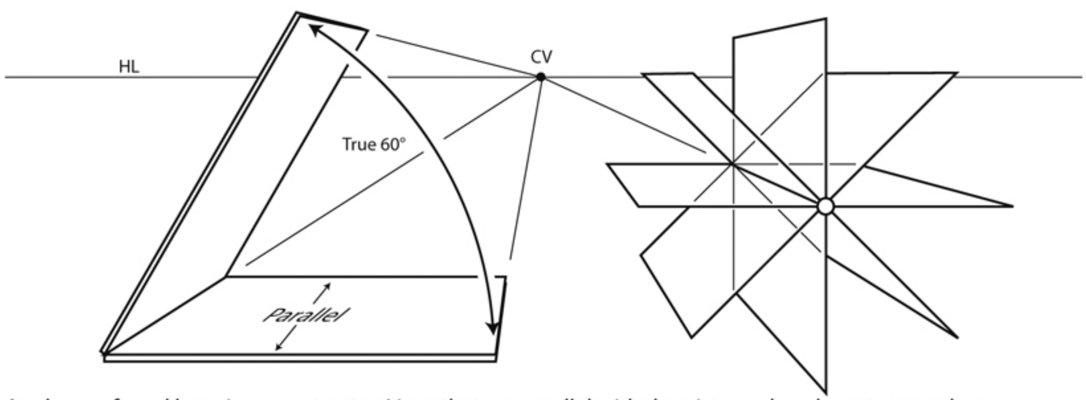


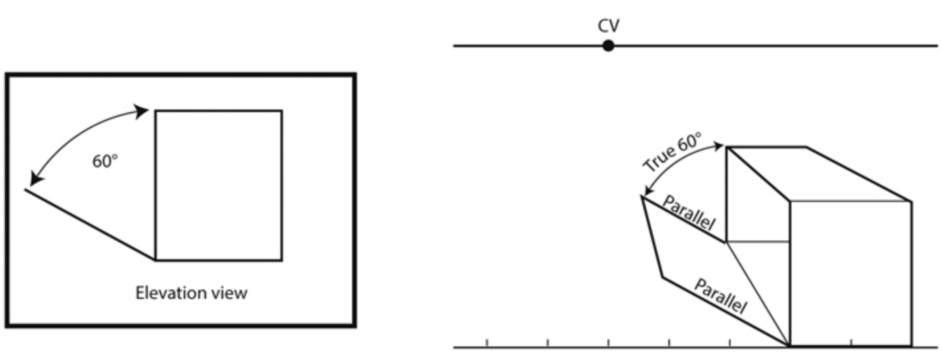
Figure 9.1 Jan Vredeman de Vries, *Perspective*, 1604. Notice the vanishing points for the inclines are erroneously placed on the horizon line.



One-point inclines parallel with the picture plane



Angles are found by using a protractor. Lines that are parallel with the picture plane have true angles.



This incline is not foreshortened. The angle is a true 60°.

Figure 9.2 Angles parallel with the picture plane have no vanishing points and are drawn as true angles.

One-Point Inclines

There are two types of one-point inclines: angles that are foreshortened (lines that are not parallel with the picture plane), and angles that are foreshortened (lines that are parallel with the picture plane). Angles parallel with the picture plane have no vanishing point. They are drawn as true angles (Figure 9.2). These are the simplest, yet the most deceiving, as many erroneously attempt to locate a vanishing point where none exists.

Lines angling toward or away from the viewer are foreshortened. These angles have vanishing points. These vanishing points are located above or below the horizon line and are called auxiliary vanishing points (AUX. VP). One-point perspective auxiliary vanishing points are aligned vertically with the center of vision (Figure 9.3).

Upward Inclines

Whether a slope angles up or down depends on the point of view. If standing at the top of the slope, the incline angles down. If standing below, the incline angles up. Inclines with auxiliary vanishing points above the horizon line are referred to as upward inclines, and inclines with auxiliary vanishing points below the horizon line as downward inclines (Figure 9.3).

The farther away the auxiliary vanishing point is from the horizon line, the steeper the incline. To draw an incline at a specific degree, a point for true angles is needed.

Vanishing Point

True angles for horizontal lines are found at the station point. The station point is used for angles parallel with the ground plane. Inclines can't use the station point. True angles for inclines are found at the measuring point. Any angle drawn at the measuring point creates an auxiliary vanishing point that draws that same angle in perspective. For example, if the desired slope is 30°, use a 30° angle at the measuring point. Extend that angle until it intersects a point directly above (or below, depending on the incline) the center of vision (Figure 9.4–5).

Auxiliary Measuring Point

To measure the incline requires a measuring point. Geometry determines its placement. From the auxiliary vanishing point, draw a horizontal line. This line is called an auxiliary horizon line (AUX. HL). The auxiliary horizon line functions as a placeholder for the measuring point. The auxiliary measuring point is placed on this line.

Measure the distance from the auxiliary vanishing point to the one-point measuring point. Transfer that distance to the auxiliary horizon line (Figure 9.6).

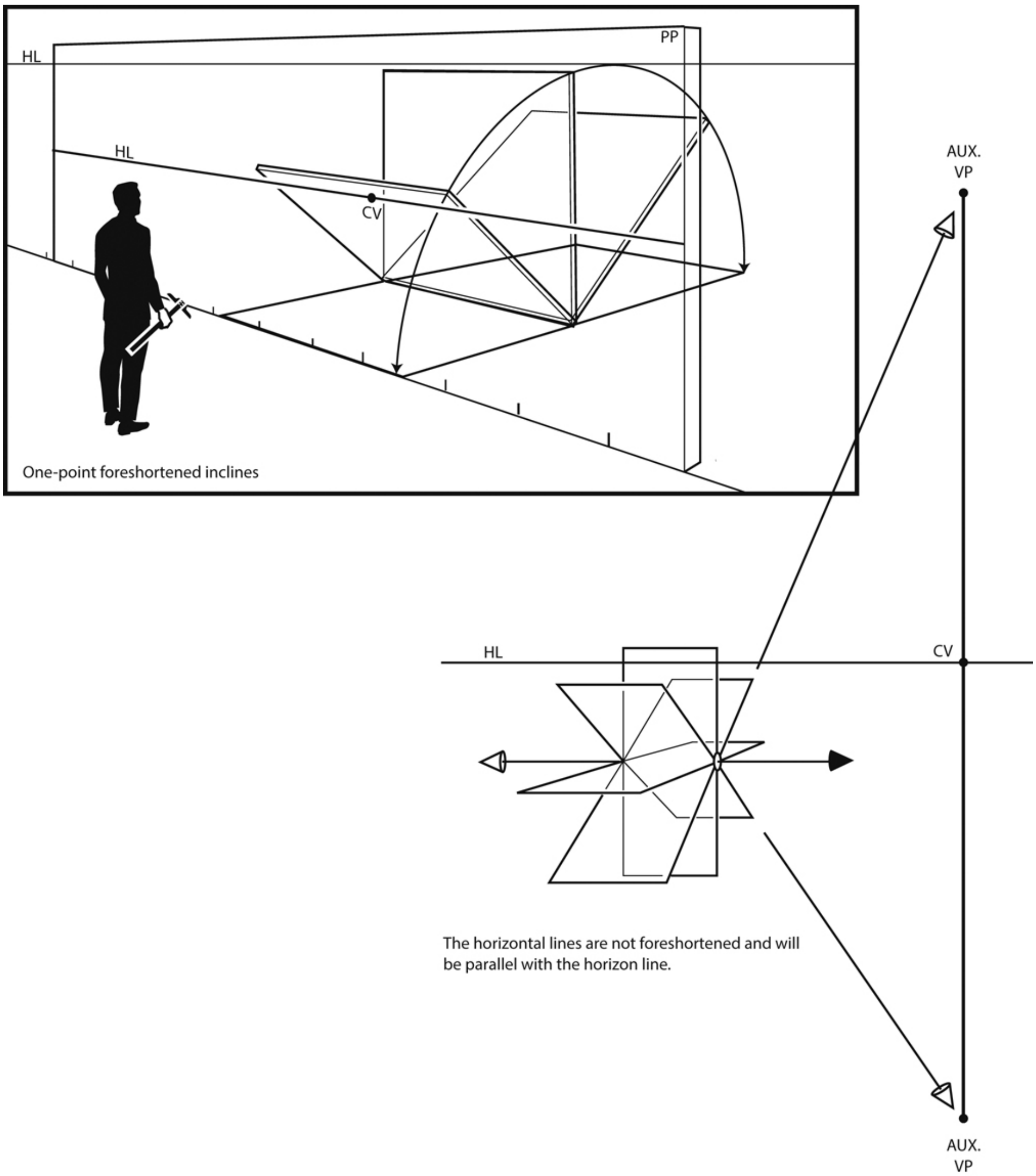


Figure 9.3 Inclines not parallel with the picture plane are foreshortened and have vanishing points above or below the center of vision.

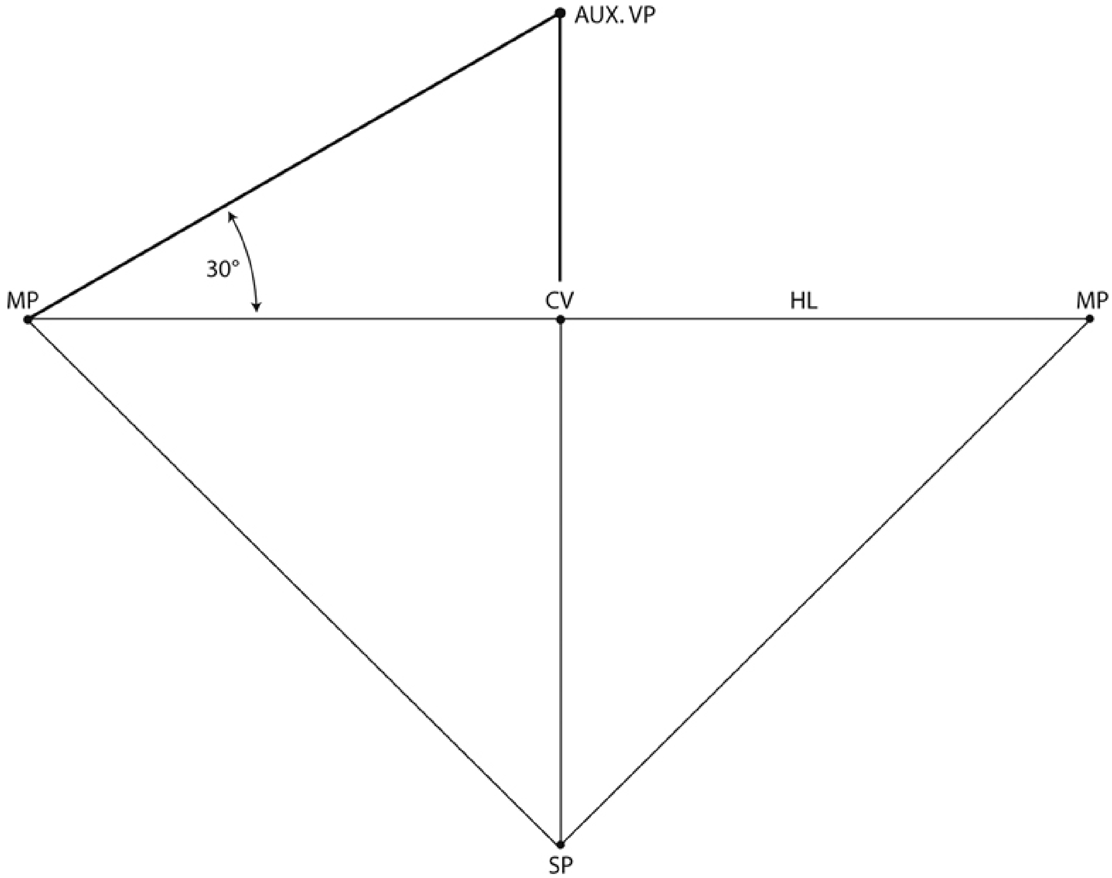


Figure 9.4 The measuring point is the true angle for inclines. Use the measuring point to establish the location of auxiliary vanishing points.

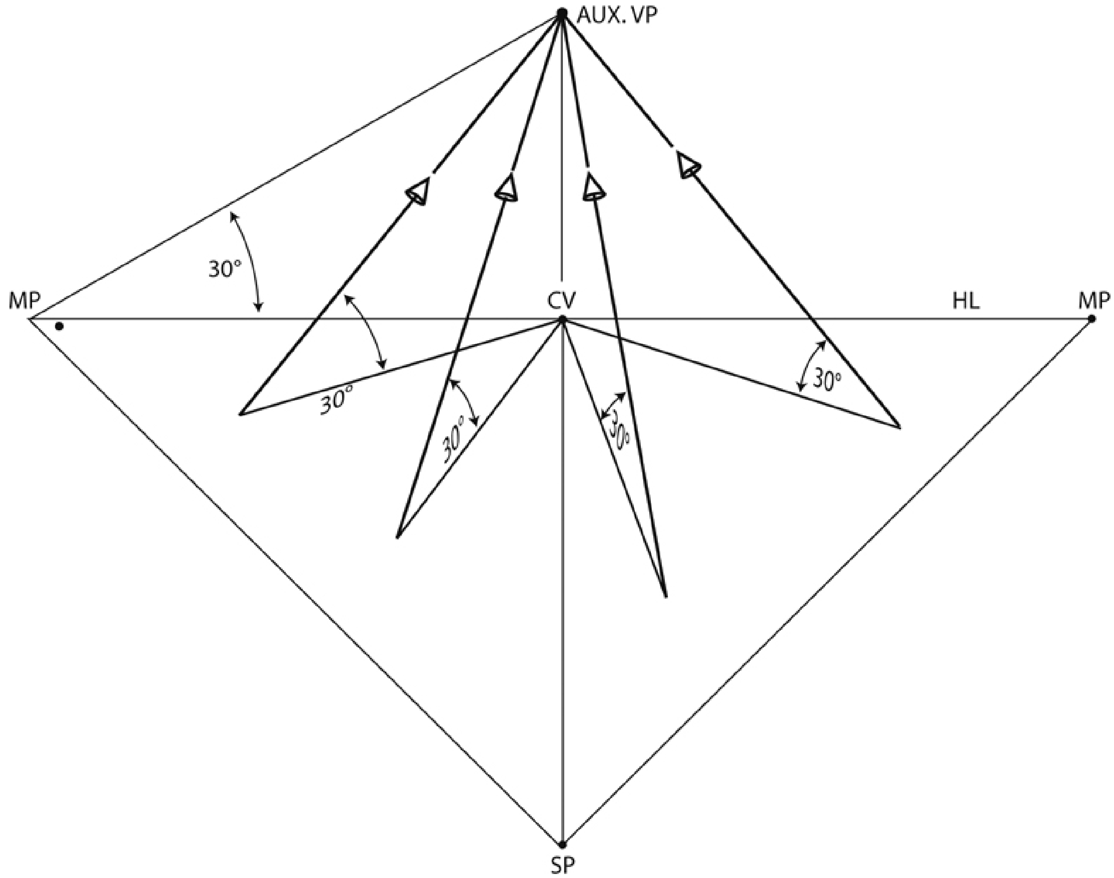


Figure 9.5 Any line drawn from the 30° auxiliary vanishing point creates an incline 30° from the ground plane.

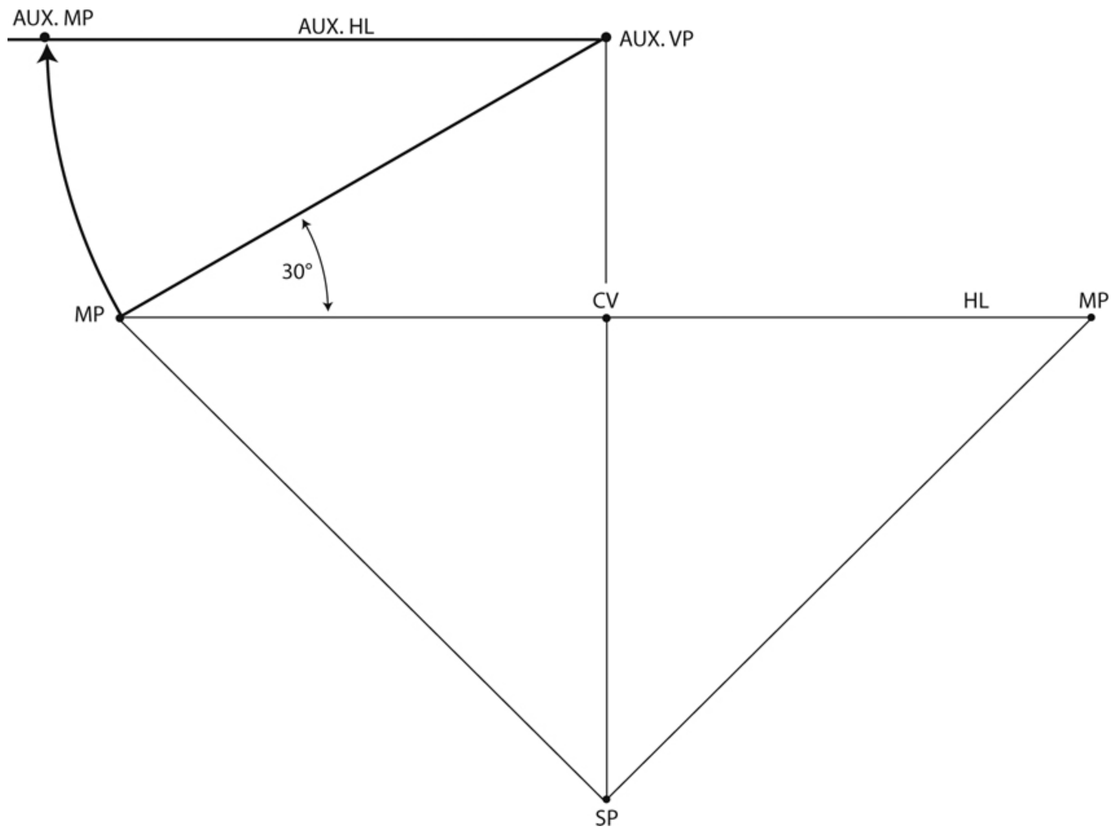
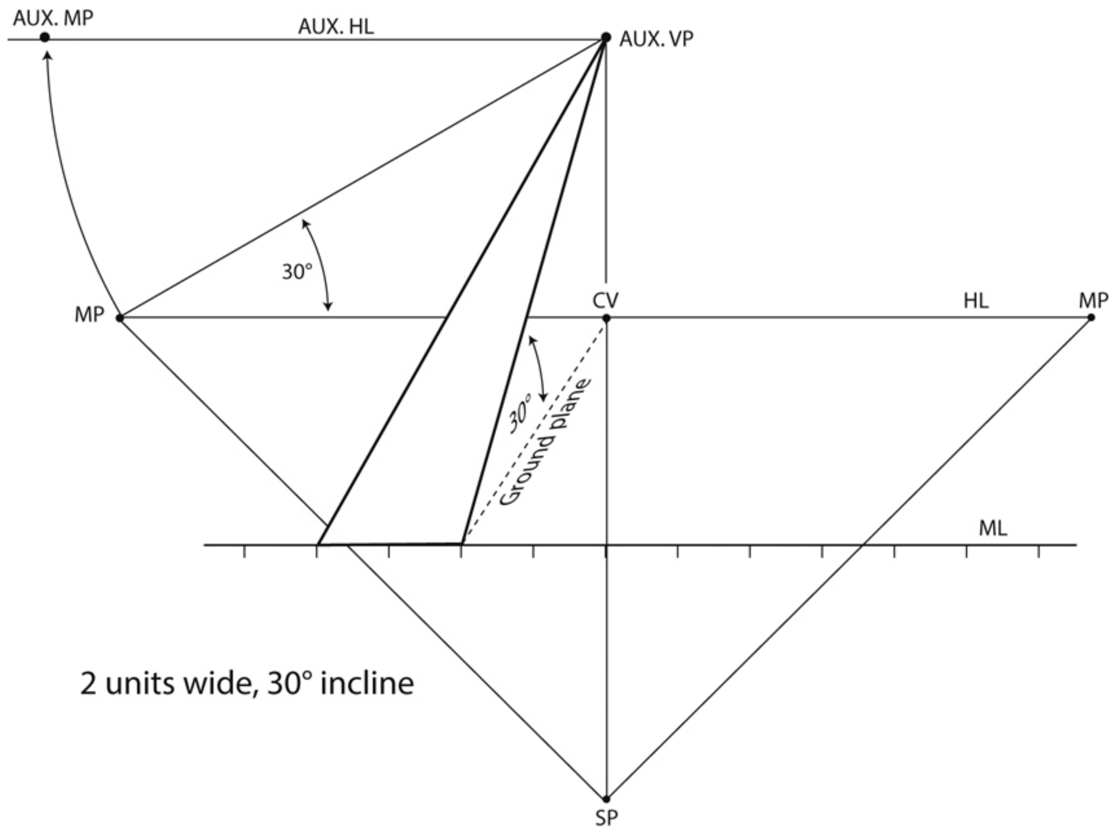


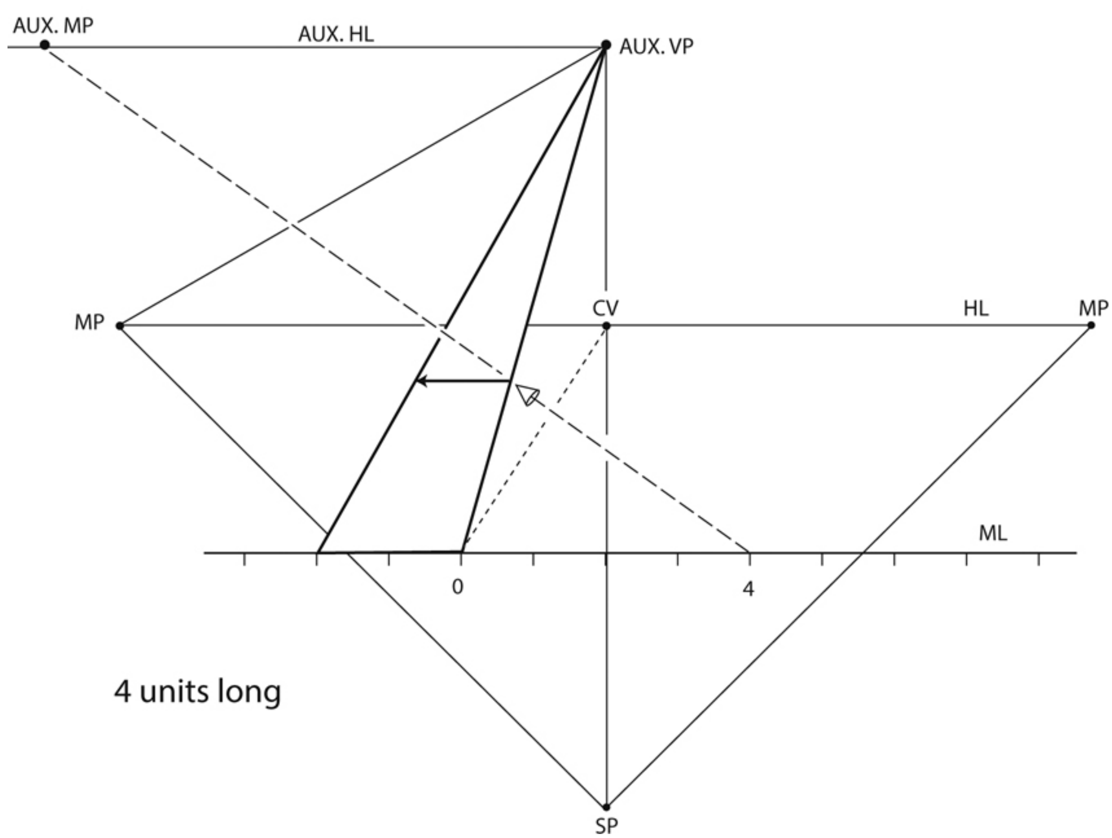
Figure 9.6 The distance from the auxiliary vanishing point to the measuring point is the same as the distance from the auxiliary measuring point to the auxiliary vanishing point.

Measuring the Incline

There is an important difference between measuring a line parallel with the ground plane and measuring a line angled to the ground plane. When measuring inclines, the measuring line must touch the line being measured (Figure 9.7). The geometry required to measure lines will be incorrect if this requirement is not met (see Chapter 11). It is often necessary to use a reference point to reposition the measuring line.



2 units wide, 30° incline



4 units long

Figure 9.7 **Measuring one-point inclines. Creating a 30° incline (above), and measuring a 4 unit length (below).**

Downward Inclines

Downward inclines have an auxiliary vanishing point below the horizon line. The steps are the same as upward inclines, except they are done upside-down. Use the measuring point to establish the auxiliary vanishing point (Figure 9.8). Plot the auxiliary measuring point (Figure 9.9). Draw the incline (Figure 9.10). Measure the incline (Figure 9.11).

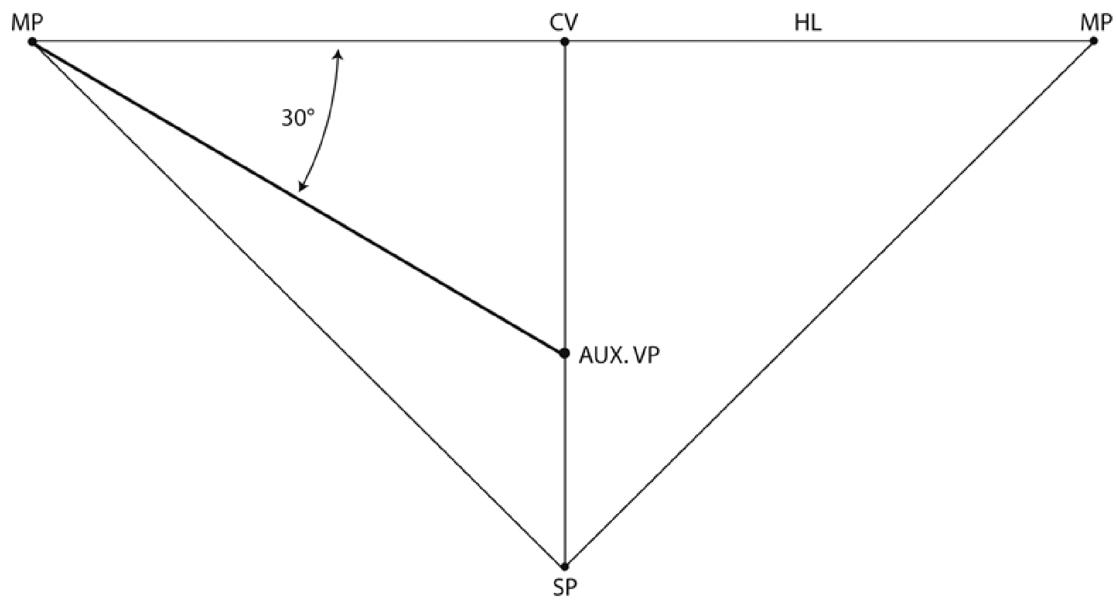


Figure 9.8 **Use the measuring point to find the auxiliary vanishing point.**

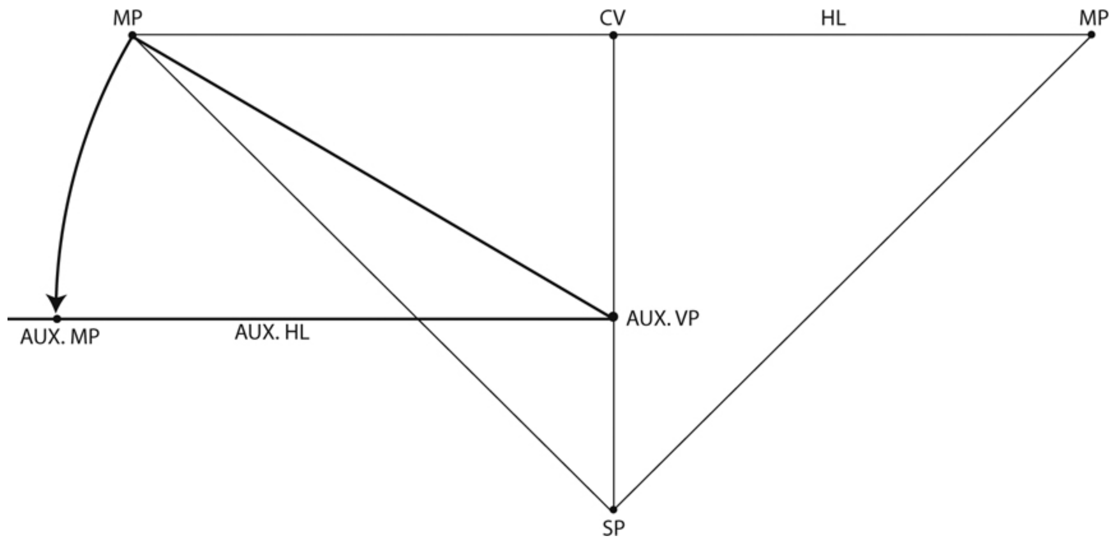


Figure 9.9 Create an auxiliary horizon line, then measure the distance from the auxiliary vanishing point to the measuring point and transfer that distance to the auxiliary horizon line.

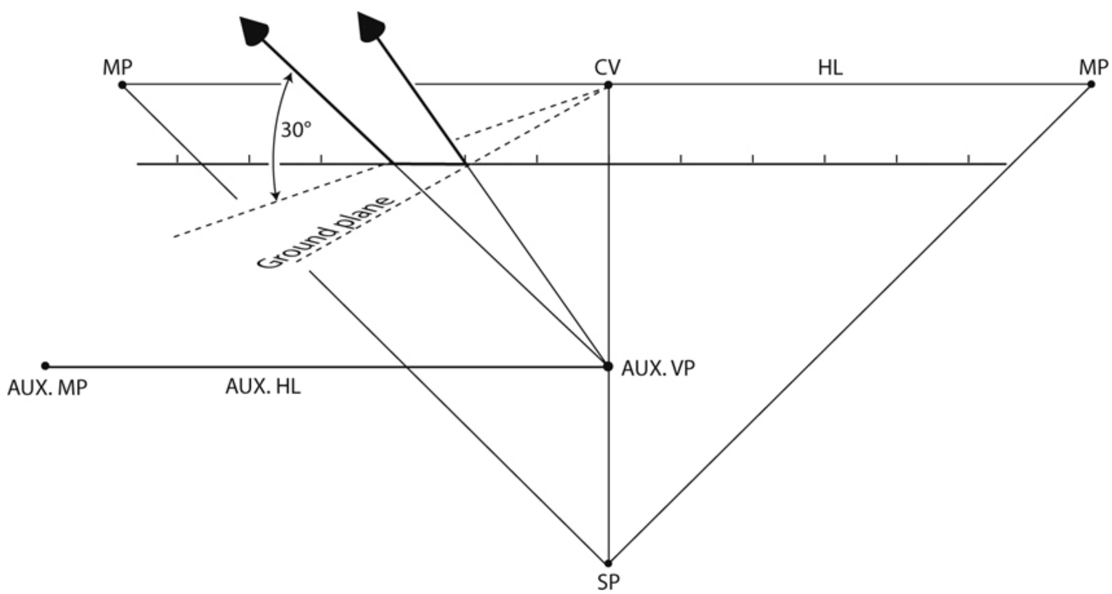


Figure 9.10 This auxiliary vanishing point draws inclines at a 30° angle to the ground plane.

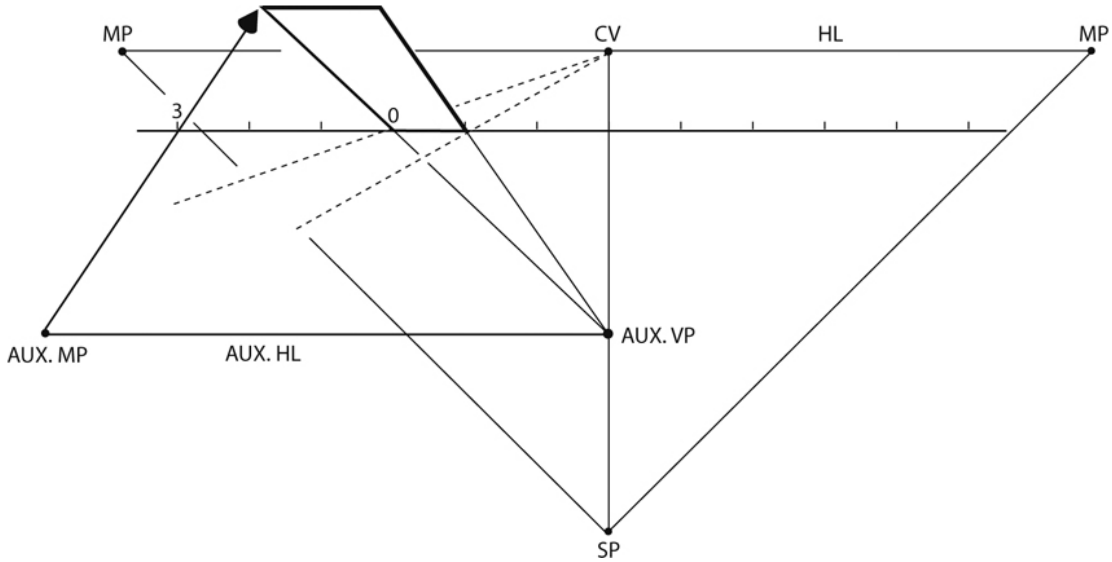


Figure 9.11 Use the auxiliary measuring point to measure the incline. Horizontal lines are parallel with the horizon line.

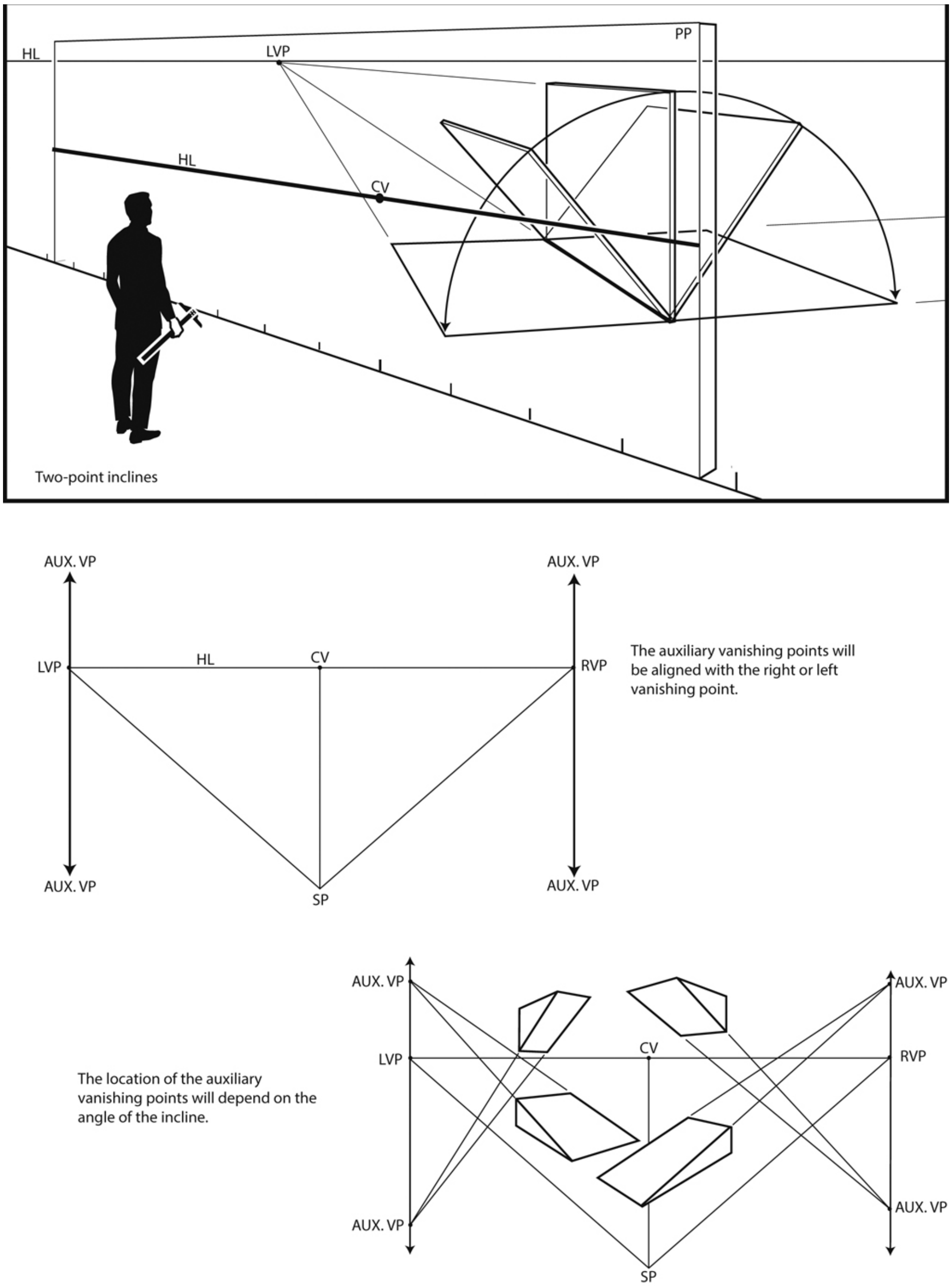


Figure 9.12 Two-point perspective auxiliary vanishing points are aligned with the left or right vanishing point.

Two-Point Inclines

Auxiliary vanishing points are aligned with vanishing points on the horizon line. In two-point perspective there is a left and a right vanishing point. So, which one should be used? Is the auxiliary vanishing point aligned with the right vanishing point or the left vanishing point? It can be confusing. More choices mean more opportunities to make mistakes. Fortunately, when a mistake is made, it is usually obvious and the drawing looks amiss. When this happens, it is time to reevaluate the choices made. With practice, choosing the location of the auxiliary vanishing point becomes intuitive. Until that time, there are guidelines to assist in their selection.

Placement

Auxiliary vanishing points can be above or below the left or right vanishing point. As a guide, consider how inclines recede in space. Think about **diminution**. Things closer to the viewer are larger; things farther away are smaller. How forms behave in perspective gives important clues for correctly locating the auxiliary vanishing point (Figure 9.12).

Another helpful hint in locating the auxiliary vanishing point is to consider the incline's axis. If the axis of an incline connects to the left vanishing point, the auxiliary vanishing point must be above or below the *right* vanishing point. Conversely, if the axis of an incline is aligned with the right vanishing point, then the auxiliary vanishing point must be on the *left* side (Figure 9.13).

Practice drawing inclines until confident with the correct positioning of the auxiliary vanishing points. When comfortable that these points can be properly located, it is time to draw specific angles.

Upward Inclines

Once the proper location of the incline's vanishing point is determined, the next step is to find the incline's angle, or the degree of the slope.

Measuring points provide true angles for inclines. If the auxiliary vanishing point is above or below the right vanishing point, use the right measuring point to find the true angles. Conversely, if the auxiliary vanishing point is above or below the left vanishing point, use the left measuring point to find the true angles. For example, to draw a 30° incline that tilts up and to the right, draw a true 30° angle from the right measuring point (Figure 9.14).

Auxiliary Measuring Point

Establishing an auxiliary measuring point follows the same procedure used in one-point perspective. Create an auxiliary horizon line. Measure the distance between the auxiliary vanishing point and the measuring point. Transfer that distance to the auxiliary horizon line (Figure 9.15).

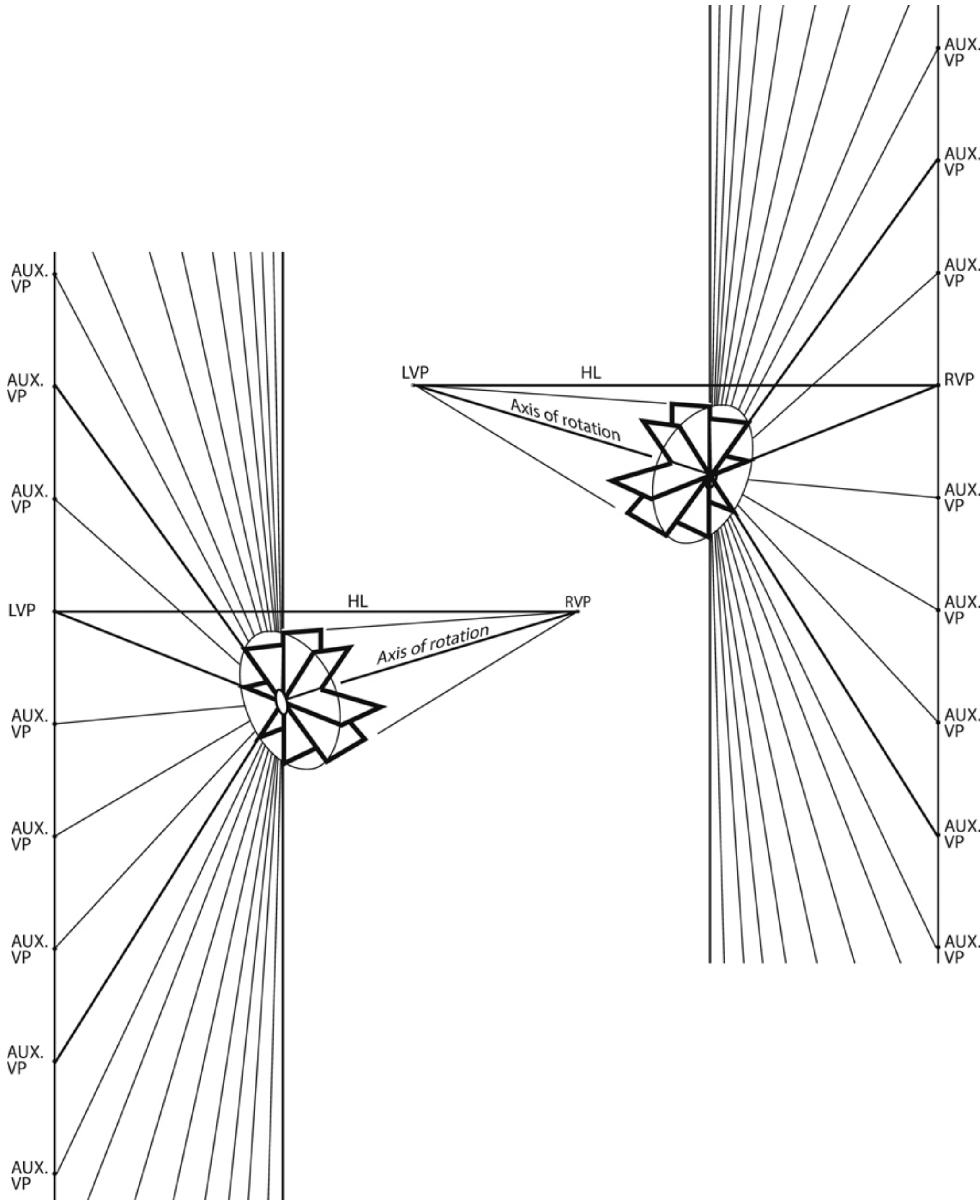


Figure 9.13 Auxiliary vanishing points are always opposite to the axis of rotation.

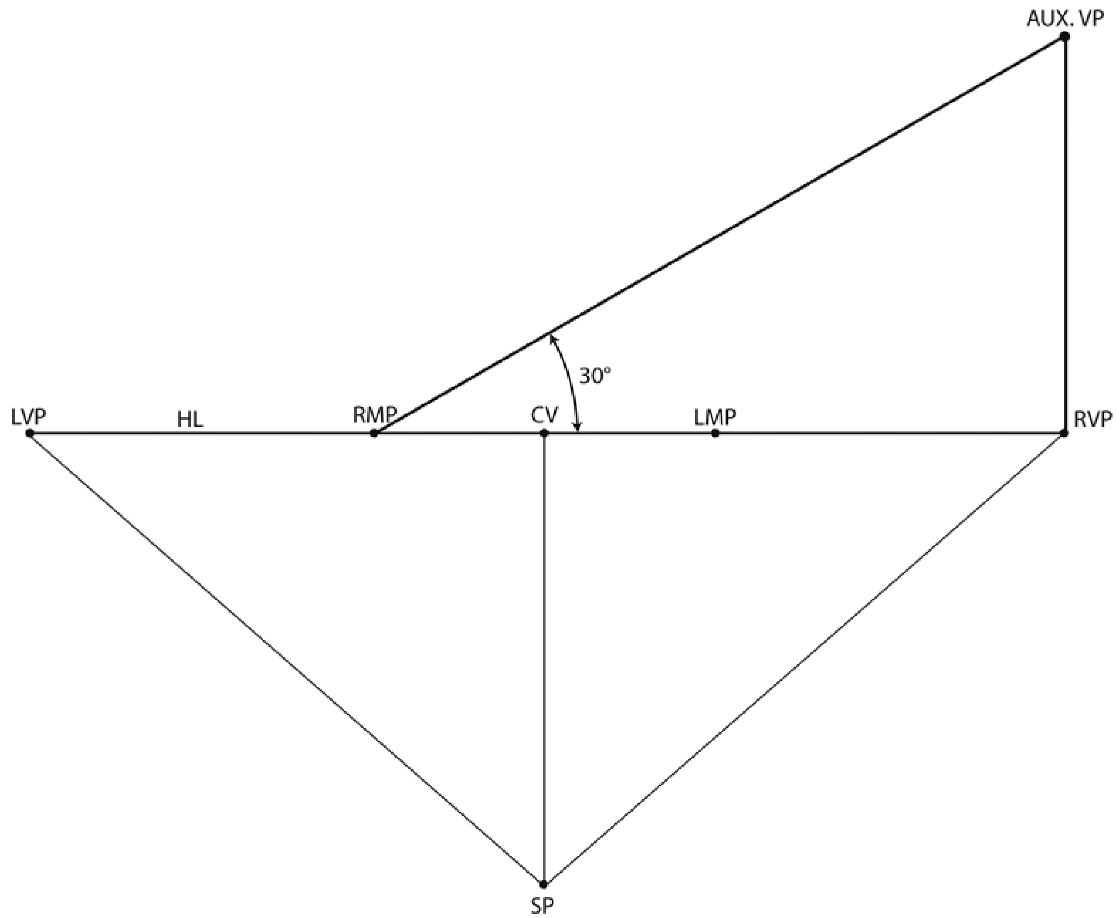
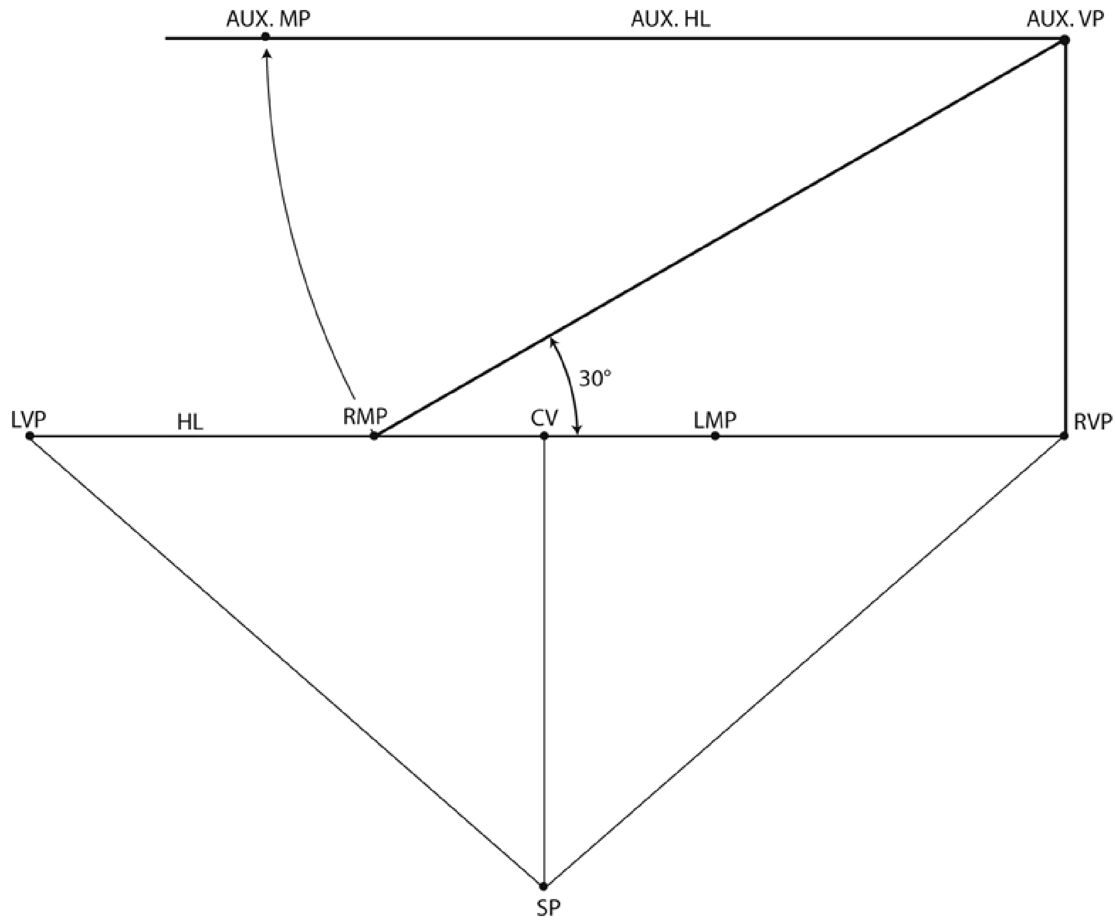


Figure 9.14 **Use the measuring point to find the location of the auxiliary vanishing point.**



[Figure 9.15](#) Use a compass or ruler to establish the auxiliary measuring point.

Measuring Line

As with all inclines, the measuring line must touch the line being measured ([Figure 9.16](#)).

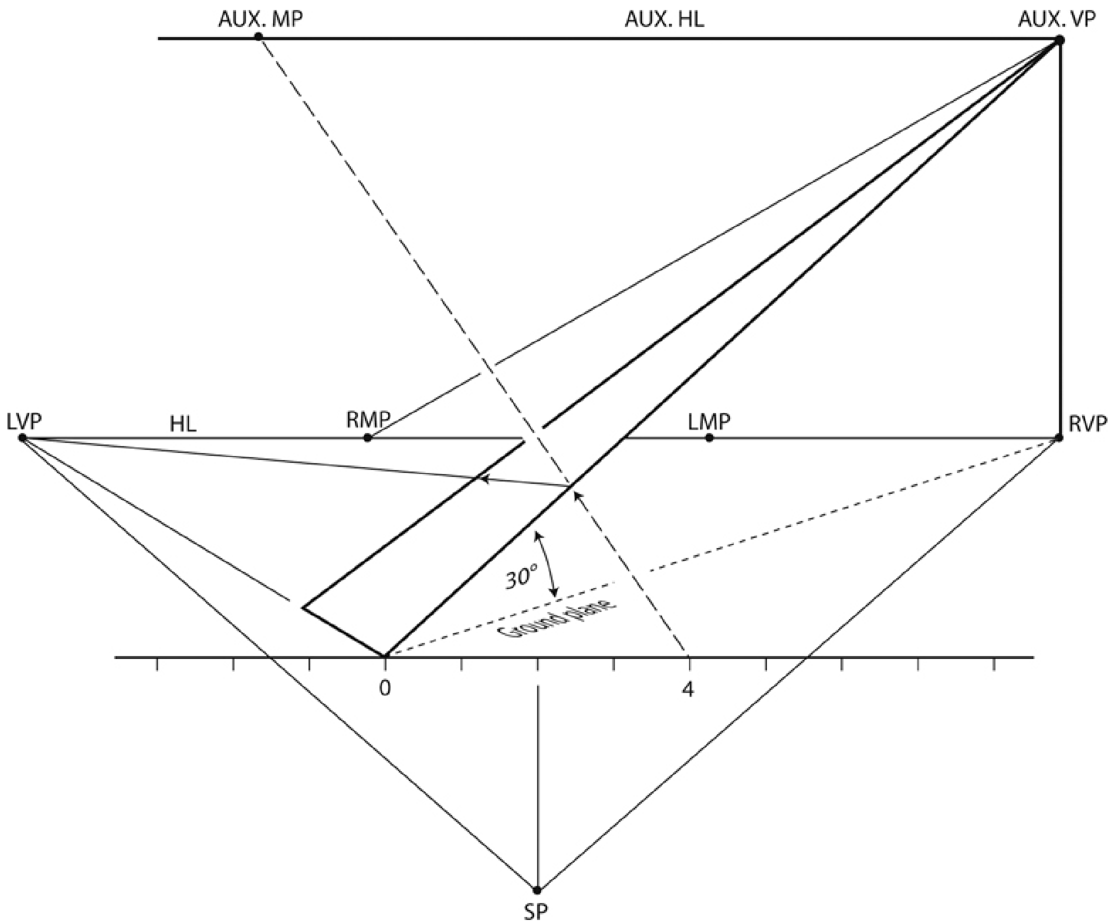


Figure 9.16 To measure an incline, the measuring line must touch the line being measured. This incline is 4 units long.

Downward Inclines

Downward inclines are approached the same as upward inclines. Do the same procedure upside-down (Figures 9.17–9.19).

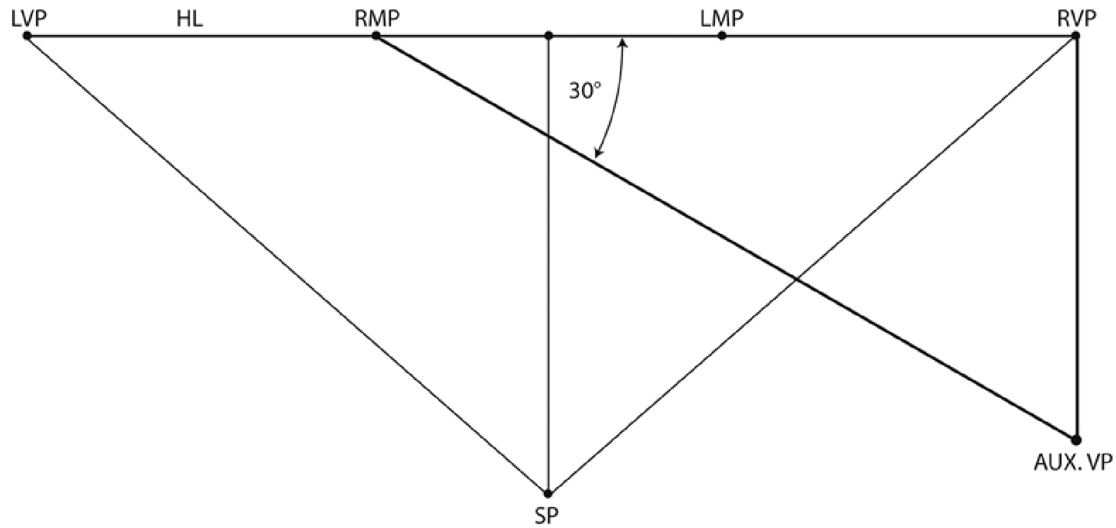


Figure 9.17 Finding the auxiliary vanishing point.

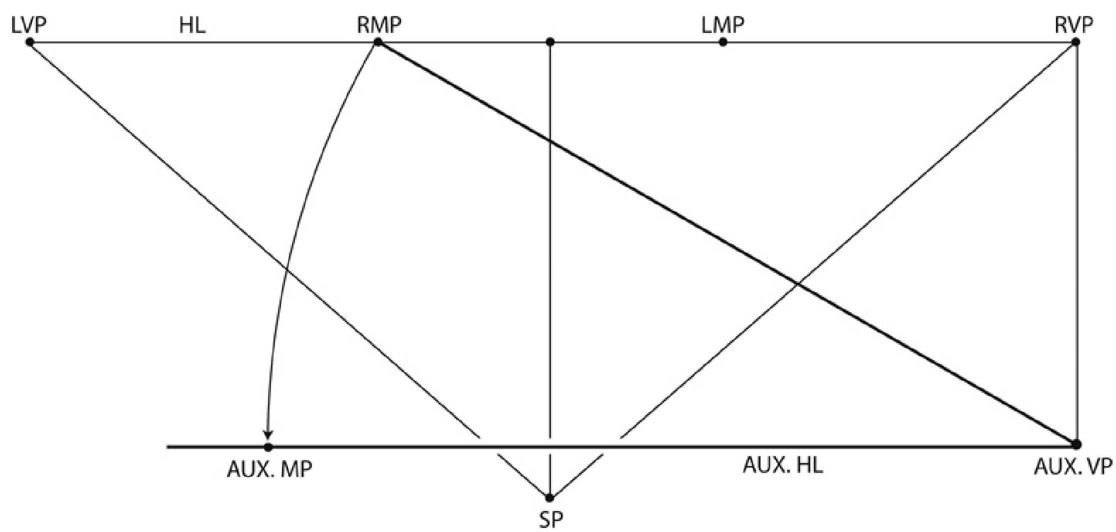


Figure 9.18 To find the auxiliary measuring point, measure the distance from the auxiliary vanishing point to the left measuring point, and transfer that distance to the auxiliary horizon line.

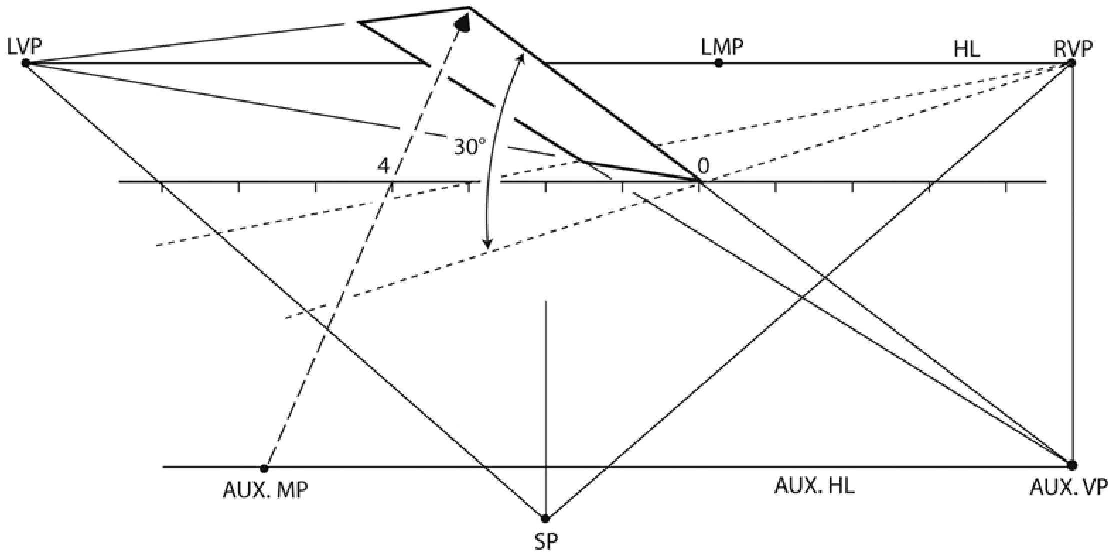


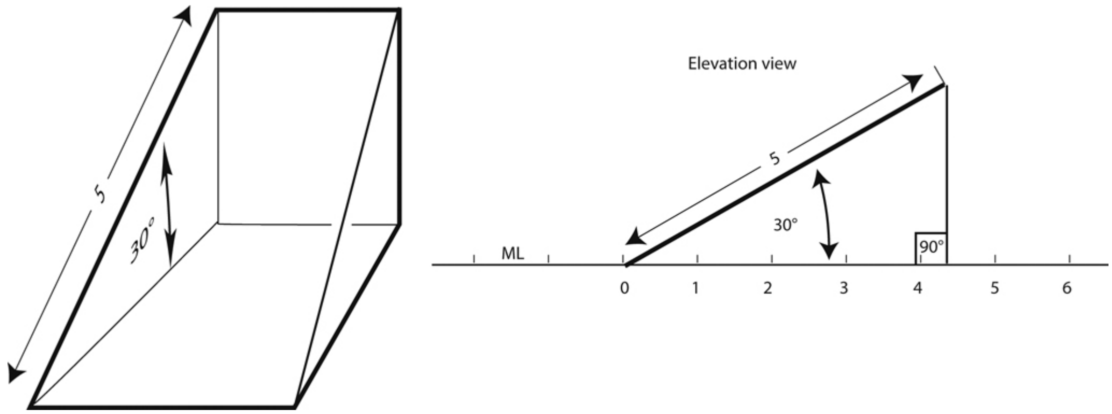
Figure 9.19 When measuring the slope, ensure the measuring line touches the line being measured. This incline is 4 units long.

An Alternative Method

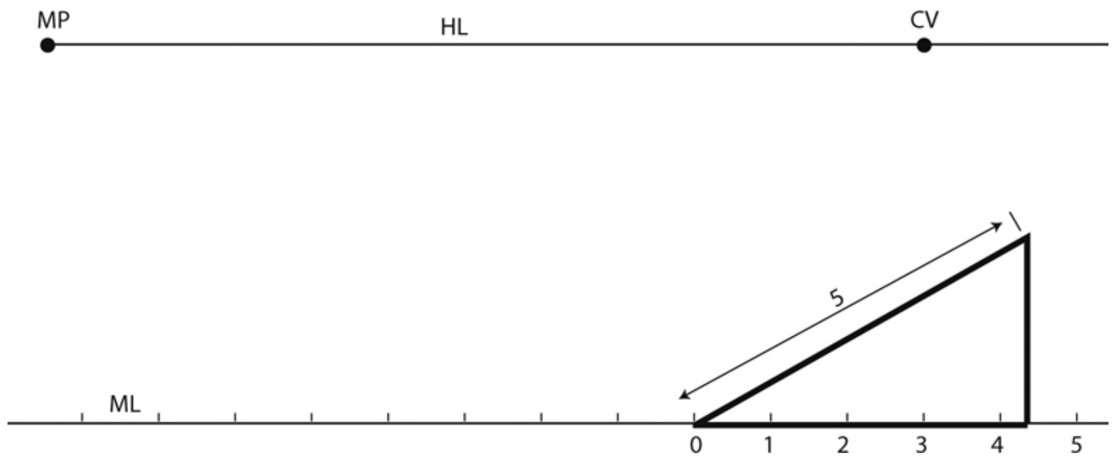
One-Point Inclines

Often, there are several methods to solve a perspective problem, each with its own advantages and disadvantages. Drawing steep inclines using auxiliary vanishing points can be inconvenient, as the vanishing and measuring points are often beyond the edge of the paper. The following alternative method is not an elegant solution, but it does keep all the points on the paper.

Any incline can be thought of as a right-angled triangle (a triangle with a 90° corner). This right-angled triangle has a horizontal and a vertical leg. The **hypotenuse** of the triangle is the incline (Figure 9.20).



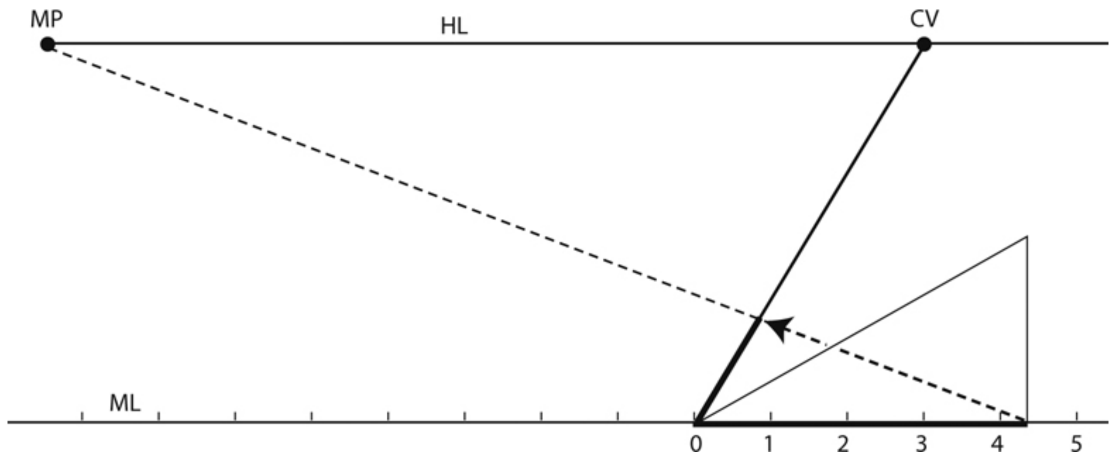
[Figure 9.20](#) **This incline is a right-angled triangle. Its hypotenuse is 5 units long, angled 30° from the horizontal leg.**



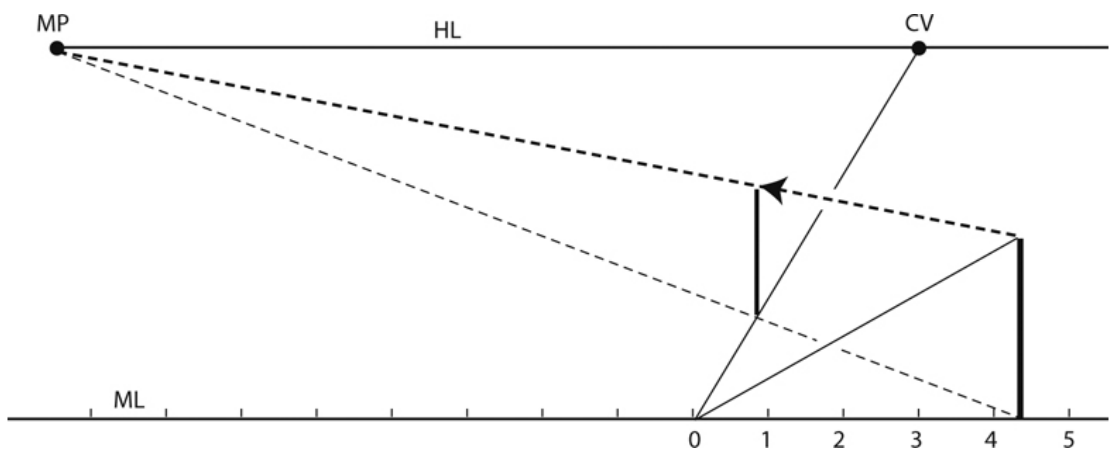
[Figure 9.21](#) **First, draw an elevation view parallel with the picture plane using true dimensions and the true angle of the incline.**

In this alternative method, first draw an elevation view of the incline. This elevation view has no perspective. It is drawn to scale, using true dimensions (Figure 9.21).

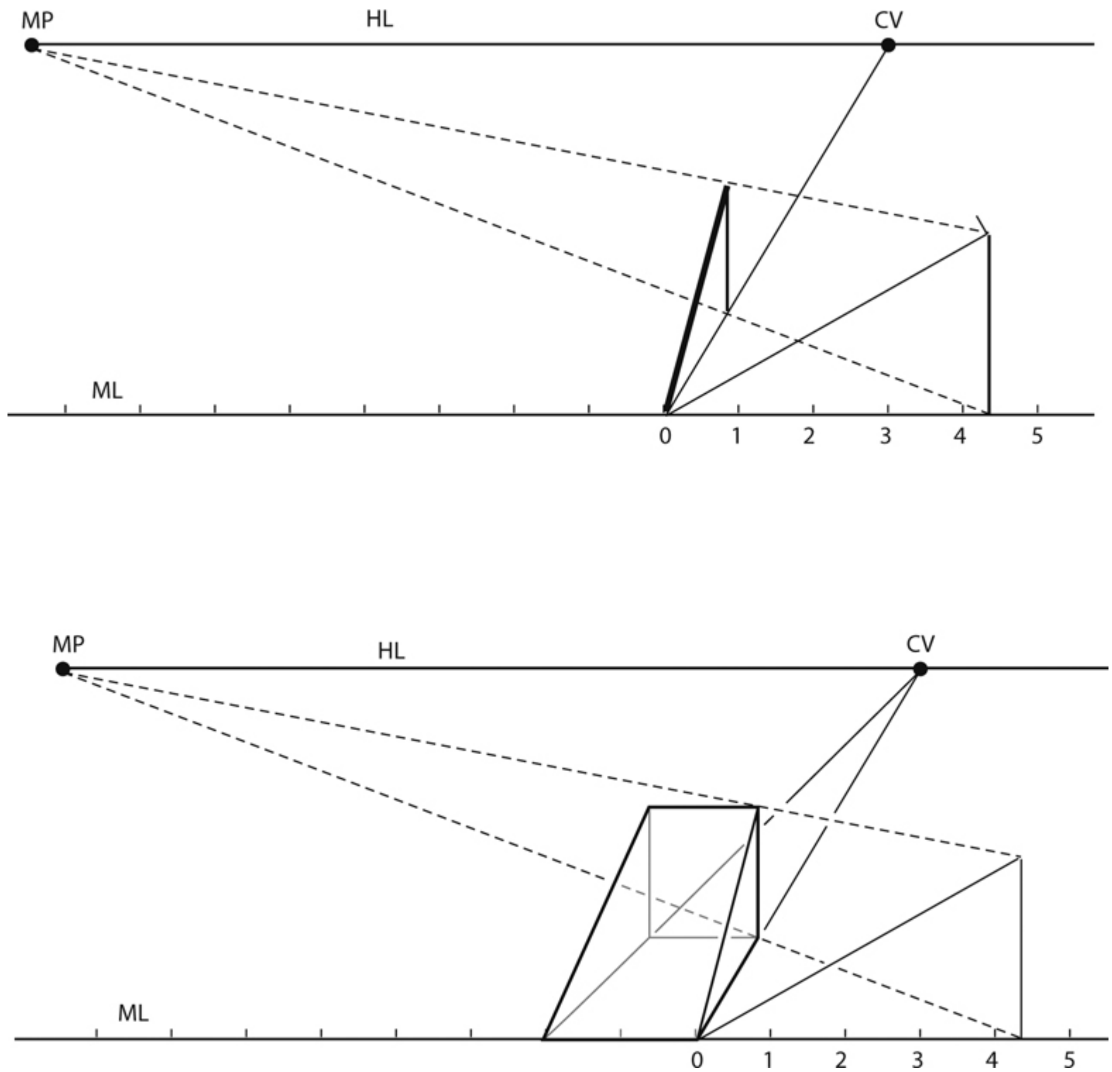
The next step is to transfer the dimensions of the elevation view into a perspective view. Use a measuring point to transfer the length of the horizontal leg to a foreshortened line (Figure 9.22). Use the same measuring point to transfer the height of the vertical leg into position (Figure 9.23). Connect the ends of the two legs to create the incline (Figure 9.24).



[Figure 9.22](#) Transfer the horizontal leg in perspective.



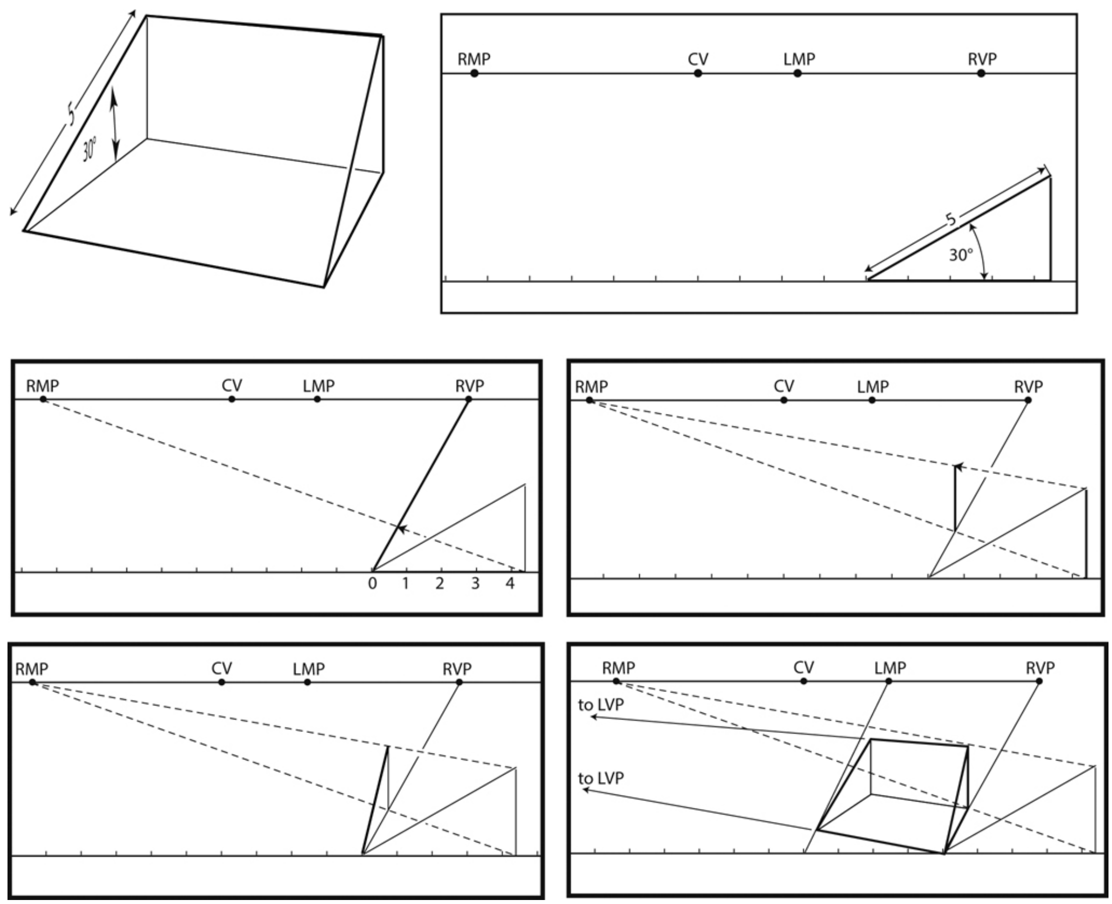
[Figure 9.23](#) Transfer the vertical line in perspective.



[Figure 9.24](#) **Completing the incline.**

Two-Point Inclines

Using the alternative method in two-point perspective follows the same basic procedures as one-point. The difference being that the right or left vanishing point is used instead of the center of vision. Draw the incline in an elevation view, and then project those dimensions to a perspective view ([Figure 9.25](#)).



[Figure 9.25](#) **Projecting the dimensions of an elevation view to a perspective view is an alternative method to draw two-point inclines.**

10
Inclined Cuboids

The previous chapter outlined the procedures to draw an inclined plane. A plane has only two dimensions. This chapter adds the missing dimension and begins where Chapter 9 left off. As such, it is helpful to have a good understanding of inclined planes before progressing to drawing inclined cuboids.

One-Point Perspective

Draw a one-point perspective inclined plane (Figure 10.1). See Chapter 9 for step-by-step instructions.

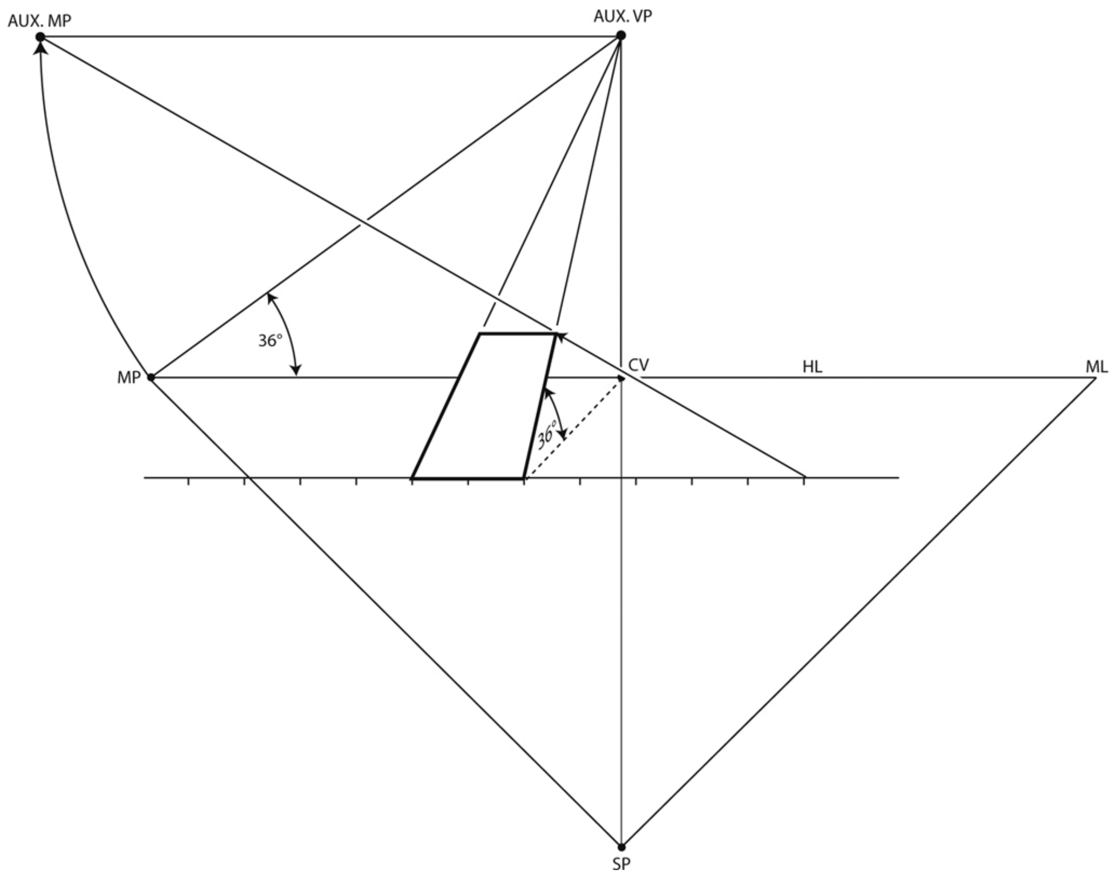


Figure 10.1 **A one-point perspective inclined plane, 2 units wide, 5 units long, at a 36° incline.**

Auxiliary Vanishing Point

A third dimension is extruded from this inclined plane. All the corners are right angles. The thickness projects 90° from the inclined plane. To draw the thickness, another auxiliary vanishing point is needed, a point 90° from the original auxiliary vanishing point. Thus, the first step is to locate the point of true angles—the measuring point. At the measuring point, draw a 90° angle from the original auxiliary vanishing point. There are now two auxiliary vanishing points that are 90° apart (Figure 10.2).

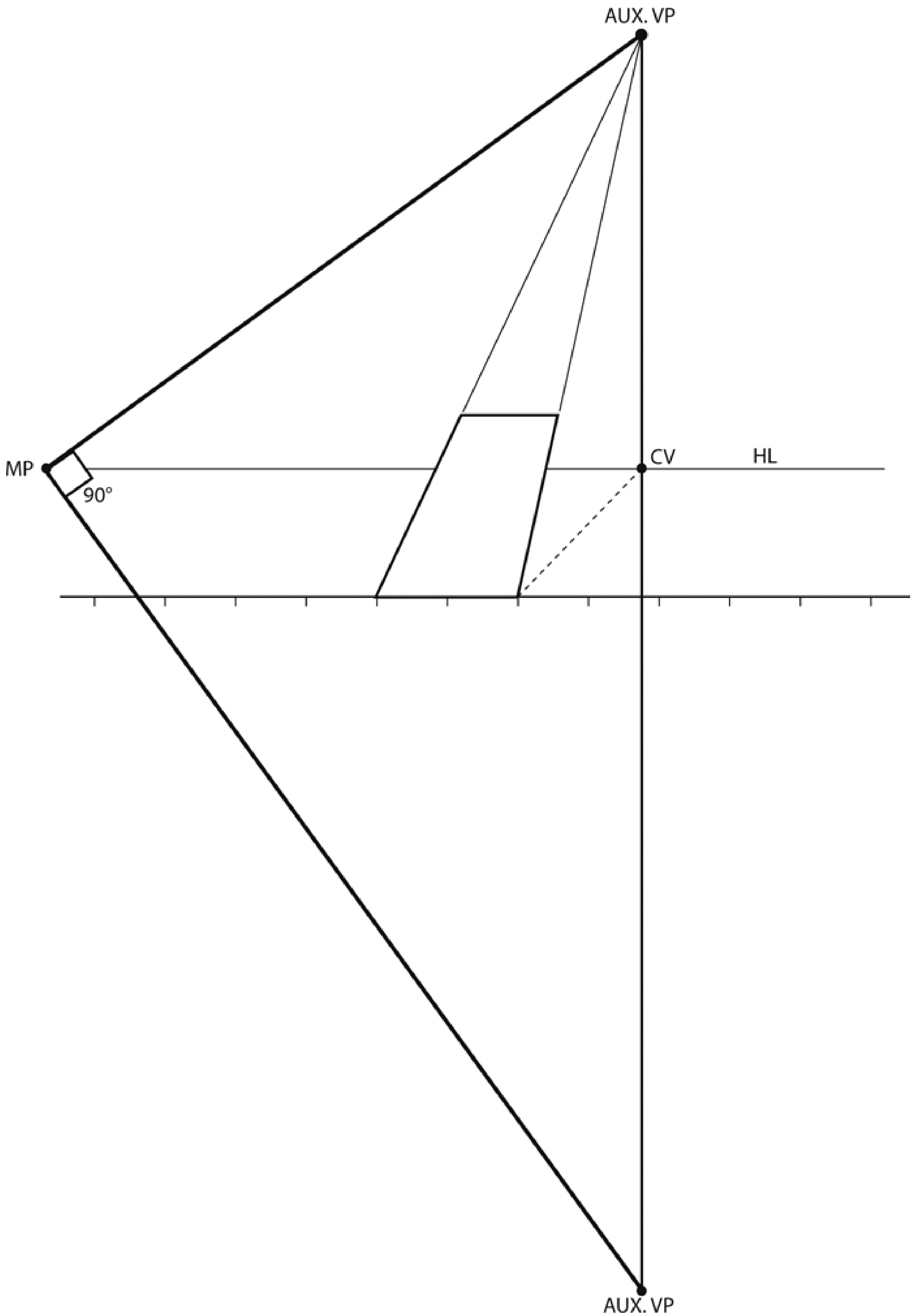
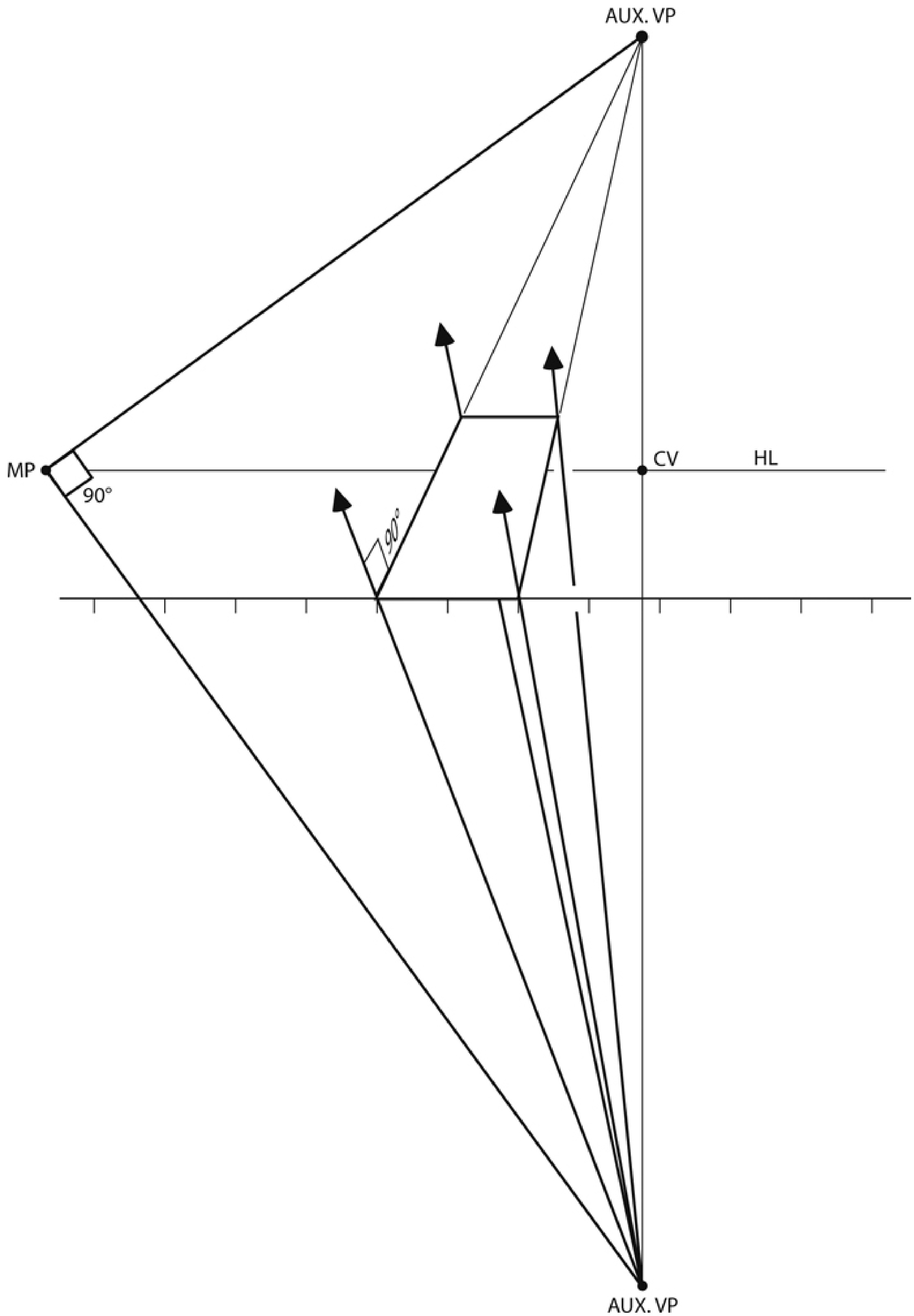


Figure 10.2 **The upper and lower auxiliary vanishing points are 90° apart at the measuring point.**

Any line drawn from the lower auxiliary vanishing point will create a 90° angle to any line drawn from the upper auxiliary vanishing point (Figure 10.3).



[Figure 10.3](#) **Lines drawn from the lower auxiliary vanishing point are right angles to lines drawn from the upper auxiliary vanishing point.**

Measuring the Thickness

Establish measuring points as described in Chapter 9 (Figure 10.4). Remember, the measuring line must touch the line being measured (Figure 10.5).

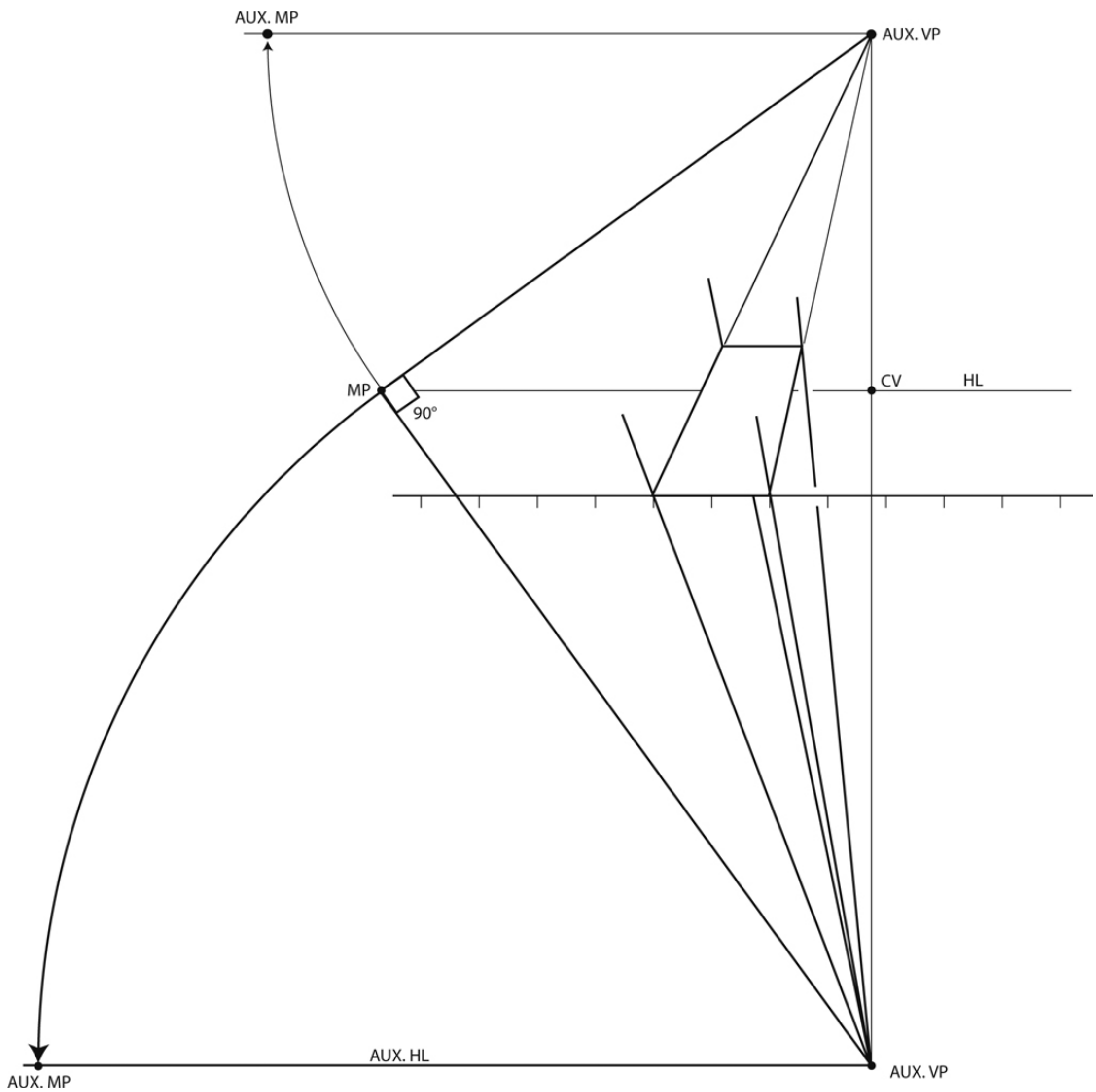


Figure 10.4 The distance from the auxiliary vanishing point to the measuring point is the same as the distance from the auxiliary vanishing point to the auxiliary measuring point.

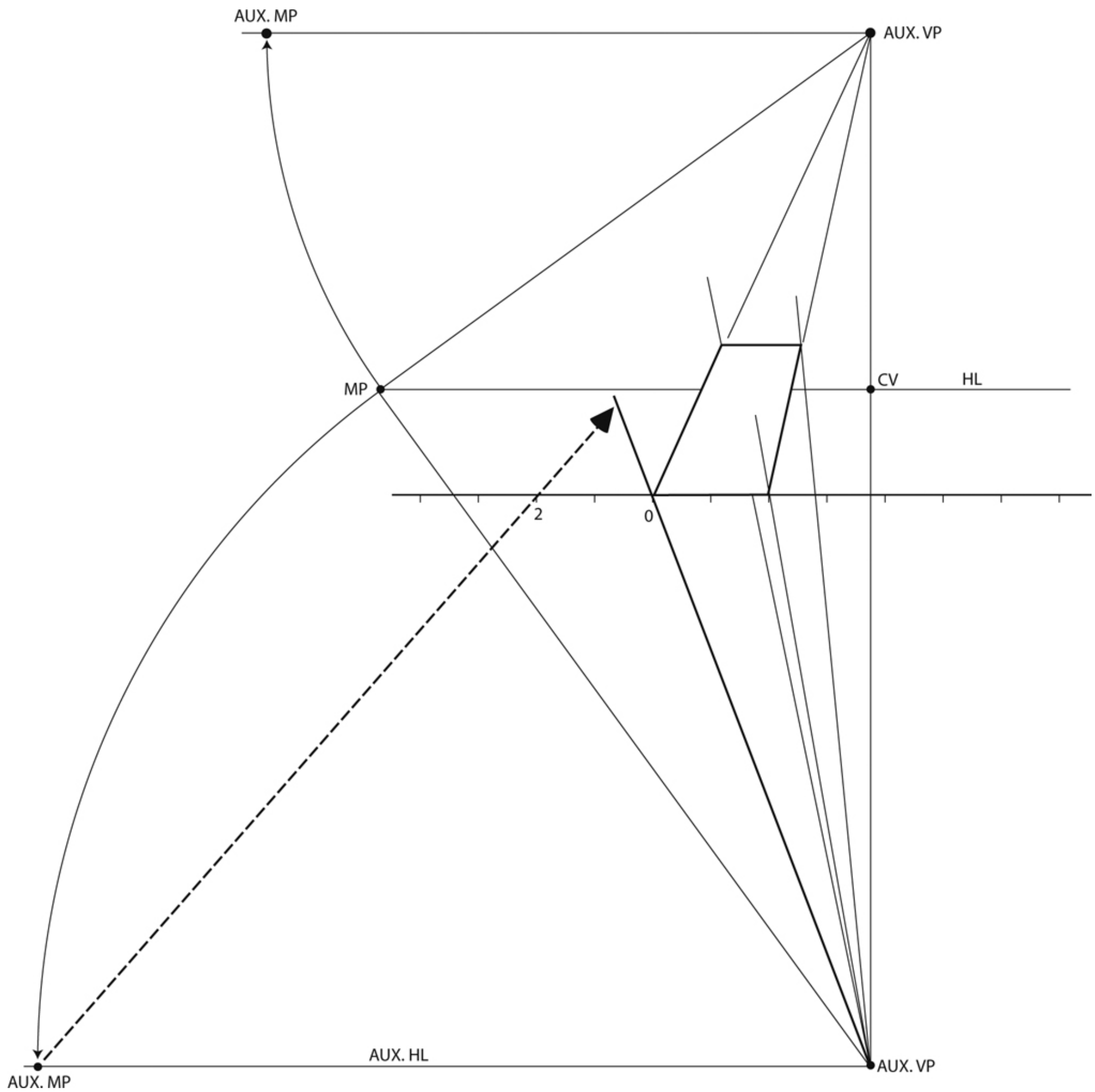


Figure 10.5 Position the measuring line so that it touches the line being measured. This illustration shows a length of 2 units.

Completing the Box

Connect the lines to the proper vanishing points to complete the box (Figure 10.6).

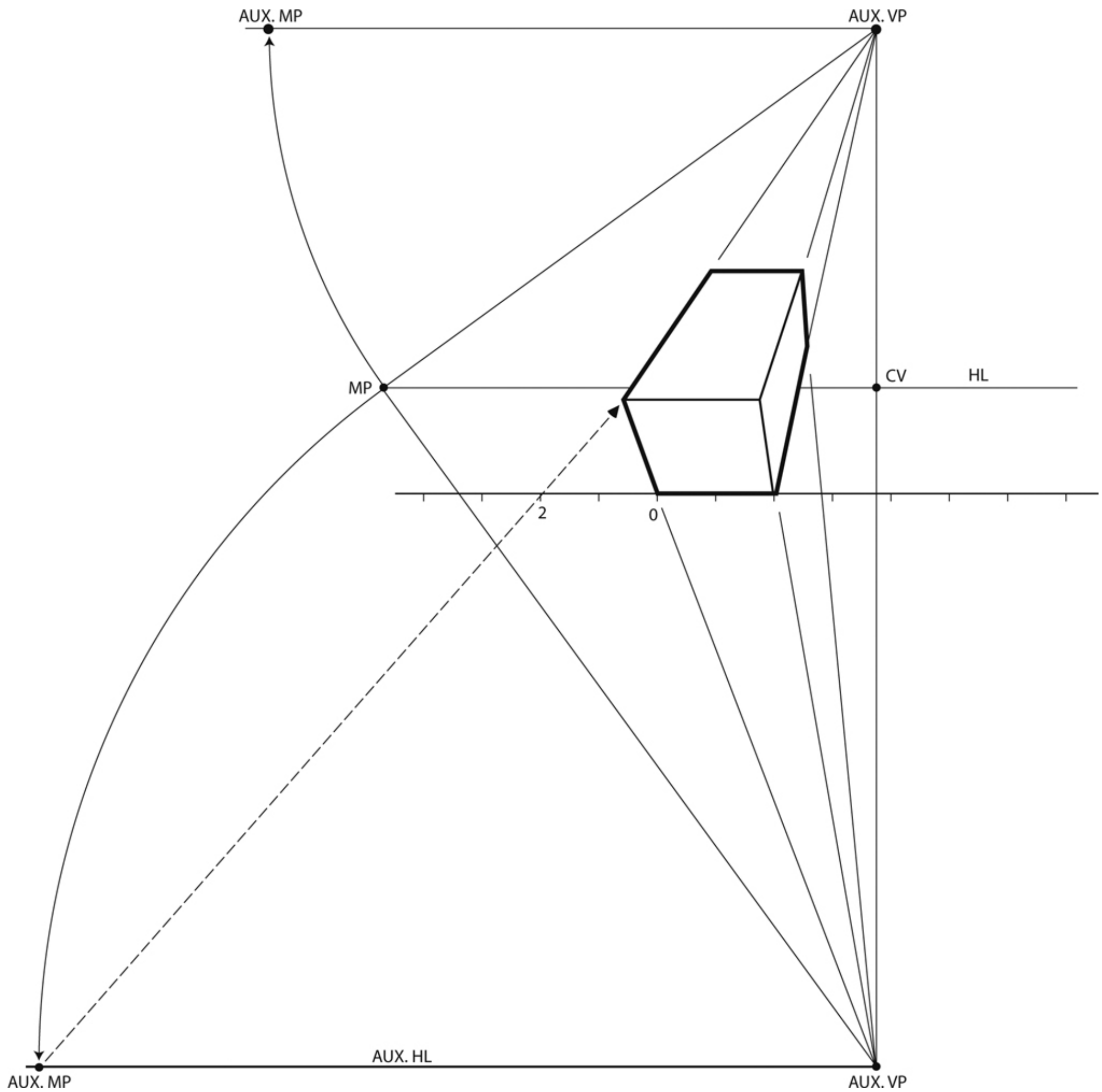


Figure 10.6 Connect lines to vanishing points to complete the box.

Inclined Two-Point Cuboids

Draw a two-point perspective inclined plane (Figure 10.7). See Chapter 9 for step-by-step instructions.

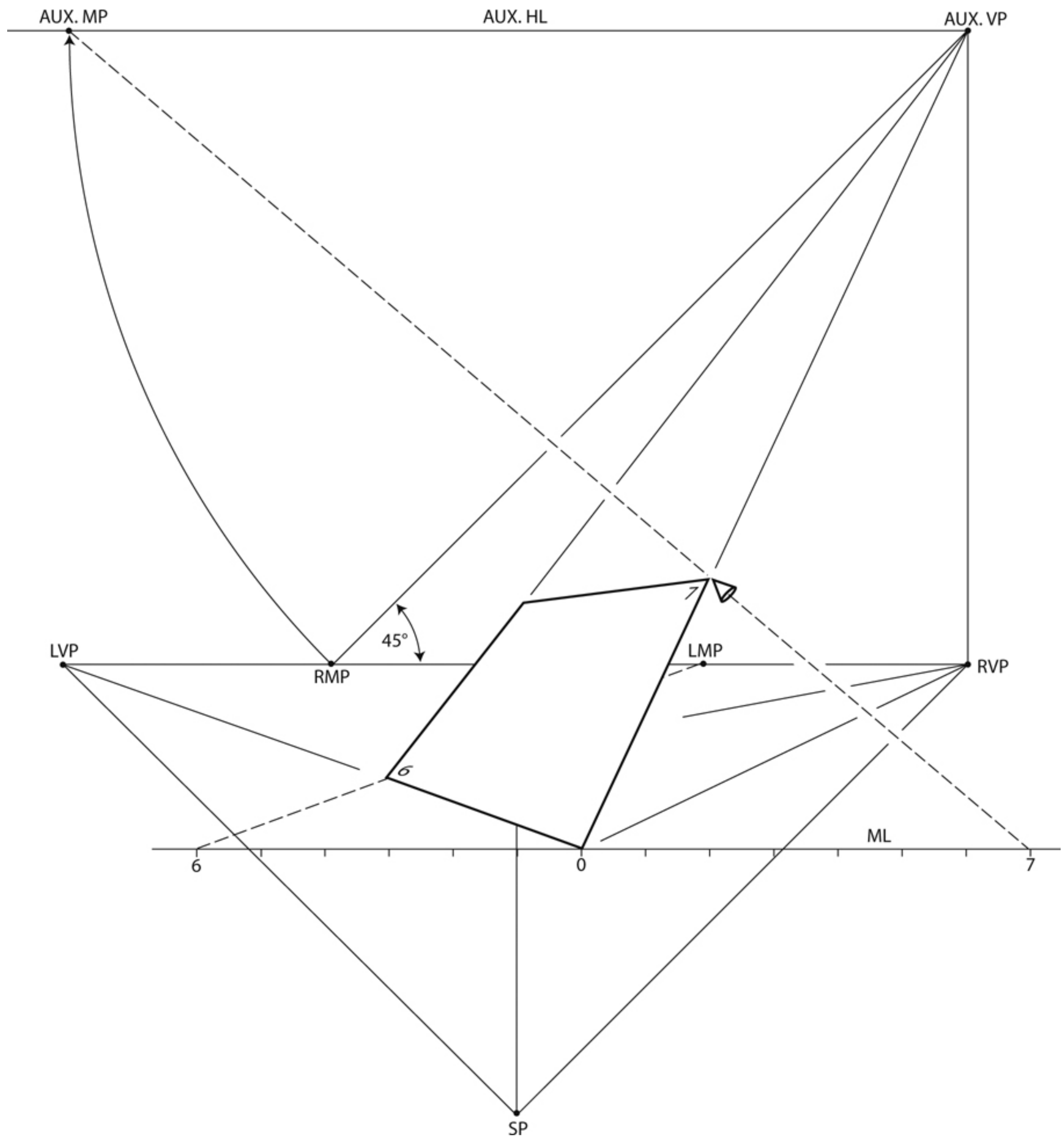


Figure 10.7 **The starting point is a two-point perspective inclined plane. It is 6 units wide and 7 units long, tilting up from the ground plane at 45°.**

Auxiliary Vanishing Points

Angled cuboids have three vanishing points: two auxiliary vanishing points and one vanishing point located on the horizon line. The auxiliary vanishing points are always aligned vertically. They are above and below the left *or* right vanishing point.

To draw 90° corners, the auxiliary vanishing points must be 90° apart. True angles for inclines are found at the measuring point. Inclines to the left use the left measuring point, and inclines to the right use the right measuring point (Figure 10.8). A right angle drawn at the measuring point creates auxiliary vanishing points that draw right angles in perspective (Figure 10.9).

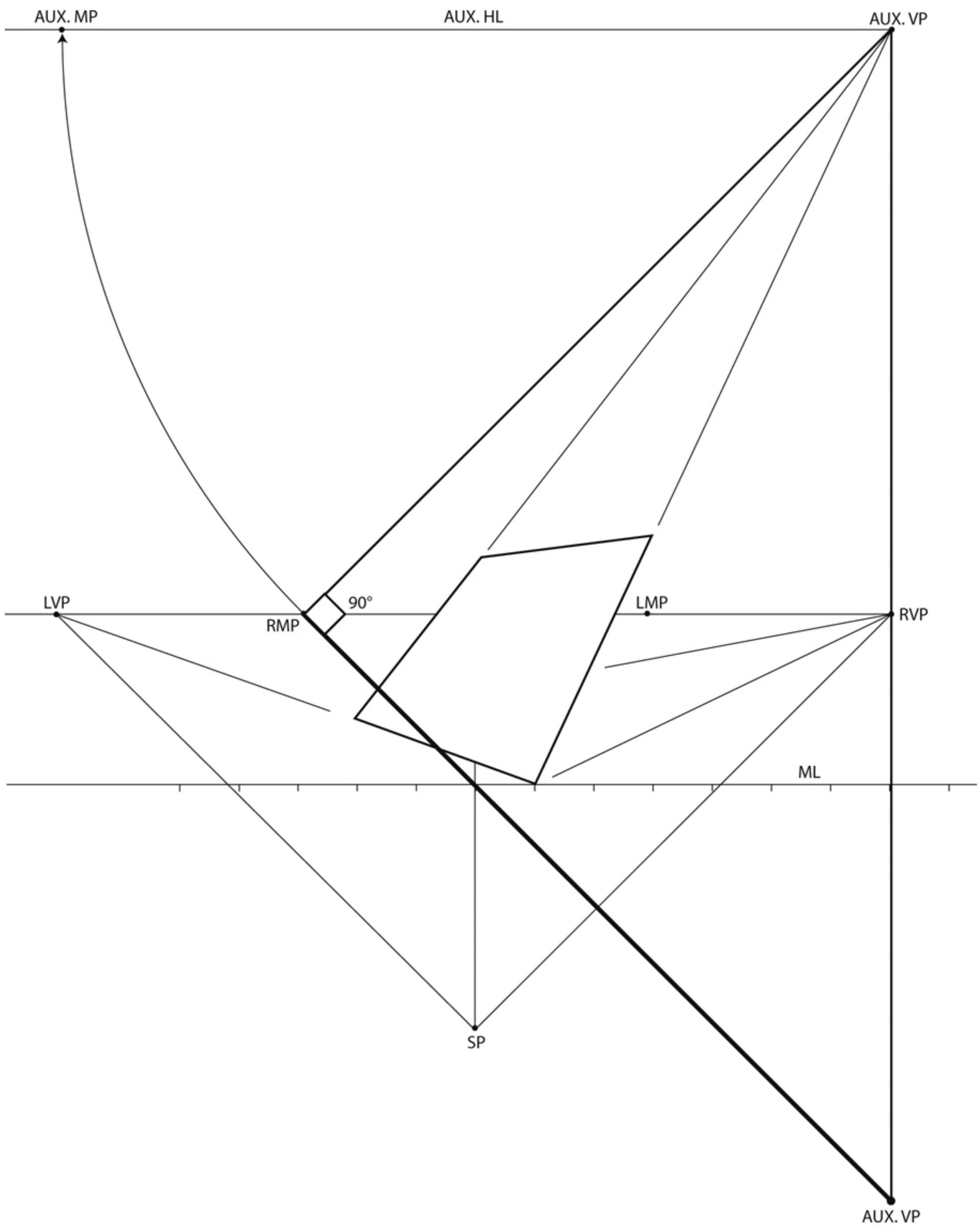


Figure 10.8 **A right angle at the measuring point creates two auxiliary vanishing points 90° apart.**

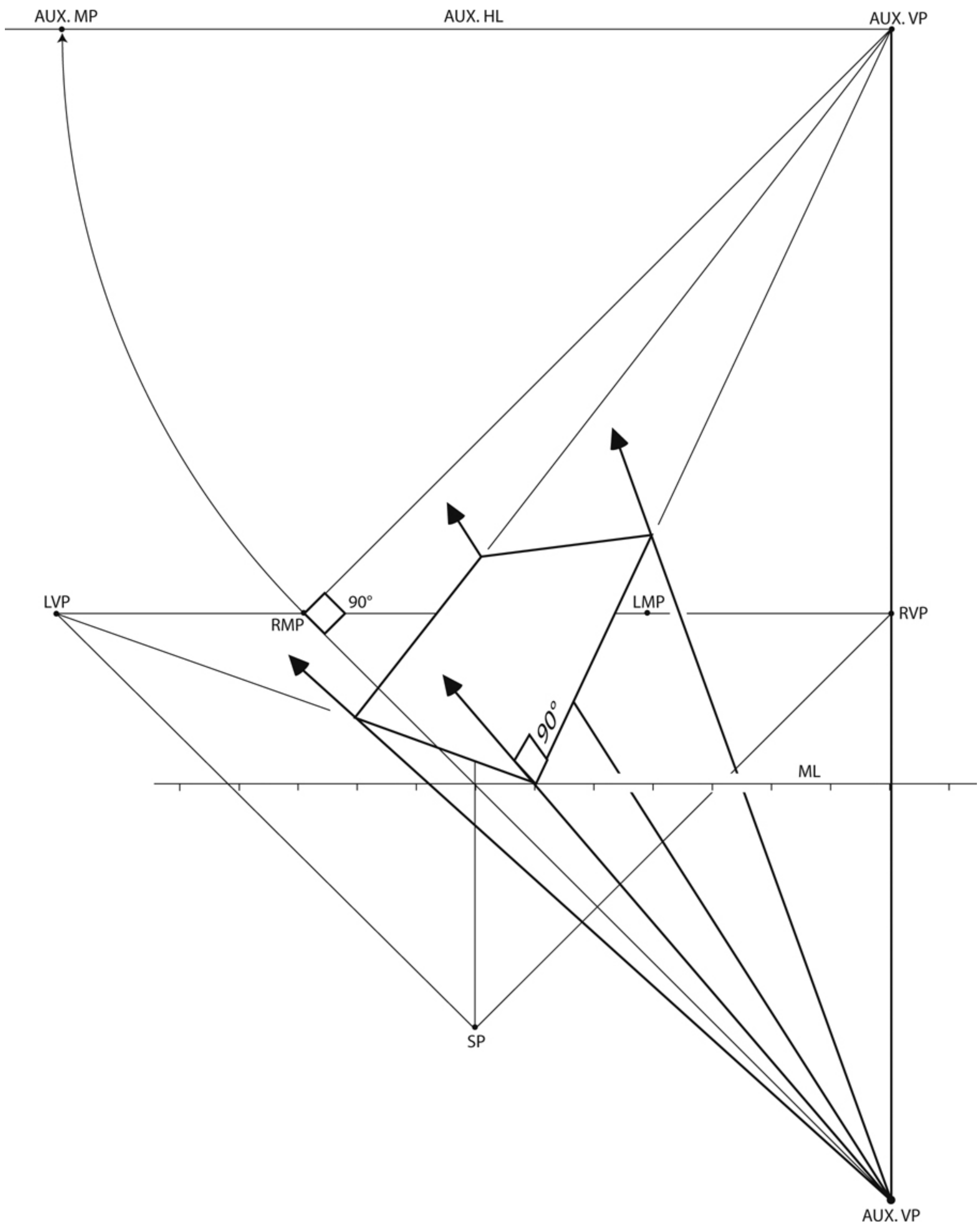


Figure 10.9 **Project lines from the lower auxiliary vanishing point to create 90° corners.**

Measuring Point

Establish the auxiliary measuring point using the procedure outlined in Chapter 9 (Figure 10.10). Measure the desired thickness of the box. Make sure the measuring line touches the line being measured (Figure 10.11).

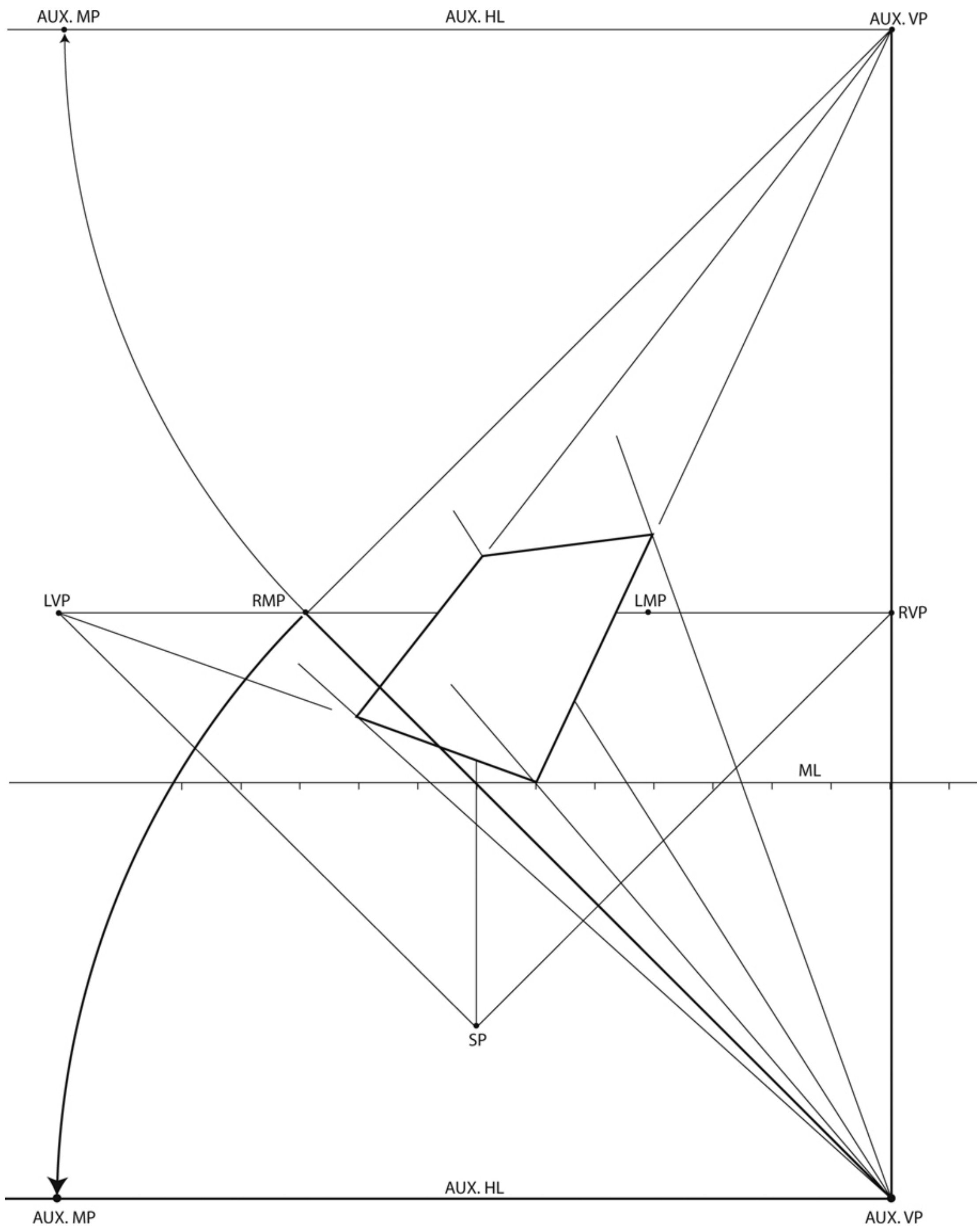


Figure 10.10 Measure the distance from the auxiliary vanishing point to the measuring point and transfer that distance to the auxiliary horizon line.

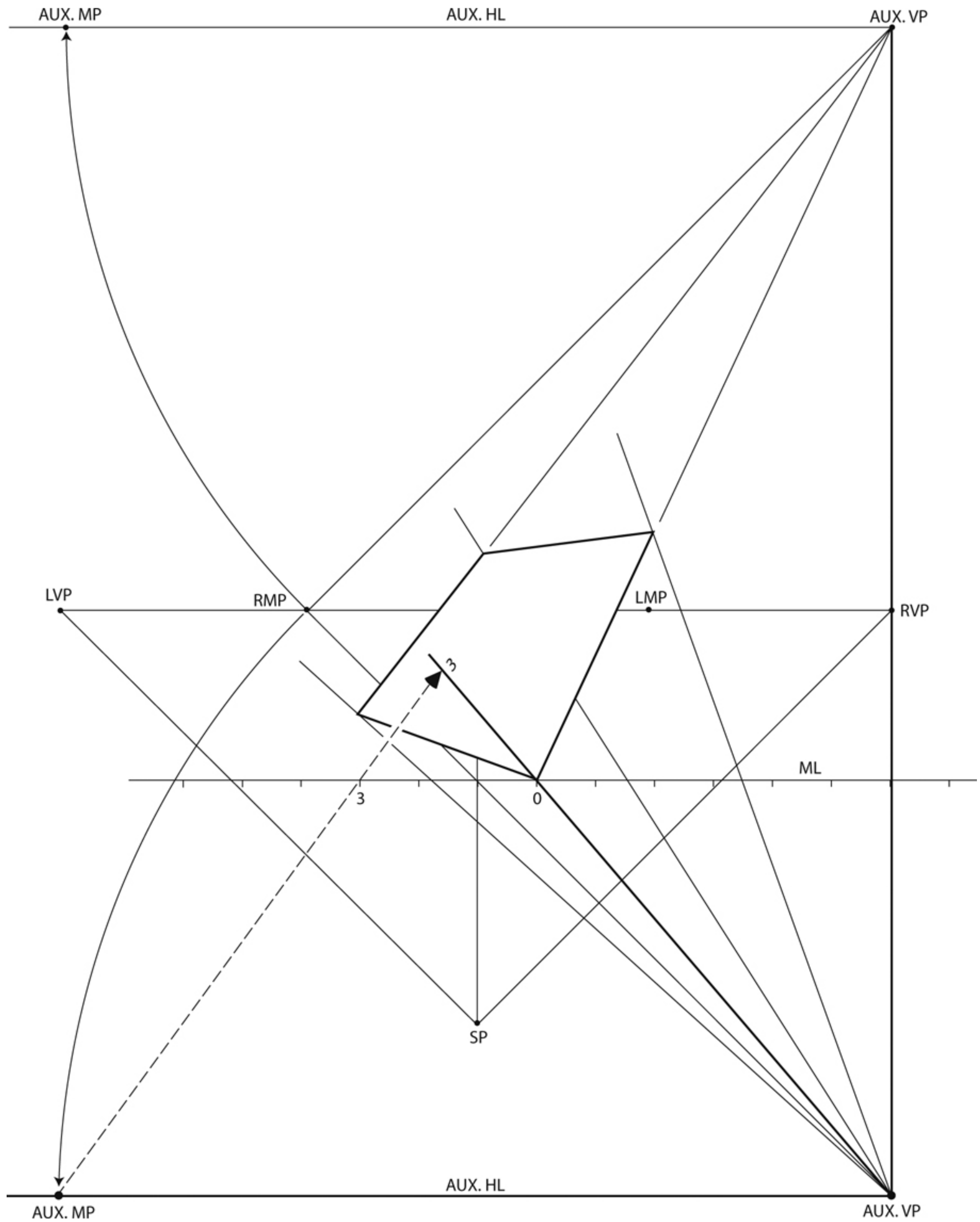


Figure 10.11 The measuring line must touch the line being measured. This example shows a length of 3 units.

Complete the Box

Connect lines to appropriate vanishing points to complete the box (Figure 10.12).

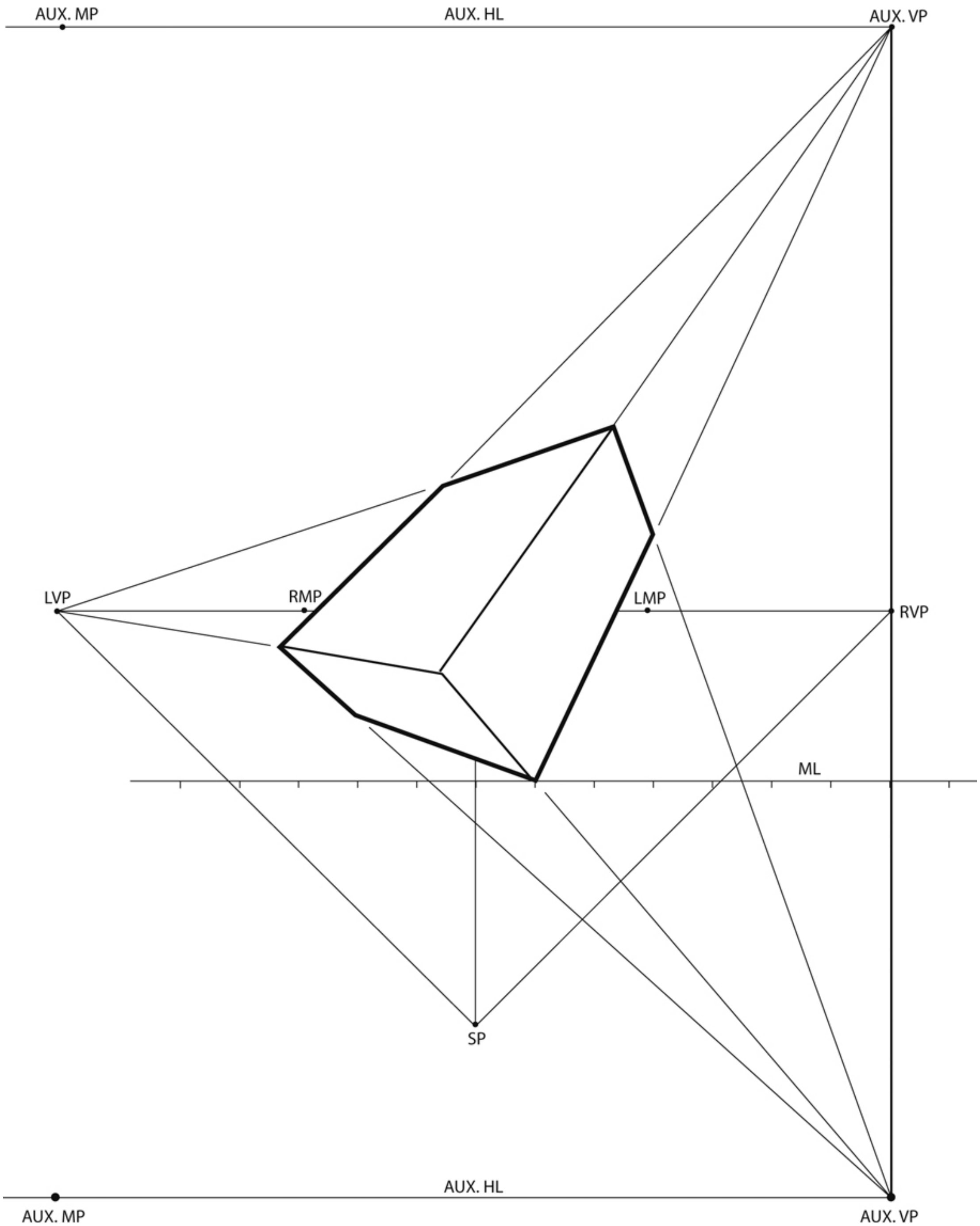


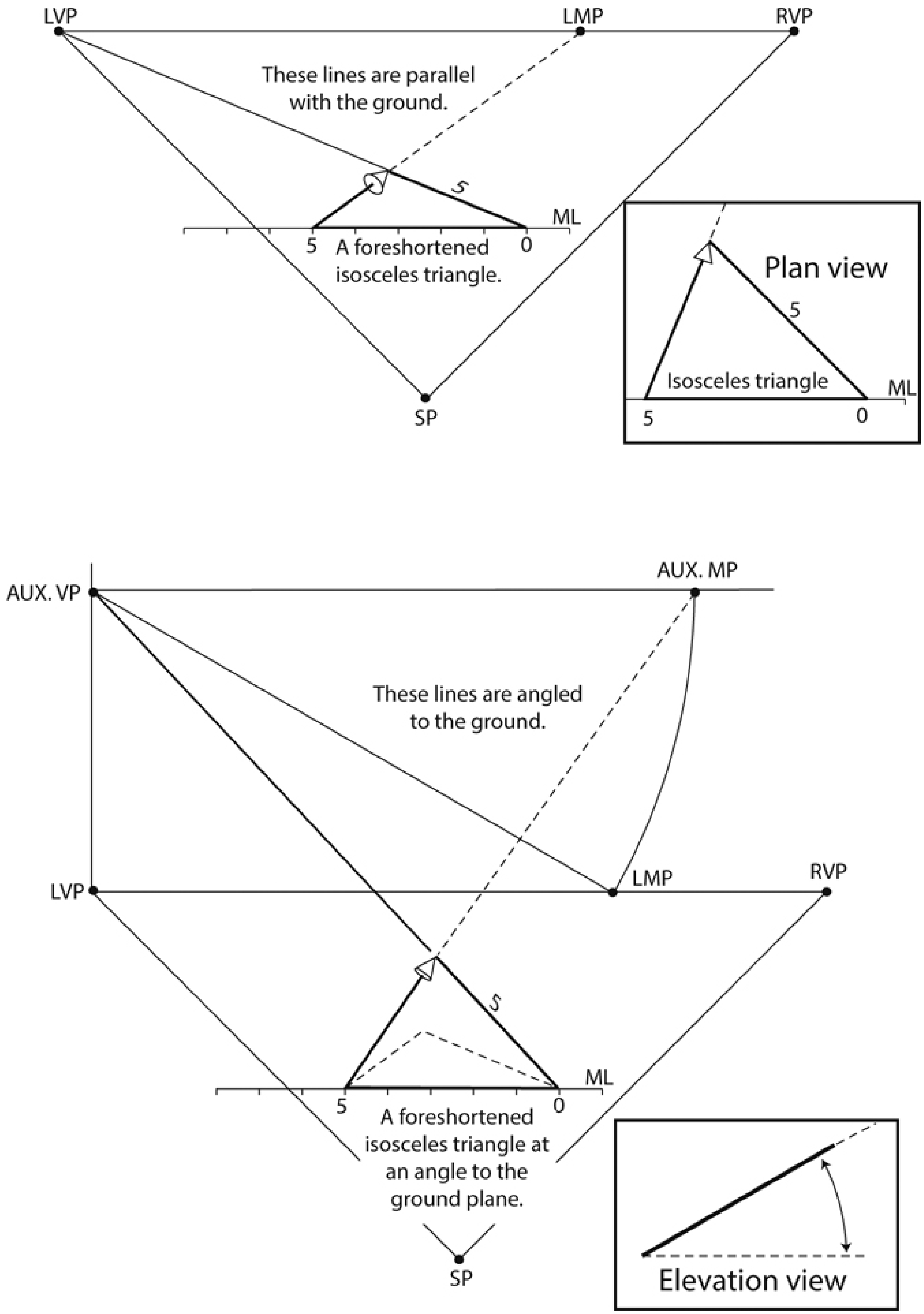
Figure 10.12 Connect lines to vanishing points to complete the box.

11
Incline Geometry

Chapter 6 explored the geometry of measuring points, the importance of isosceles triangles, and how measuring points draw isosceles triangles in perspective. Auxiliary measuring points also draw isosceles triangles, the only difference being that the isosceles triangle is at an incline. If the line being measured is parallel with the ground plane, the isosceles triangle is also parallel with the ground plane (Figure 11.1, top). If the line being measured is inclined, the isosceles triangle is also inclined (Figure 11.1, bottom).

It has been stated several times that, when measuring inclines, the measuring line must touch the line being measured. This, however, is not the case when measuring lines parallel with the ground plane. Why the aberration?

Lines drawn from auxiliary vanishing points are angled to the ground plane. Likewise, lines drawn from auxiliary measuring points are also angled to the ground plane. Lines drawn from the auxiliary measuring point would, if extended, follow a path *underground*. If the measuring line did not touch the line being measured, the line projected from the auxiliary measuring point would never contact the measuring line. Because of this, it is necessary for the measuring line to be in a very specific location—touching the line being measured (Figure 11.1). If the measuring line is not already touching the line being measured, the measuring line must be moved so that it does.



[Figure 11.1](#) **Measuring points located on the horizon line create isosceles triangles parallel with the ground plane (top). Auxiliary measuring points create isosceles**

triangles at an angle to the ground plane (bottom).

Vertical Auxiliary Measuring Points

In previous examples, the auxiliary measuring point was placed on an auxiliary horizon line (Figure 11.2, line "C," inset). Yet, auxiliary measuring points do not *need* to be on the auxiliary horizon line; there are many places the measuring point can be sited. Often, the most convenient location is on a vertical line, the line connecting the two auxiliary vanishing points (Figure 11.2, line "B," inset). However, no matter where its location, a measuring point must always draw isosceles triangles. For this to happen, the geometry must be correct. For the geometry to be correct, an important rule must be followed.

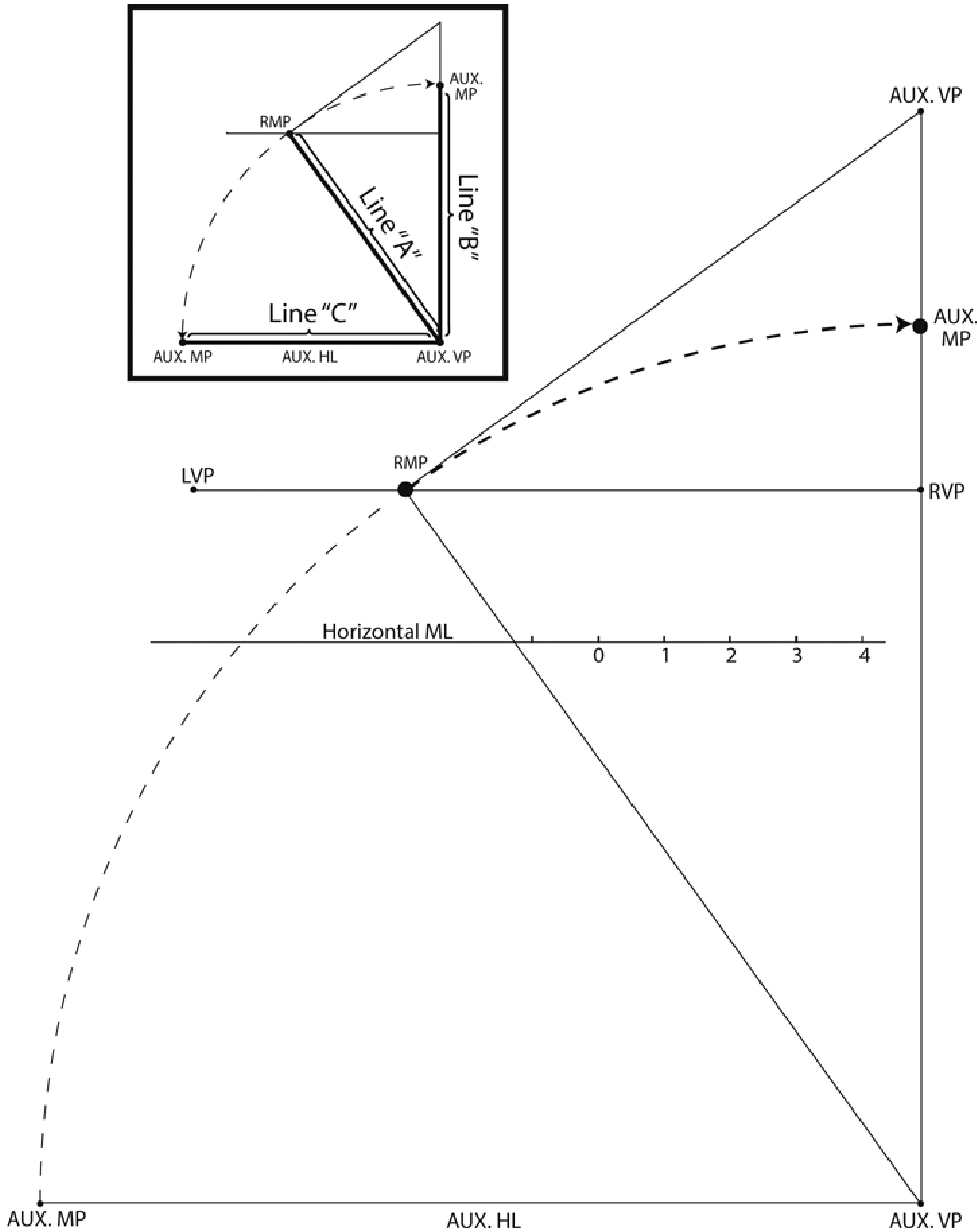


Figure 11.2 **The auxiliary measuring point can be on a vertical line as well as a horizontal. The distance from the measuring point to the auxiliary vanishing point (Line "A") must be the same as the distance from the auxiliary vanishing point to the auxiliary measuring point (Line "B" or "C").**

The relationship between a measuring point and the measuring line is critical. For a measuring point to draw an isosceles triangle, the measuring line must be parallel with the line the measuring point is on. If the measuring point is on a horizontal line (e.g., the horizon line, or the auxiliary horizon line) the measuring line must also be horizontal. If the measuring point is on a vertical line, the measuring line must also be vertical (Figure 11.3).

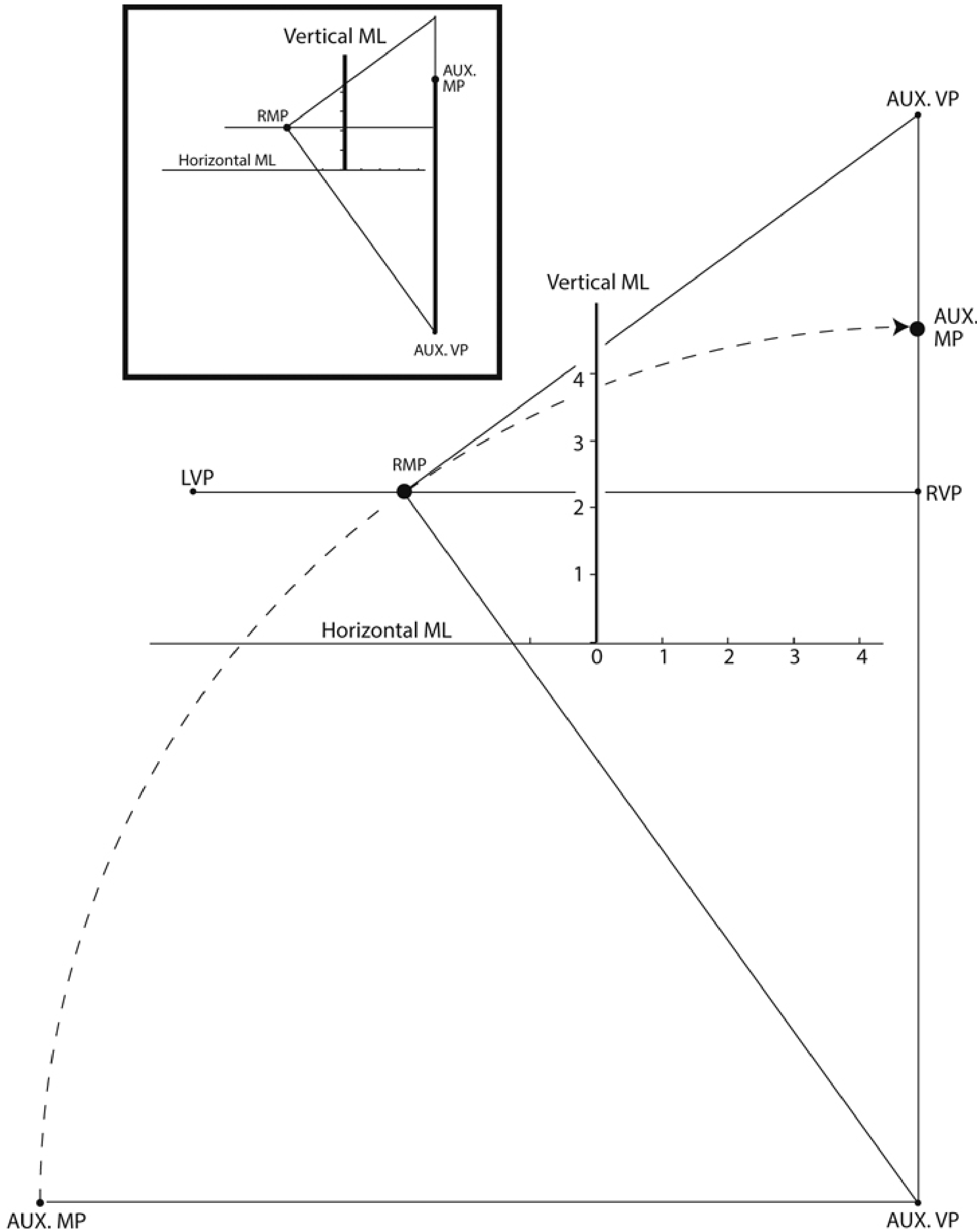


Figure 11.3 The measuring line must be parallel with the line the measuring point is on. If the measuring point is on a vertical line, the measuring line must also be vertical.

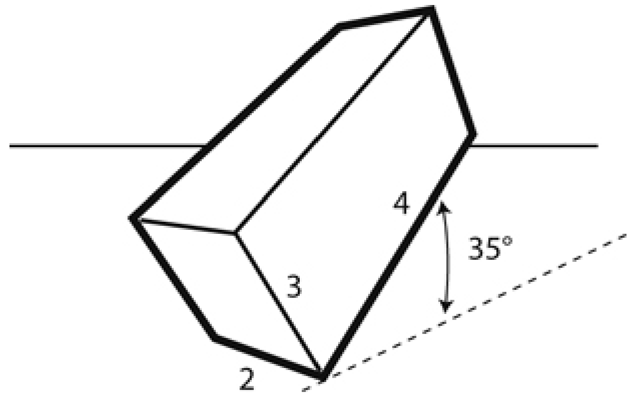


Figure 11.4 A box measuring 3 units high 4 units wide, and 2 units deep at a 35° incline.

The primary advantage of placing the measuring point on a vertical line is its proximity. Inclines often lead to points beyond the page's border. If the auxiliary vanishing point is off the page, the auxiliary measuring point will be even farther away. Placing the measuring point on a vertical line keeps the measuring points close at hand.

To demonstrate this, draw a box that is 3 units tall, 4 units wide, and 2 units deep, inclined at a 35° angle (Figure 11.4).

Using a vertical measuring point (VMP) and vertical measuring line, measure the height and width (Figures 11.5–11.6). Then connect the lines to the appropriate vanishing points to complete the box (Figure 11.7).

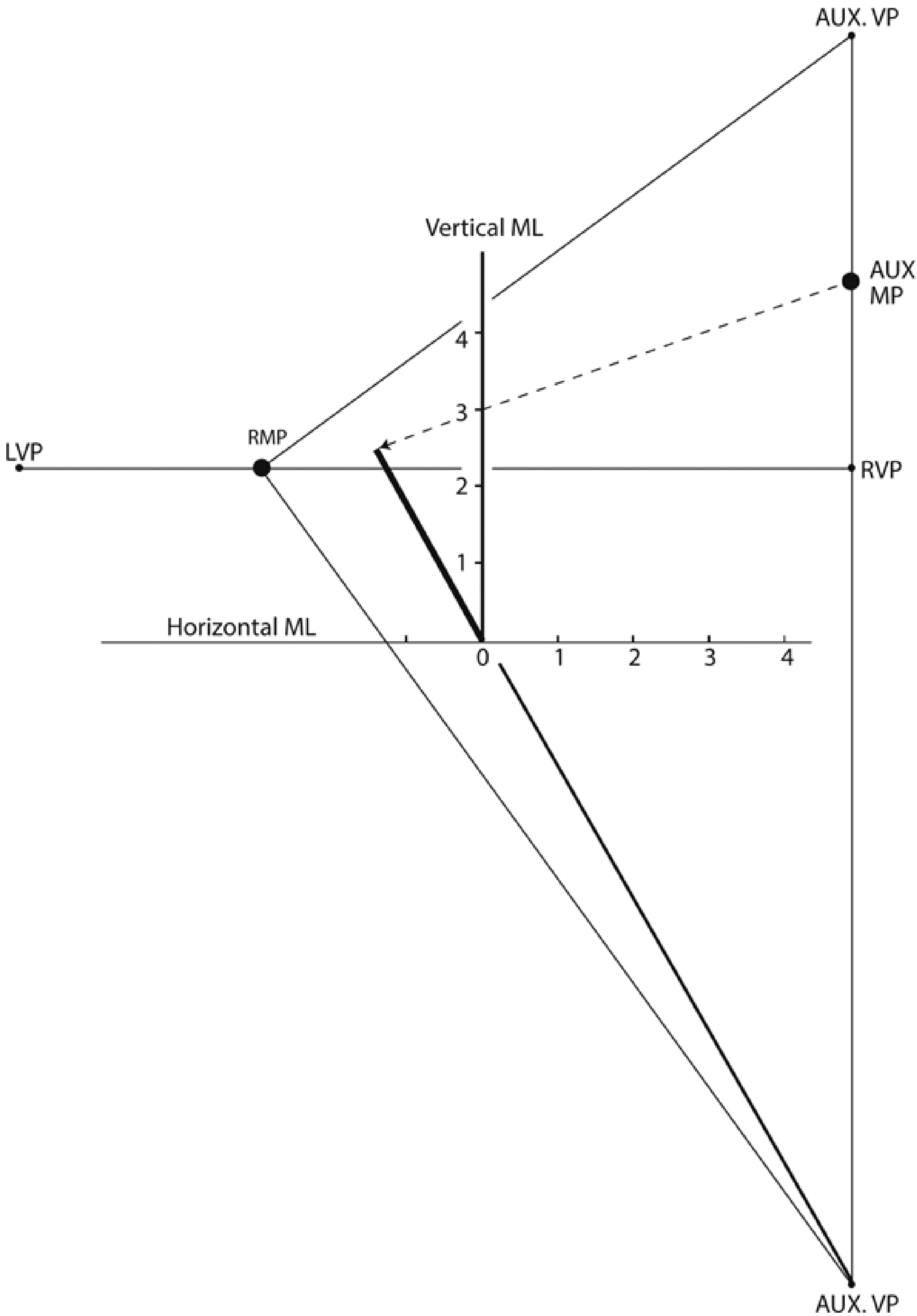


Figure 11.5 Using a vertical measuring point and a vertical measuring line to measure a 3 unit length.

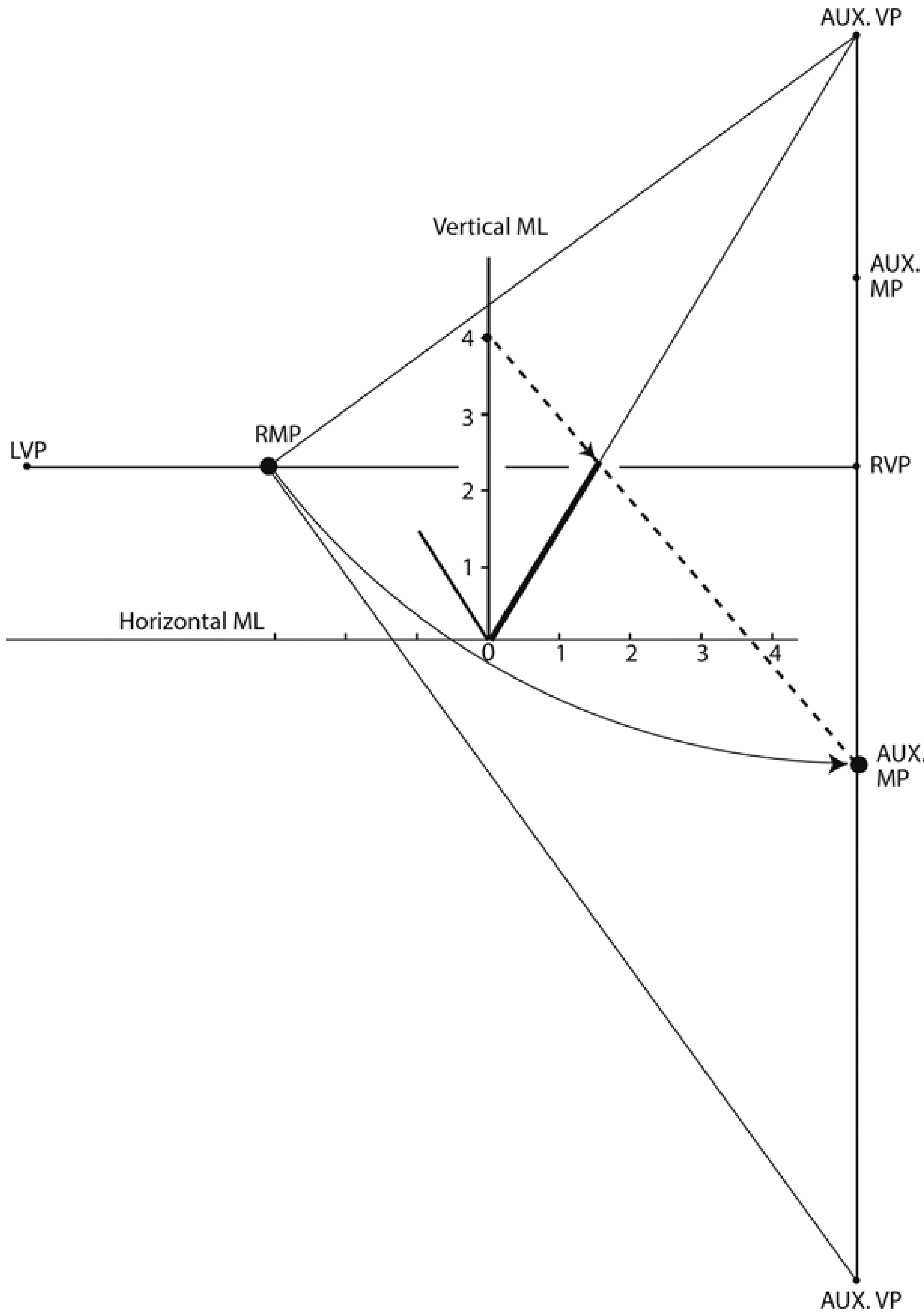


Figure 11.6 Using a vertical measuring point and a vertical measuring line to measure a 4 unit length.

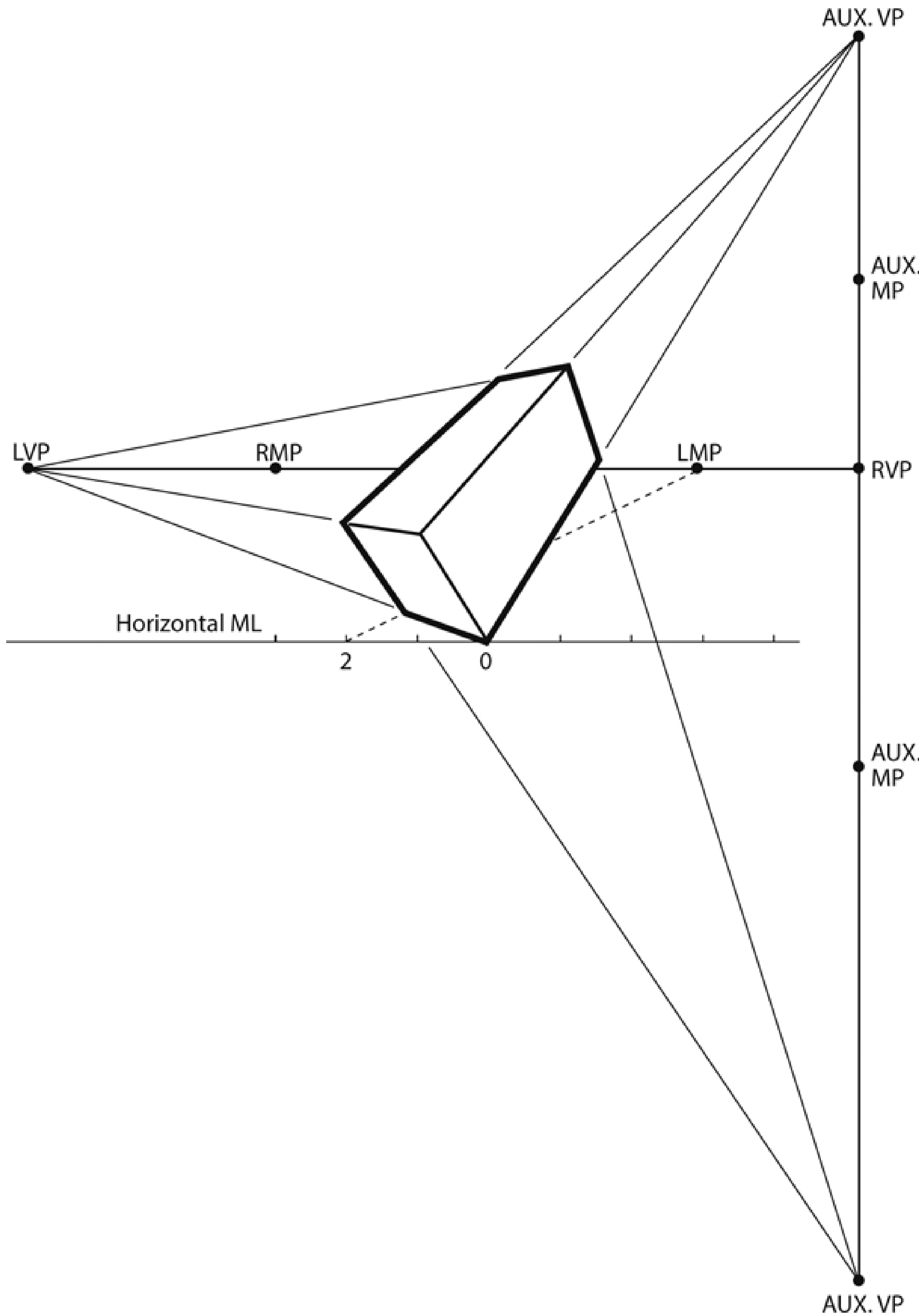


Figure 11.7 Connect corners to vanishing points to complete the box. Use the horizontal measuring line to measure the depth (2 units).

More on Measuring Point Geometry

Measuring points can be on horizontal lines or vertical lines. These are the two most logical and practical locations. Placing them elsewhere is not advised—but it is possible.

The measuring point can be anywhere, if two crucial rules are adhered to: 1) the measuring point must be the same distance from the vanishing point as the vanishing point is from the station point; and 2) the measuring line must be parallel with the line the measuring point is on. If these two rules are followed, the measuring point will draw an isosceles triangle (Figures 11.8–11.10).

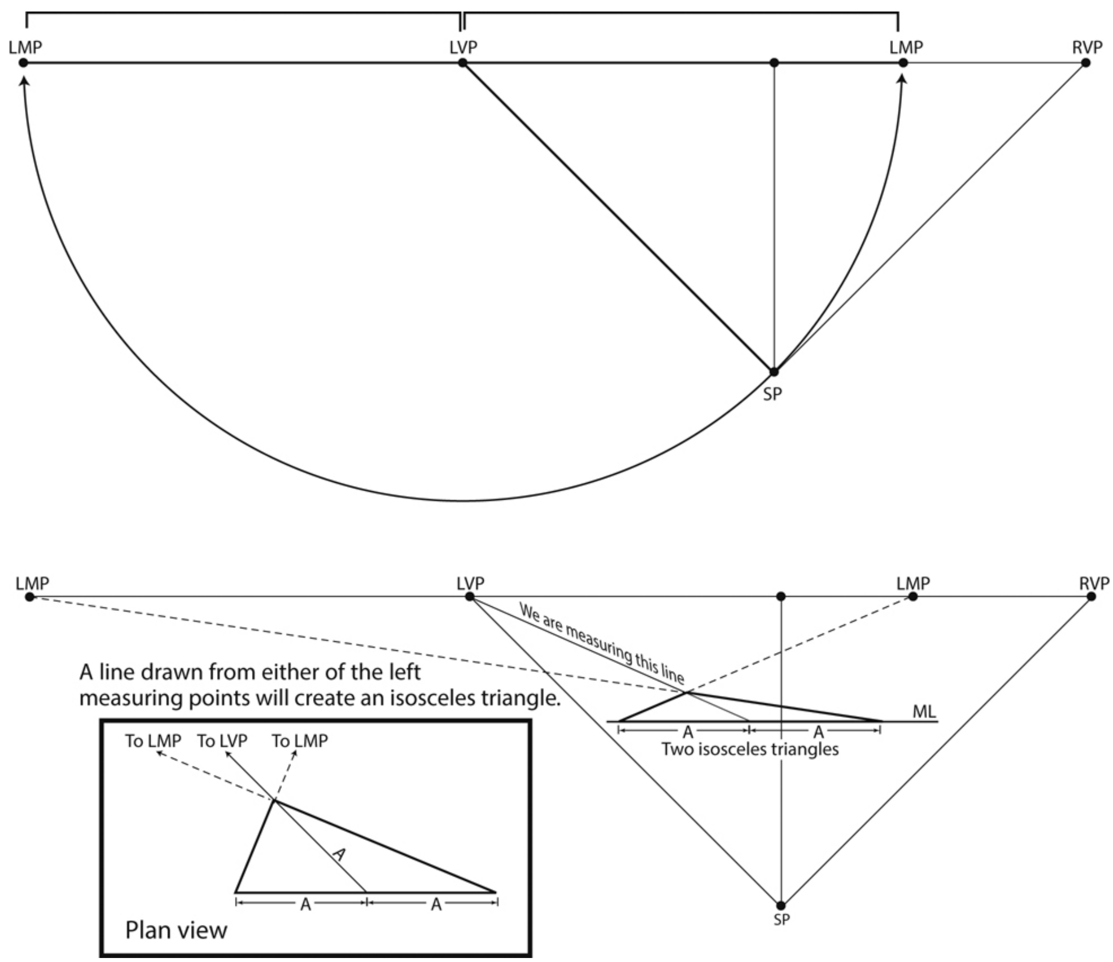


Figure 11.8 The measuring point can be placed to the right or the left of the vanishing point. Both measuring points draw isosceles triangles. Both can be used to measure lines that connect to the left vanishing point.

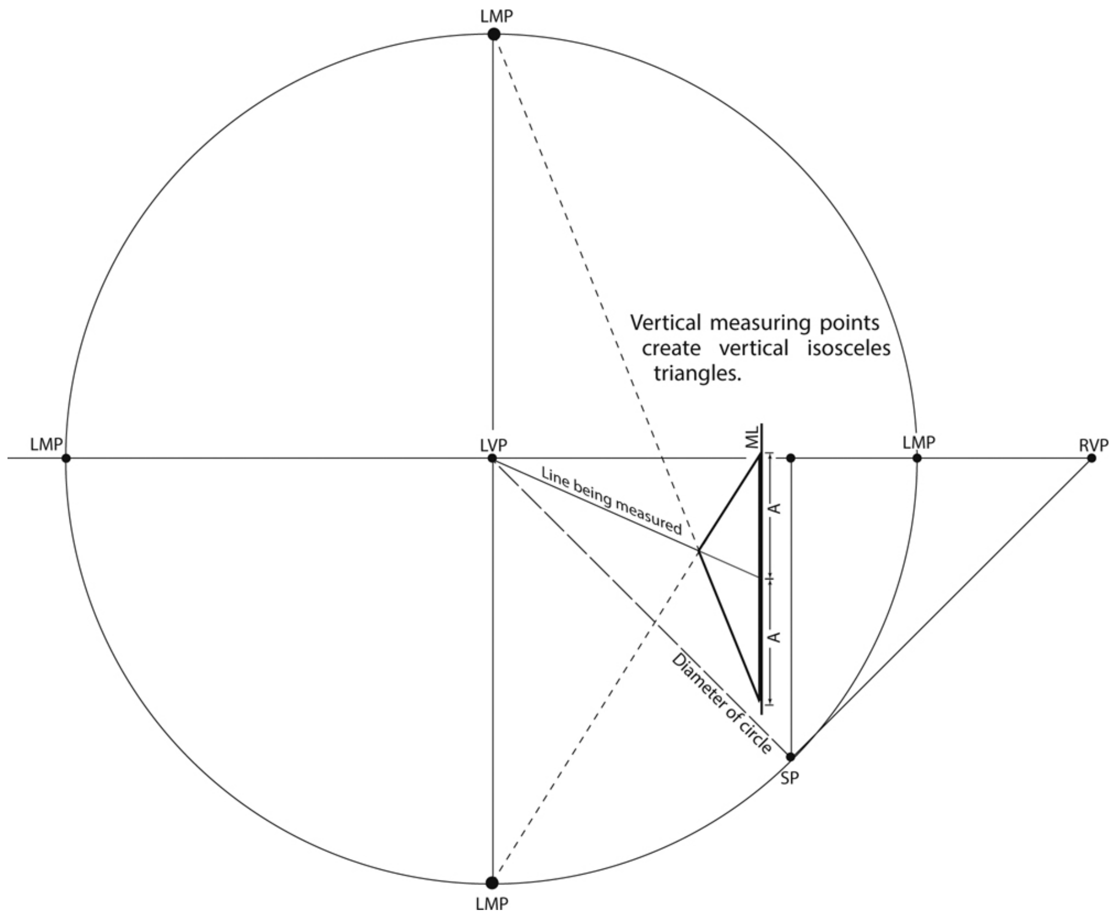
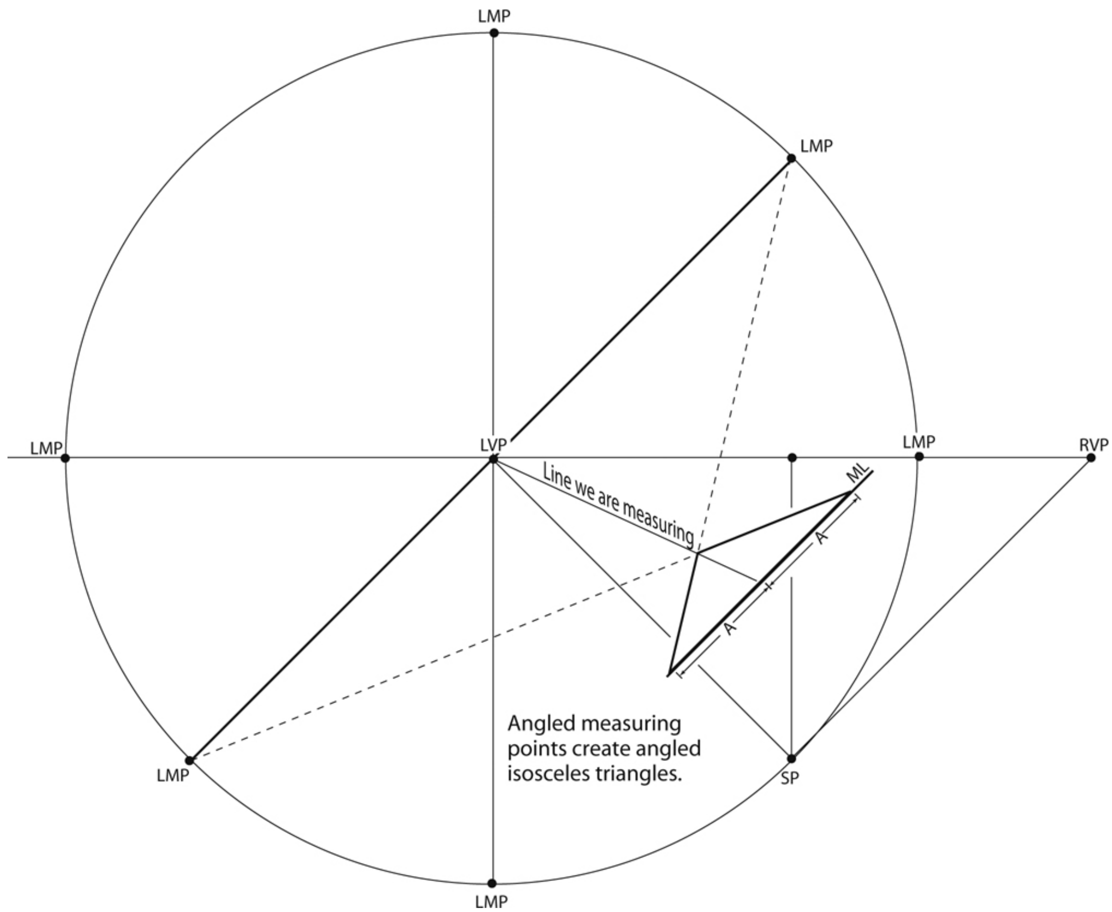


Figure 11.9 Measuring points can be on a vertical line, but the measuring line must also be vertical.



[Figure 11.10](#) **Measuring points can be on a diagonal line, if the measuring line is at the same angle.**

12 The Problem of Distant Vanishing Points

When plotting vanishing points, they can, from time to time, fall off the page, sometimes very far off the page. Before buying a very long desk and ruler, and taping paper together end to end, there is another solution. It borrows a page from the grid approach to perspective.

The Grid Approach

If the vanishing point is off the page, create a grid directing the lines to that distant point. This grid is based on the fact that evenly spaced parallel lines remain proportional as they recede.

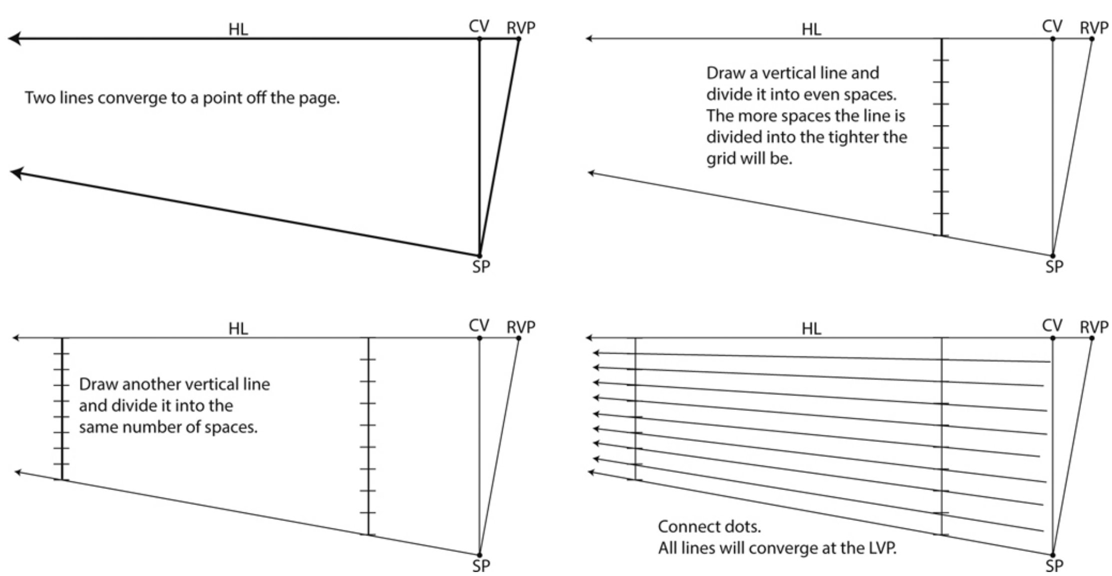


Figure 12.1 Creating a grid to find a vanishing point off the page.

The line drawn from the station point leads to a vanishing point far off the page. Knowing this, divide the distance between that line and the horizon into evenly spaced segments. The number of divisions used are at the artist's discretion. The more divisions, the tighter the grid (Figure 12.1). This grid serves as a guide to the distant vanishing point. Once the grid is in place, use it to gauge the direction of diminution. Create as fine a grid as needed to guide the drawing (Figure 12.2).

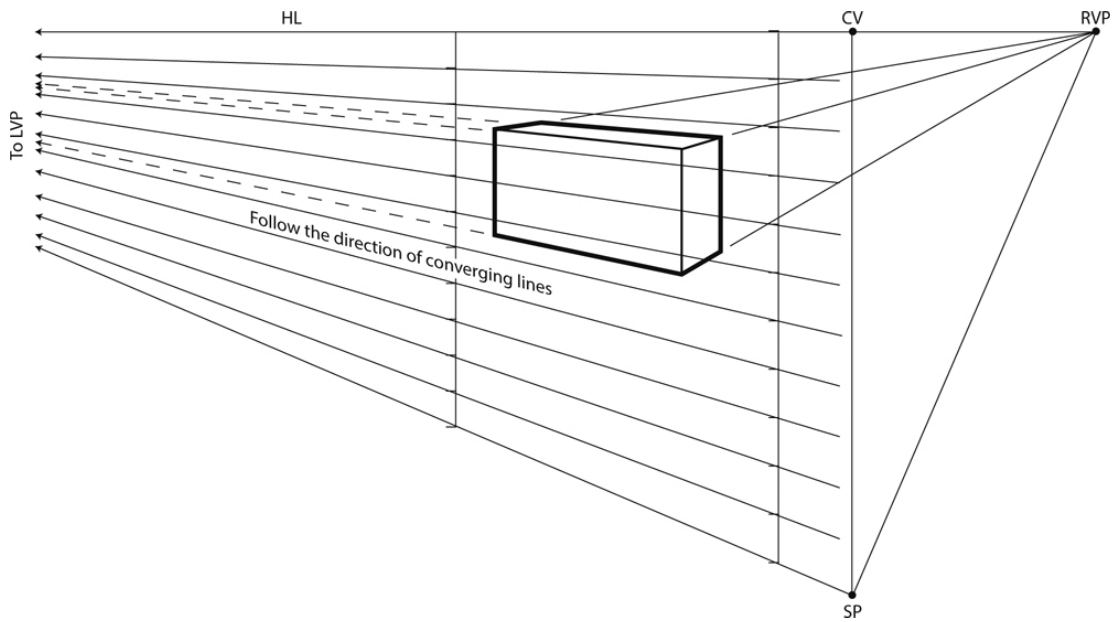


Figure 12.2 Drawing a box using the grid. The grid creates a guide. Align the foreshortened lines of the object to the grid.

Shortcut

To save some time—and some lines—here is a shortcut. Instead of dividing the length of each line into a series of even spaces, divide the line in half. Continue to divide into halves until there are enough lines to do the job. Often only a few lines are needed (Figure 12.3).

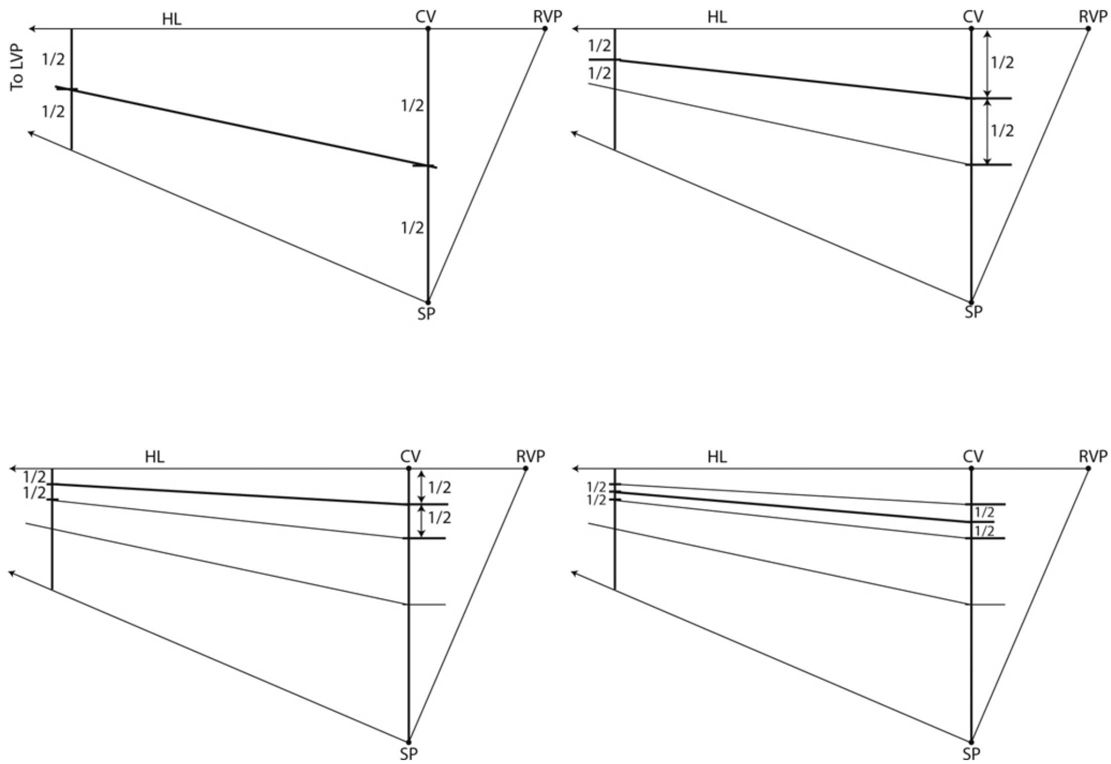


Figure 12.3 This shortcut is a fast and simple way to create a grid.

Measuring

Foreshortened lines are now able to be drawn without a vanishing point. Next, a way to measure those lines is required. But without the vanishing point, how is the measuring point located? There are a few solutions. This first method is a three-step process. First, measure the distance from the station point to the center of vision. Then transfer that distance to the line connecting the station point to the vanishing point. From this intersection, draw a right angle, projecting it to the horizon line (Figure 12.4).

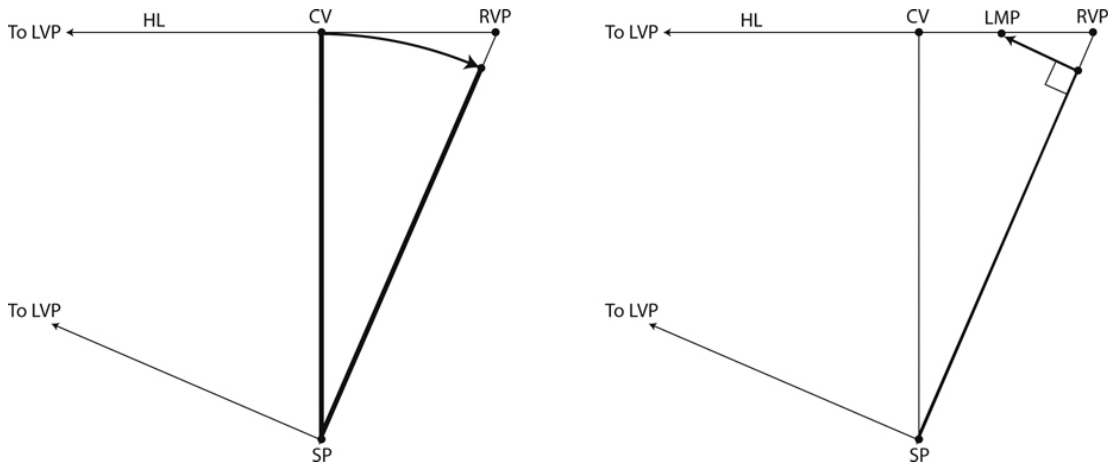


Figure 12.4 Finding a measuring point without using a vanishing point.

Inclines

This method is useful when working with inclines, where the auxiliary vanishing point is often off the page. The procedure is the same, but the diagram is turned 90°. This method necessitates the auxiliary measuring point being on a vertical line (Figure 12.5).

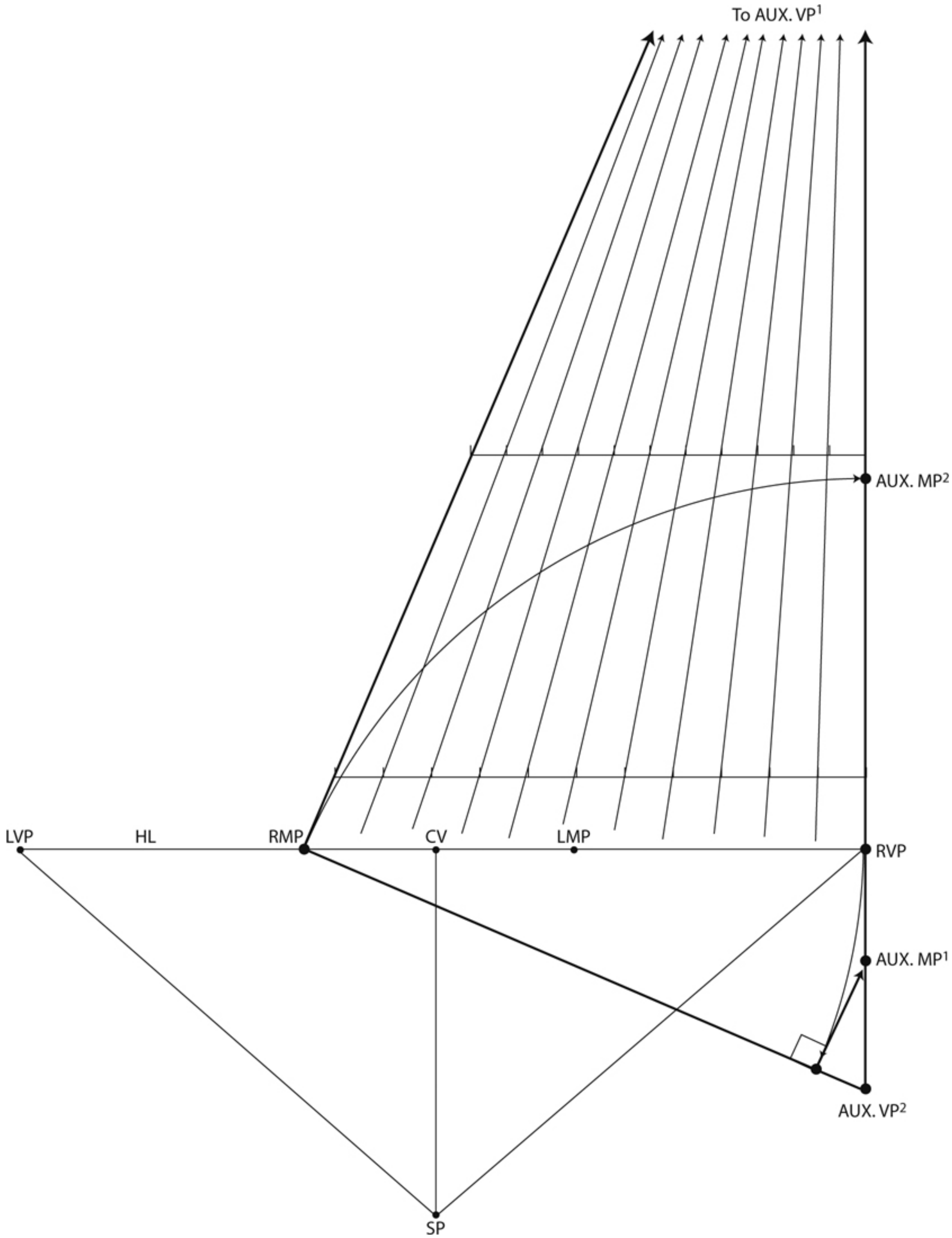


Figure 12.5 Using a grid to access a distant auxiliary vanishing point. The auxiliary measuring point is on a vertical line, so the measuring line will also need to be vertical.

Using a Protractor

An alternative method to locating the measuring point is using a protractor. The measuring point is half way (in degrees) between the center of vision and the vanishing point. For example, if the angle between the line of sight and the right vanishing point is 24° then the measuring point is at 12° (Figure 12.6).

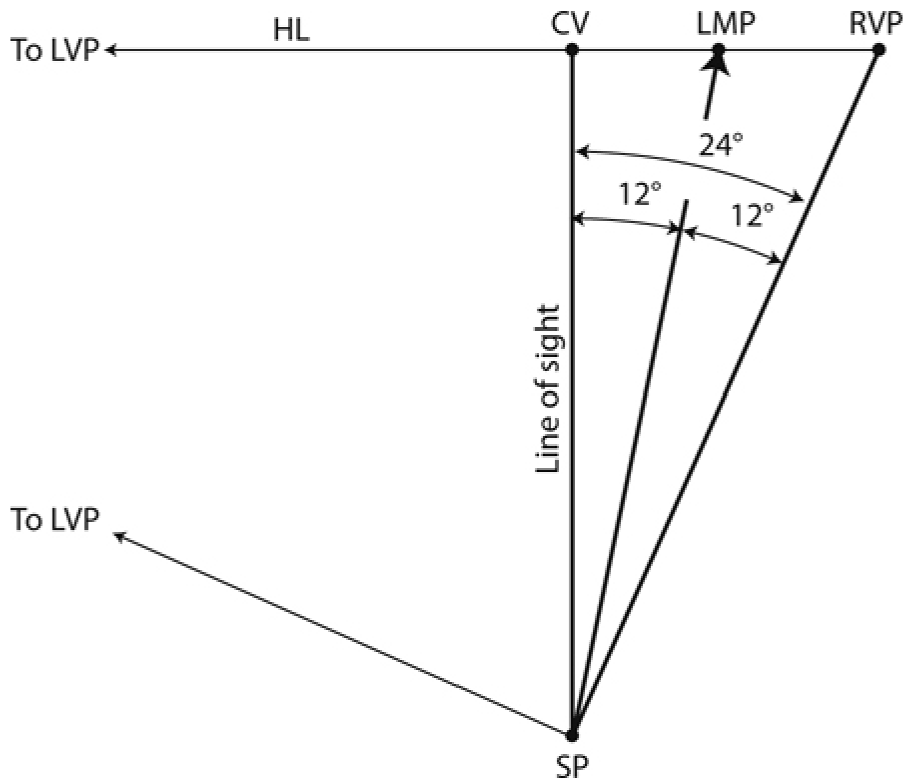


Figure 12.6 Using a protractor to find a measuring point.

13
Falling and Rotating Forms

Auxiliary vanishing points can be daunting. Practice simple planes first, then advance to three-dimensional forms. When comfortable drawing inclines at various angles, apply that knowledge to more elaborate scenes. Try incorporating several objects in a drawing, relating one object's movements to another.

Rotating Objects Example

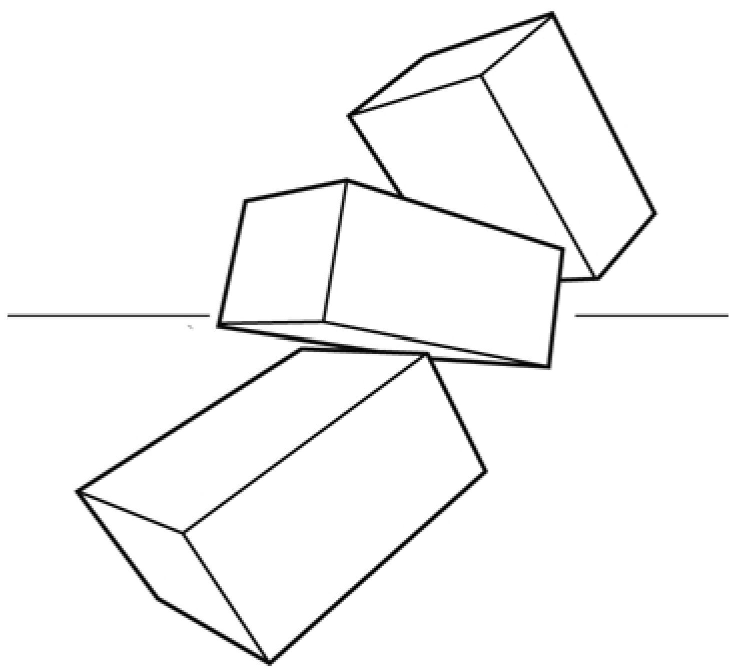


Figure 13.1 The final drawing. A sequence of three boxes, each 1 unit tall, 2 units long, and 2 units deep, along a curved trajectory.

Here is an example combining information from Chapter 8 and Chapter 10. In this example, a curved line will be drawn in perspective, with a series of three objects rotating along that curve. They are spinning while they travel along a curved trajectory (Figure 13.1). This is more ambitious than previous examples, but there is nothing new in this series—the information has been covered before. Since much of this chapter is combining previous procedures, prior pages are referenced when appropriate.

The Axis Point

Assuming an object is balanced, and not heavier on one side, it will rotate along a central axis. To accurately measure the object, it must be measured from the axis point (the center of the box).

The Angles

The box is 1 unit high, 2 units long, and 2 units deep. Each sequence is separated by 20°. The first box rotates 40° counterclockwise, the second 80°, and the third 120°. It is helpful to draw an elevation view of complicated inclines, as it gives a visual of the true angles (Figure 13.2). This drawing may seem onerous, but drawing an angled box is no more difficult than drawing one angled line. Similarly, drawing three angled boxes is as easy as drawing one angled box. Draw this series one box at a time, and draw each box one line at a time.

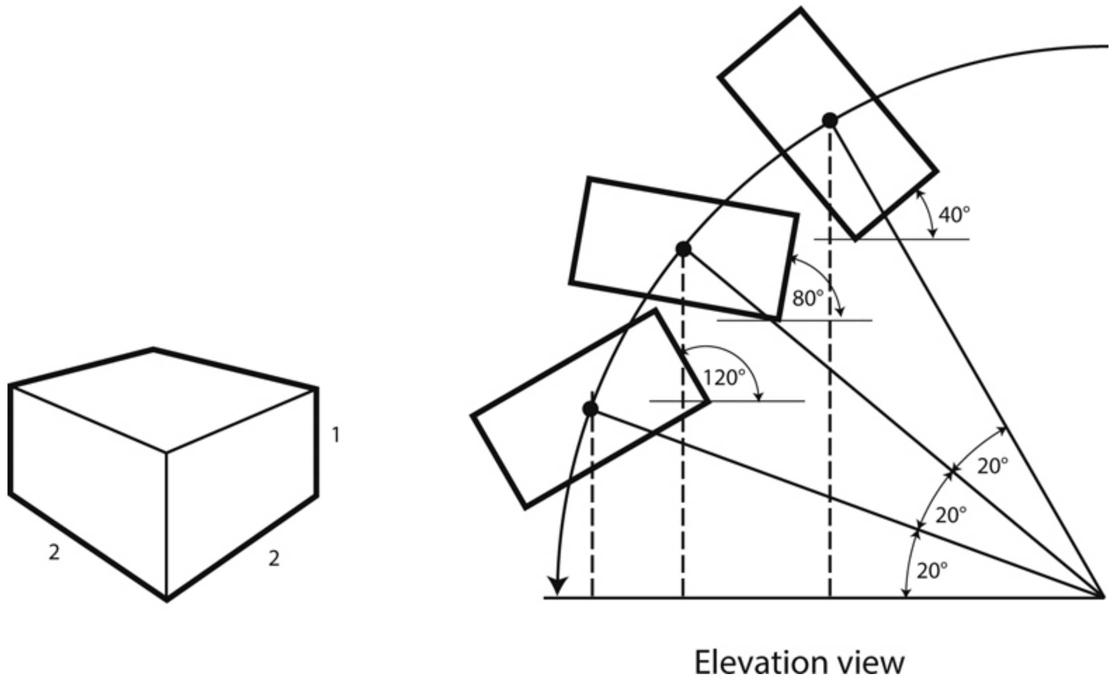


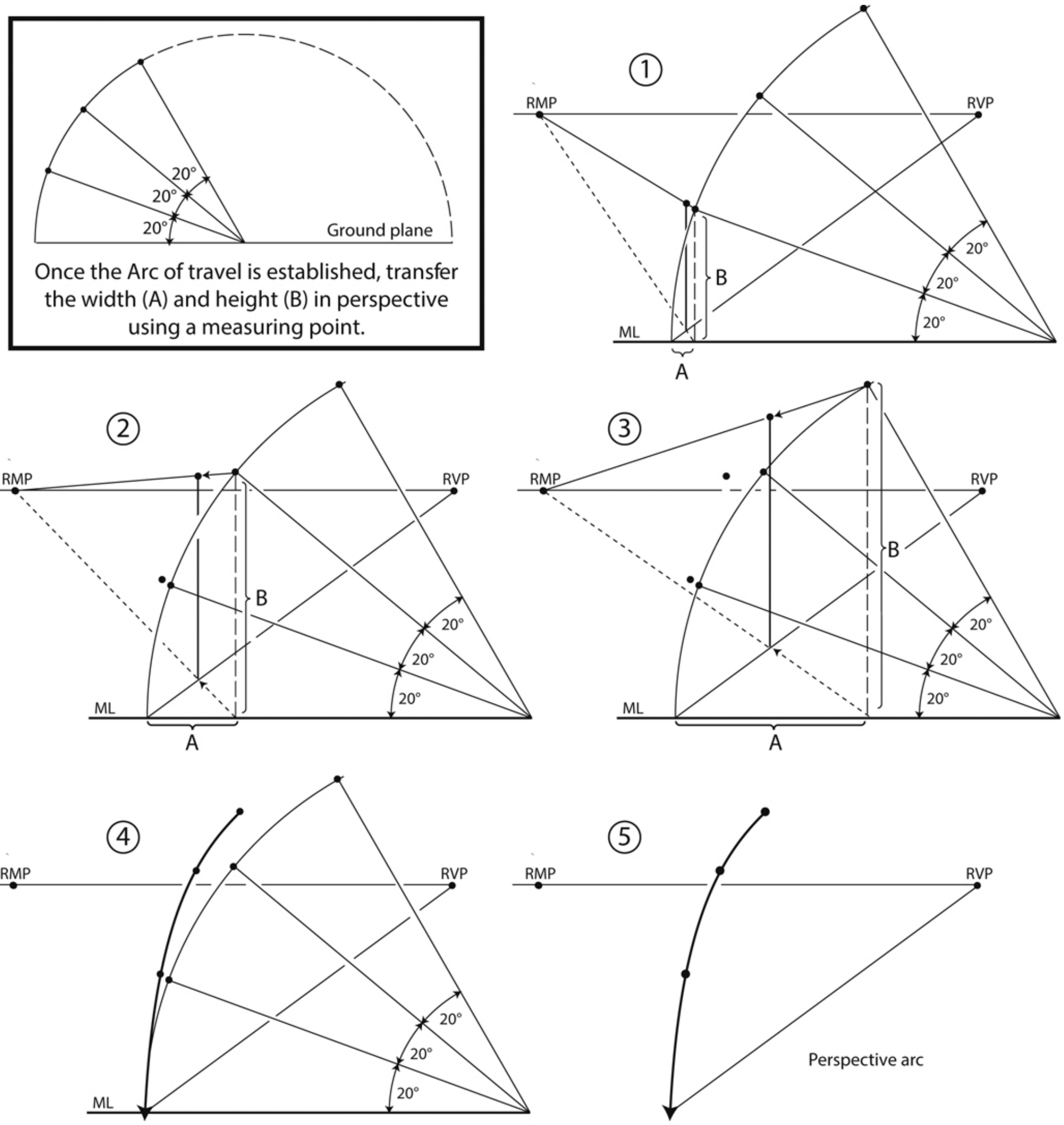
Figure 13.2 It is helpful to draw an elevation view of complicated scenarios. These are the angles and dimensions of the falling, rotating box.

The Arc

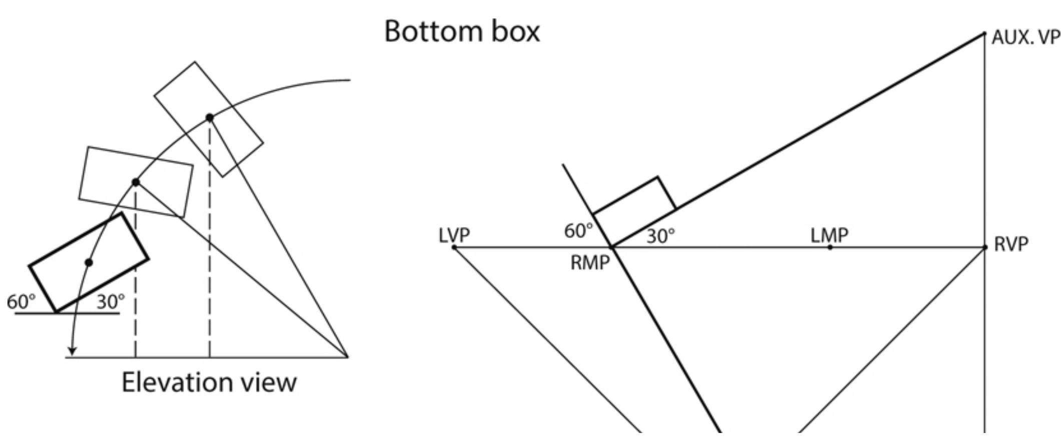
Begin by drawing the **arc**. This is the path the boxes will follow. The center of each box is aligned with this curve (Figure 13.3). Refer to Chapter 8, Figure 8.27.

Auxiliary Vanishing Points

After the arc and the center of all three boxes are plotted, the next task is to locate the auxiliary vanishing points. Each box is at a different angle, so each box will need a different pair of auxiliary vanishing points (this is best done on separate overlays). When drawing inclines, true angles are found at the measuring point. It is helpful to think of the measuring point as an elevation view of the incline. Place the angles given in the following instructions on the measuring point. Then project those angles to create auxiliary vanishing points (Figure 13.4).



[Figure 13.3](#) Drawing the arc of travel, and the center points of each box (see for [Figure 8.27](#) another example of plotting points along a curved line).



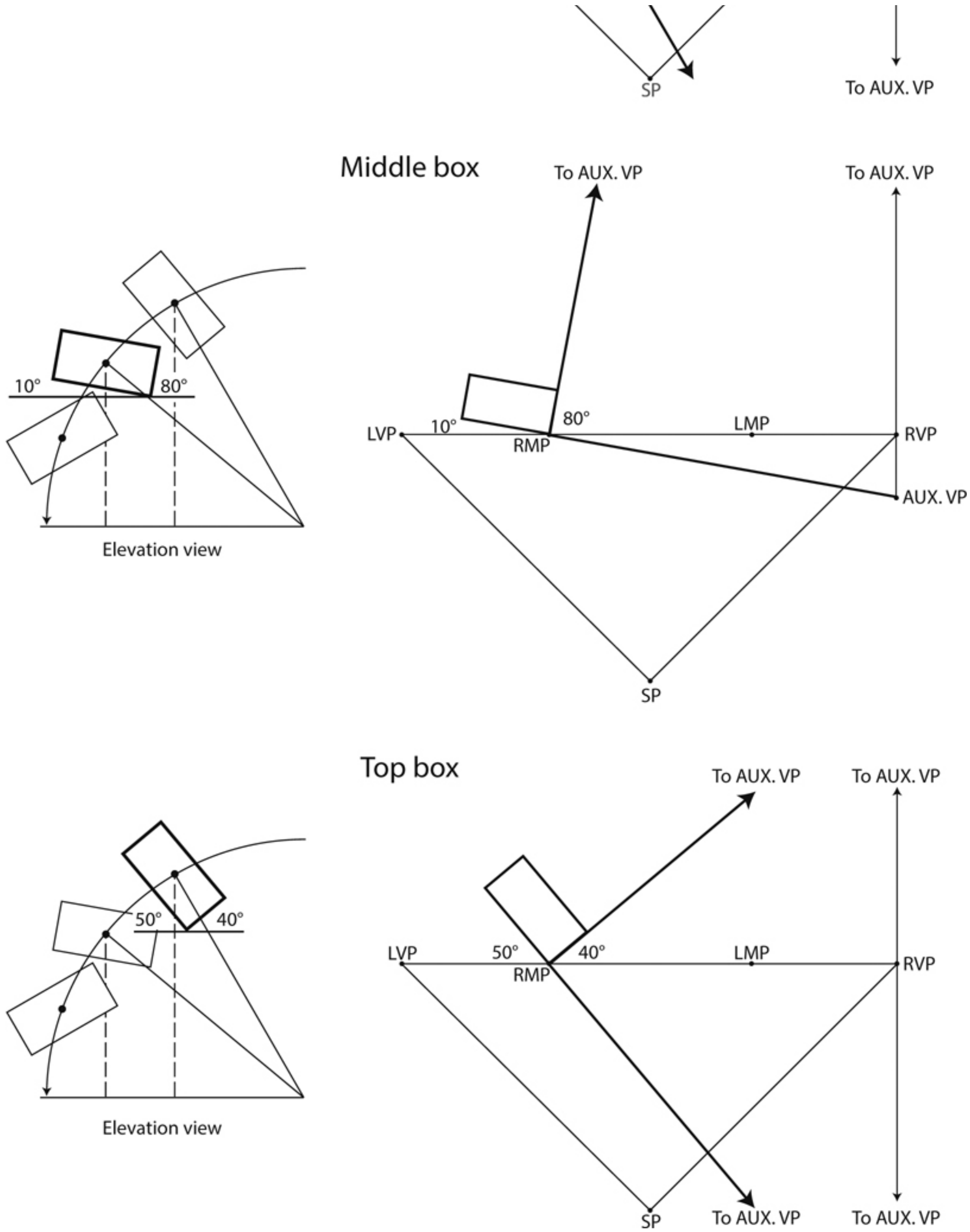


Figure 13.4 Auxiliary vanishing point angles for each box.

Bottom Box

Start with the box closest to the viewer—the bottom box. This box is tilted at a 60°/30° angle to the ground plane (Figure 13.4, top). From the measuring point, project a 30° angle to find the upper auxiliary vanishing point. Then, project a 60° angle to find the lower auxiliary vanishing point (Figure 13.5).

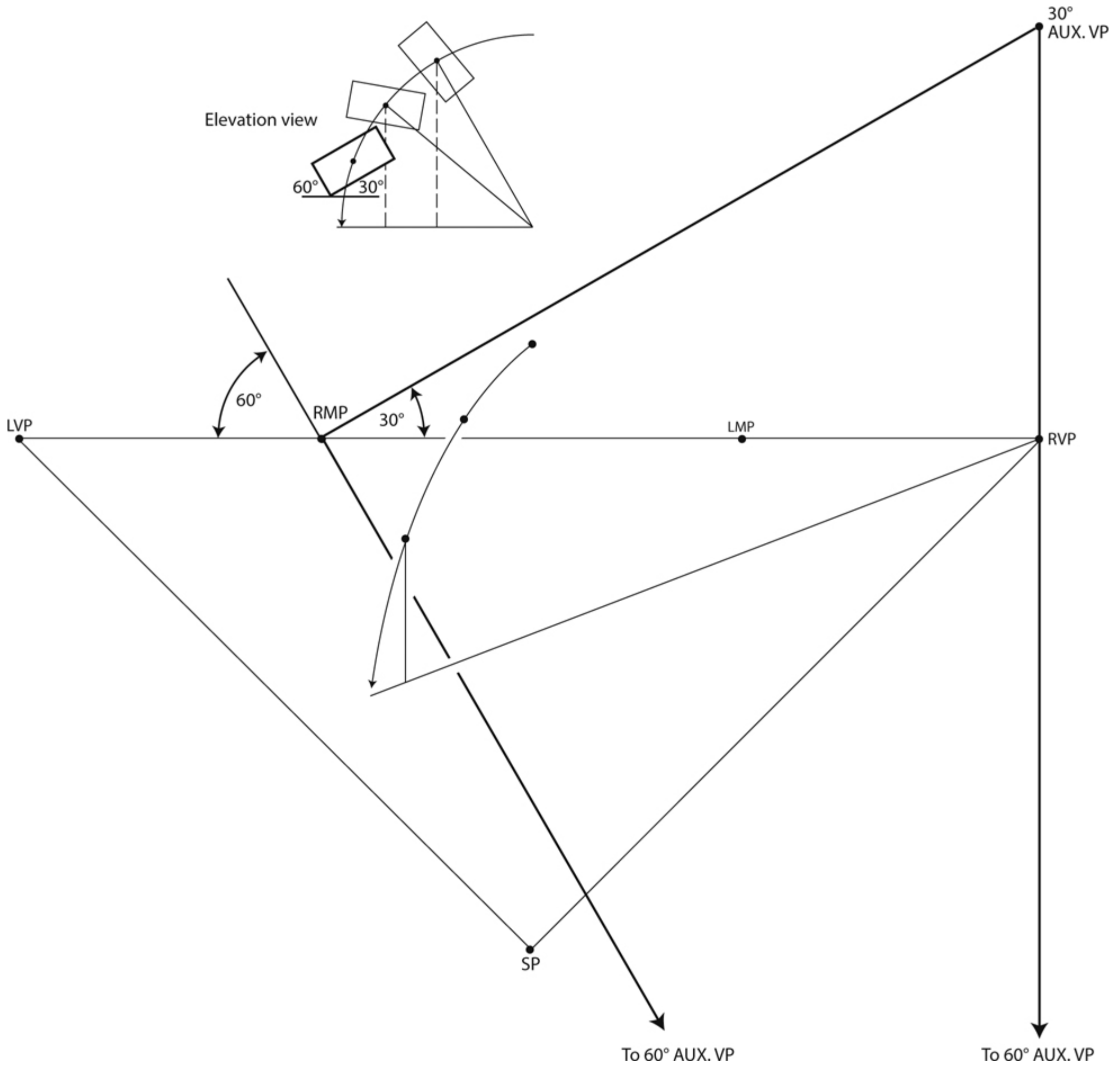


Figure 13.5 **The box closest to the viewer is angled 60°/30° from the ground plane. Place those same angles at the measuring point.**

Auxiliary Measuring Points

Next, establish the auxiliary measuring points (Figure 13.6). To save space, use vertical measuring points. Refer to Chapter 11, Figure 11.2.

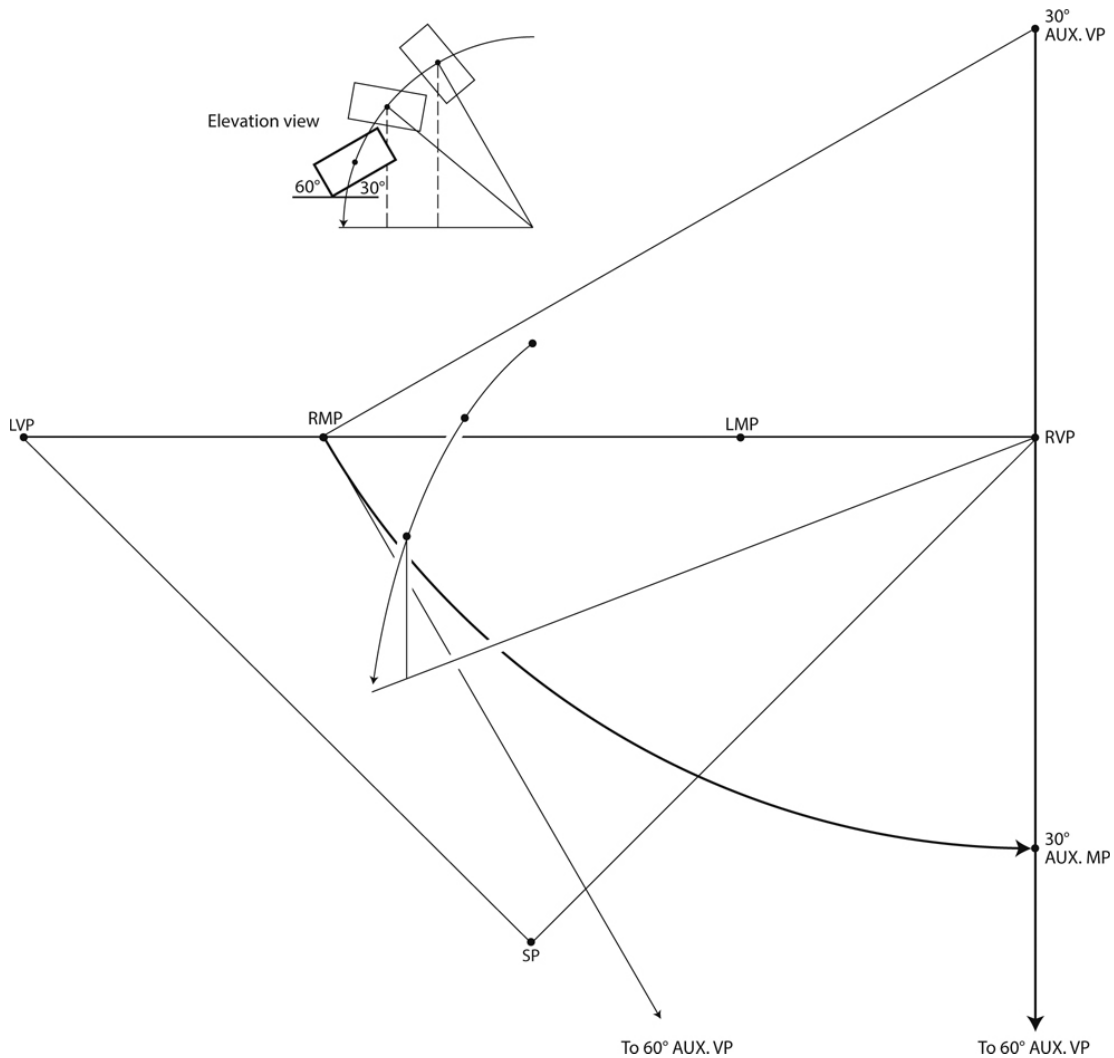
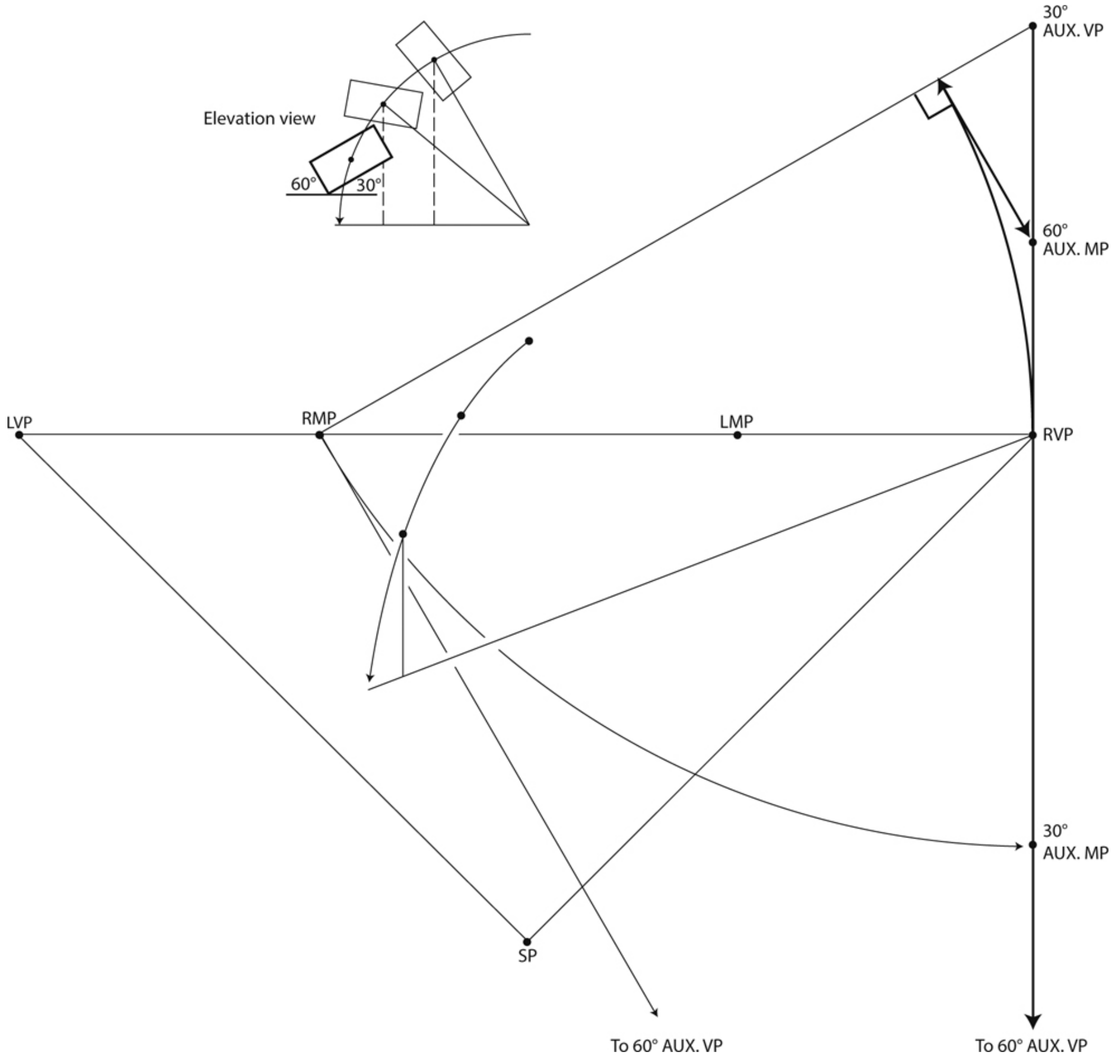


Figure 13.6 Placing the auxiliary measuring point on a vertical line keeps it nearby.

The lower auxiliary vanishing point is off the page. Use the technique outlined in Chapter 12, Figure 12.4 to compensate for this plight (Figure 13.7).



[Figure 13.7](#) **Establishing a vertical auxiliary measuring point when the auxiliary vanishing point is too far to reach.**

Measuring Depth

The depth is 2 units. Measure from the center of the box, 1 unit on each side of the axis point. If using a vertical measuring point, make sure the measuring line is also vertical and positioned at the axis point of the box, touching the line being measured ([Figure 13.8](#)).

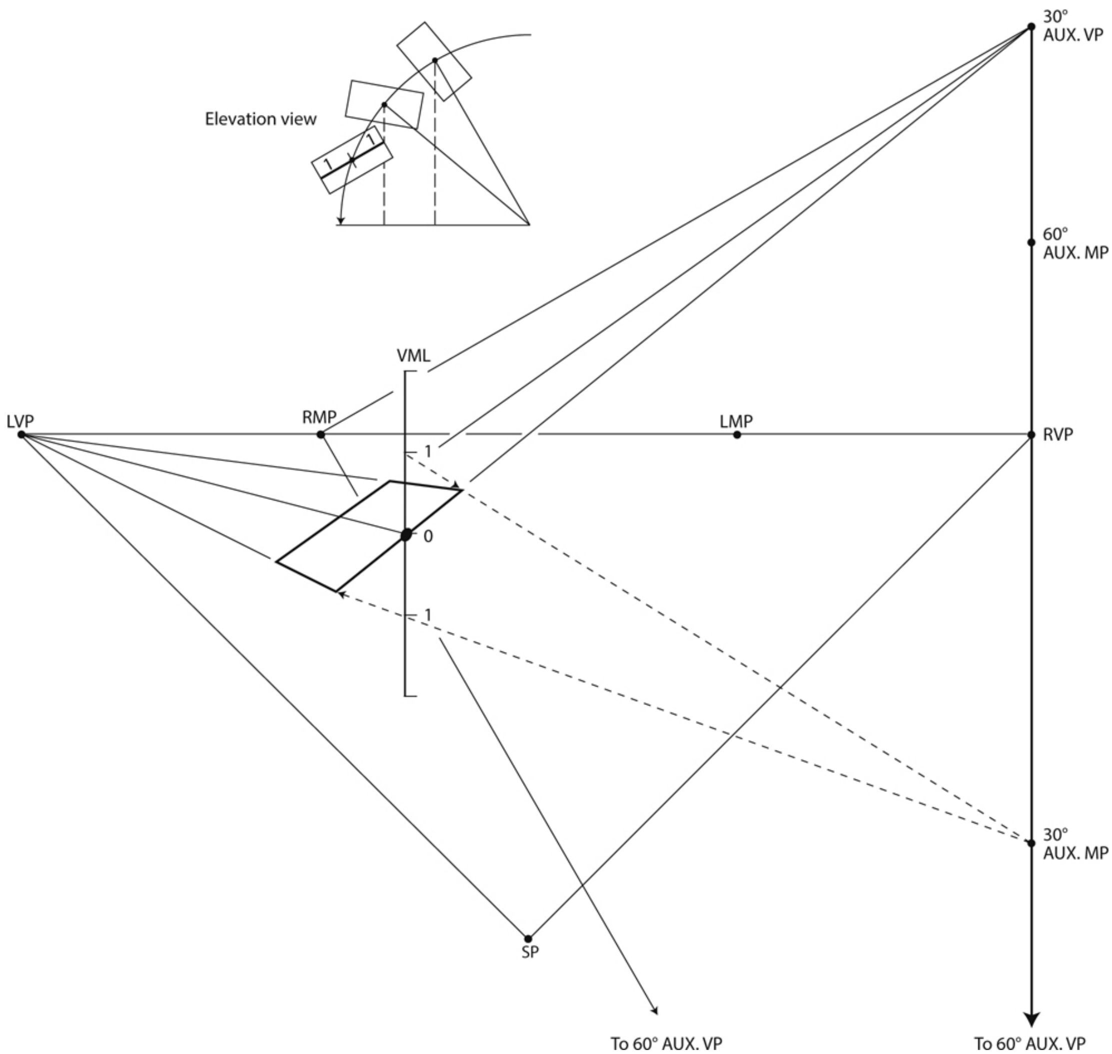


Figure 13.8 Measuring 2 units along the center plane.

Measuring Height

The height is 1 unit. Since the lower auxiliary vanishing point is off the page, use the technique outlined in Figure 12.3 to create a grid. The grid assists in guiding the direction of the lines. Make the grid as tight as needed (Figure 13.9).

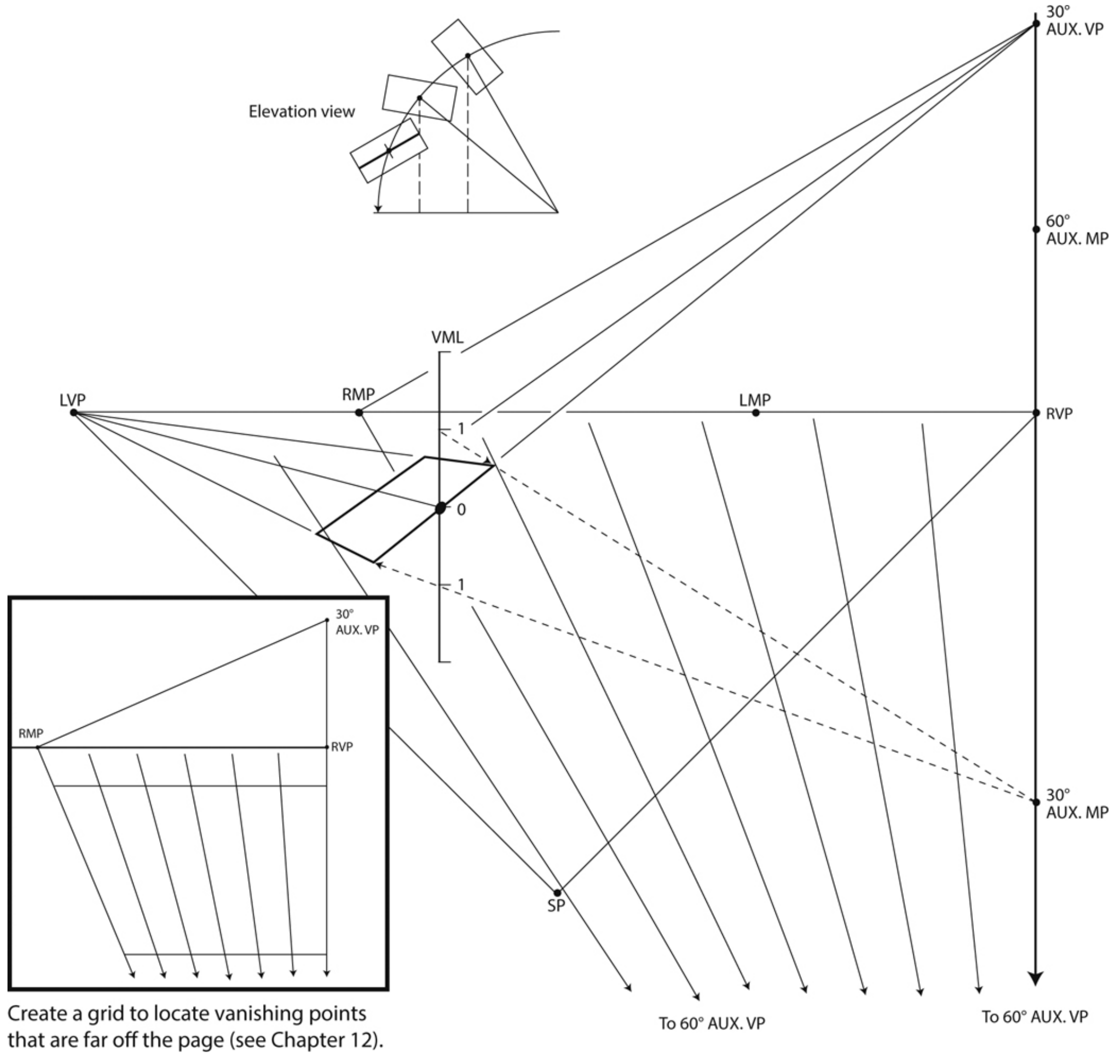
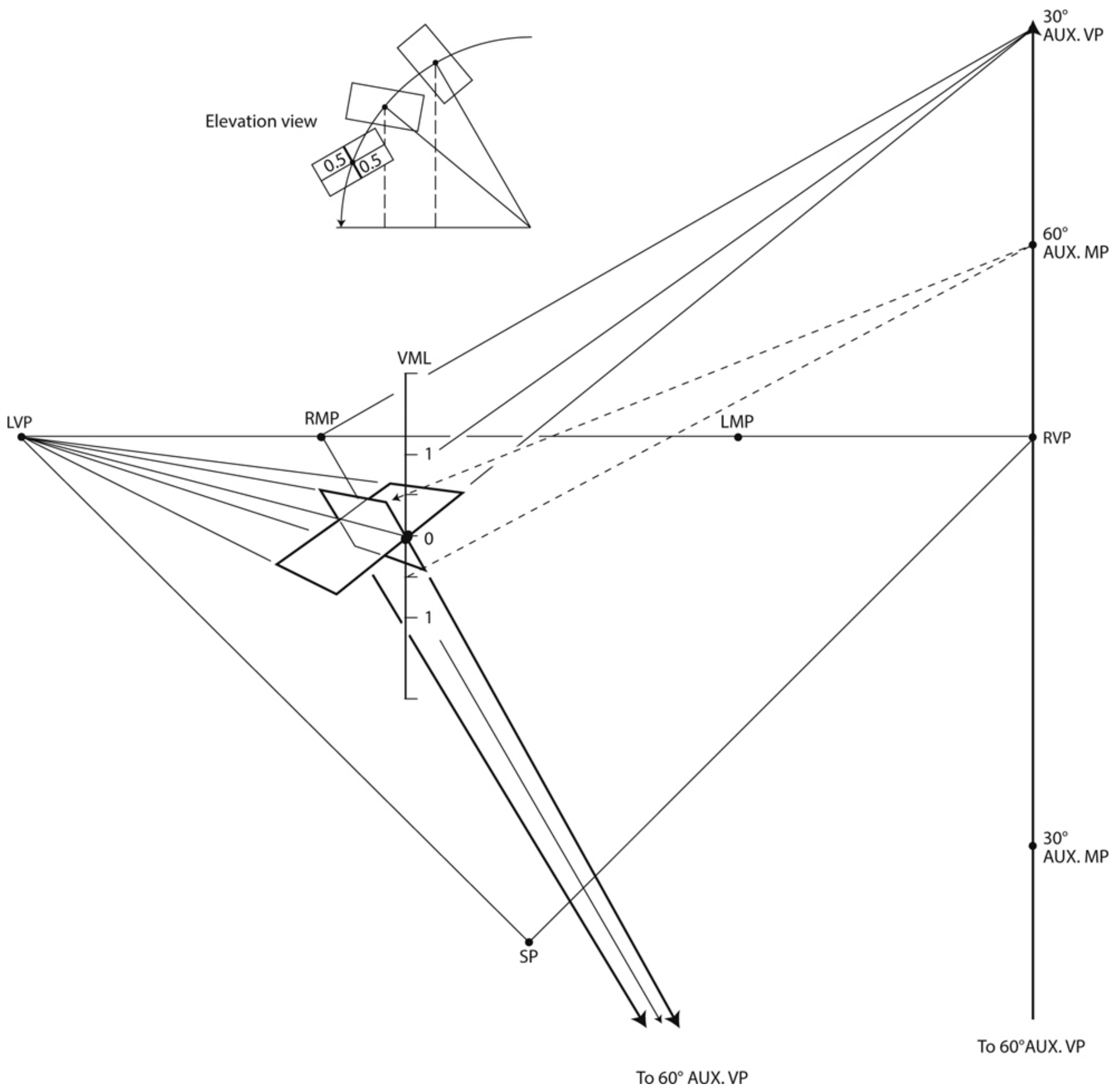


Figure 13.9 Create a grid to guide lines to the lower auxiliary vanishing point. See Chapter 12 for step-by-step instructions.

Measure 0.5 units on each side of the axis point (Figure 13.10).



[Figure 13.10](#) **Measure 0.5 units on each side of the zero point to establish the height.**

Complete the Box

Connect corners to vanishing points to complete the box ([Figure 13.11](#)).

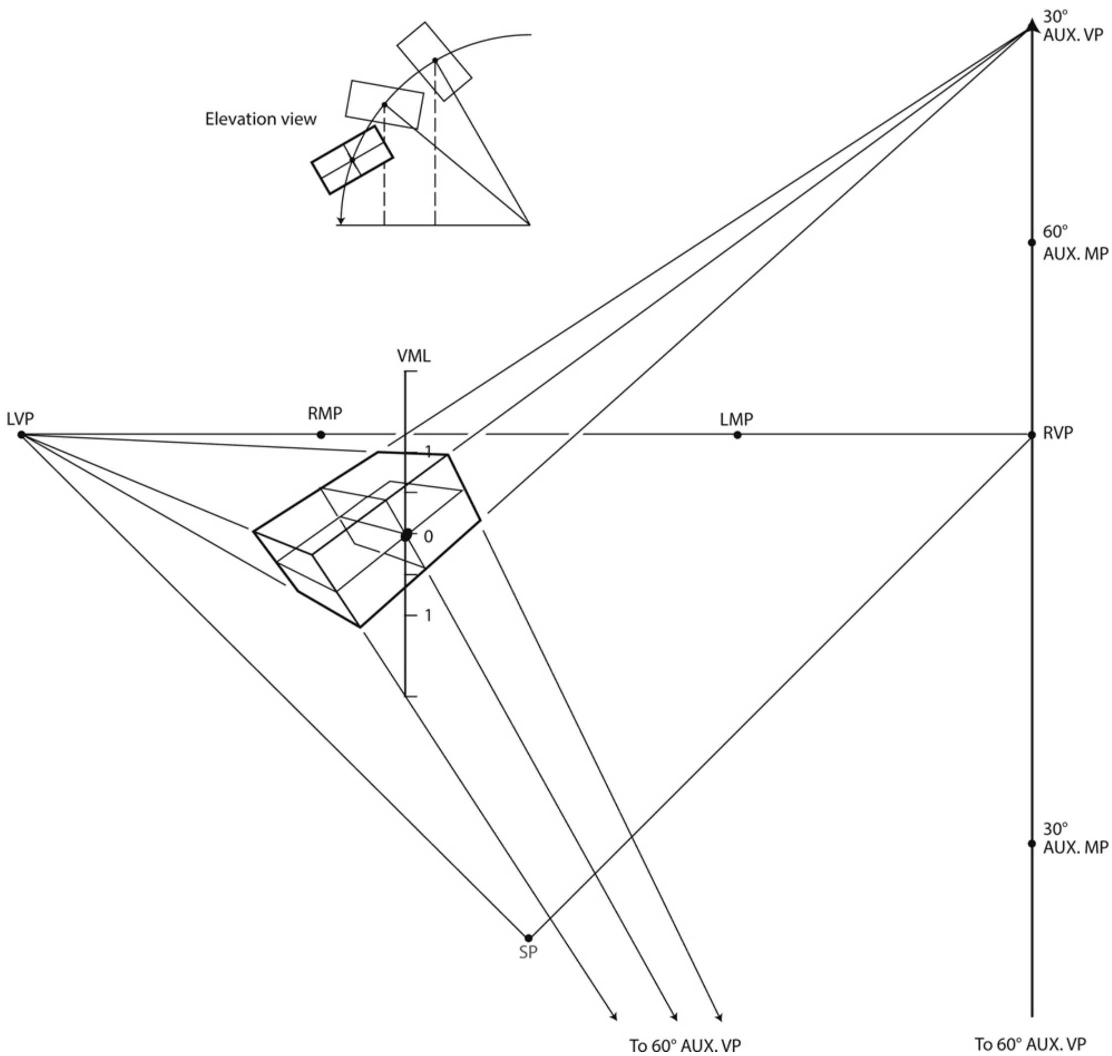
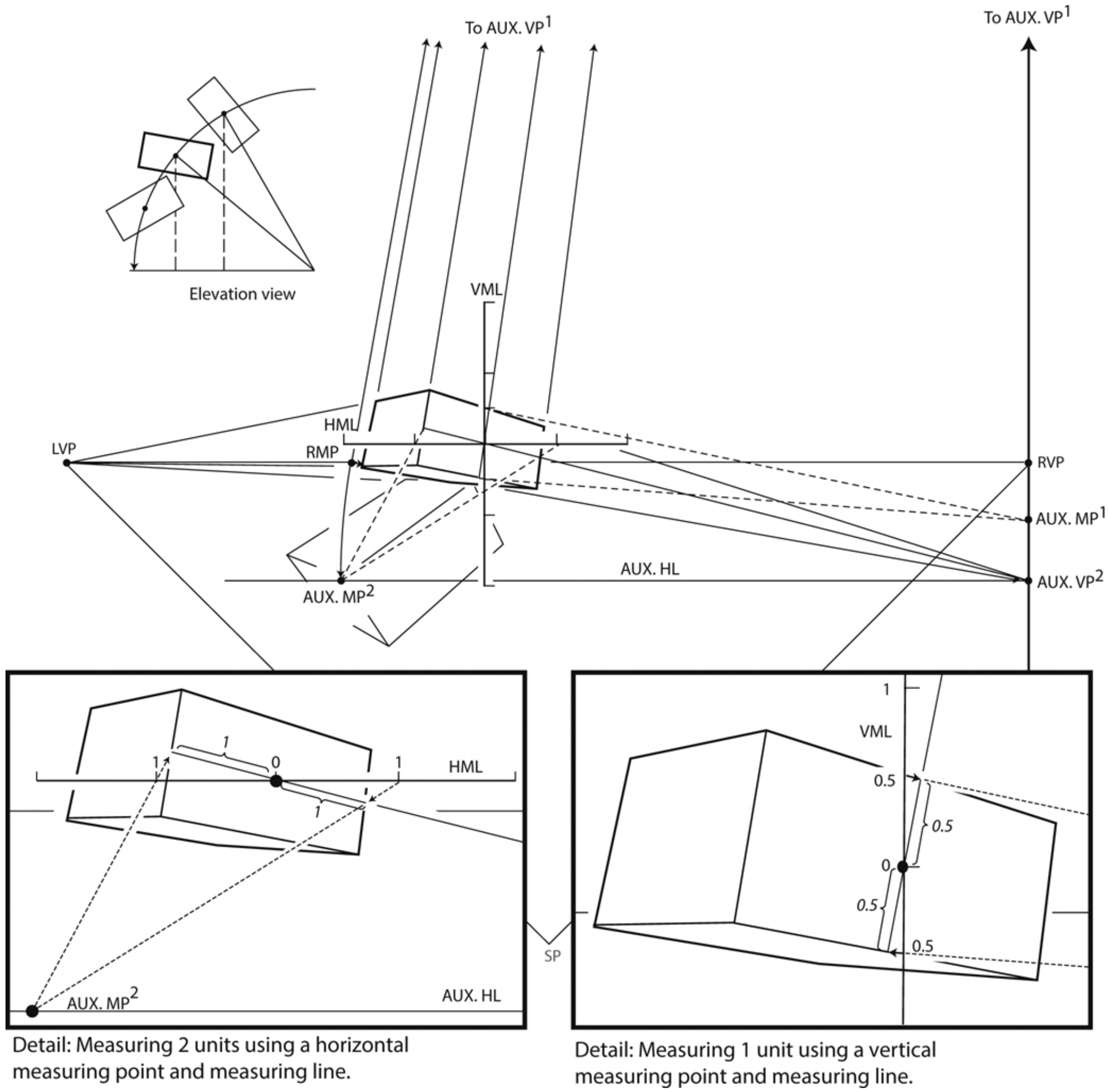


Figure 13.11 Connect to vanishing points to complete the box.

Middle Box

The middle box is drawn in the same fashion as the first. The steps are the same. However, the measuring line must be moved back in space (in perspective). The measuring line must touch the center (axis point) of the box being measured.

In this case, to measure the depth, place the auxiliary measuring point on a horizontal line. It is closer and more convenient than placing it on a vertical line (Figure 13.12). It is useful to have options. To complete this box, follow the same steps used to draw the previous box (Figure 13.13).



[Figure 13.12] **For the middle box a horizontal measuring line was used for the depth (2 units), and a vertical measuring line was used for the height (1 unit).**

Top Box

When drawing the top box, repeat the process used to draw the bottom box. Use vanishing points and measuring points that correspond to the 50° and 40° angles ([Figure 13.13]).

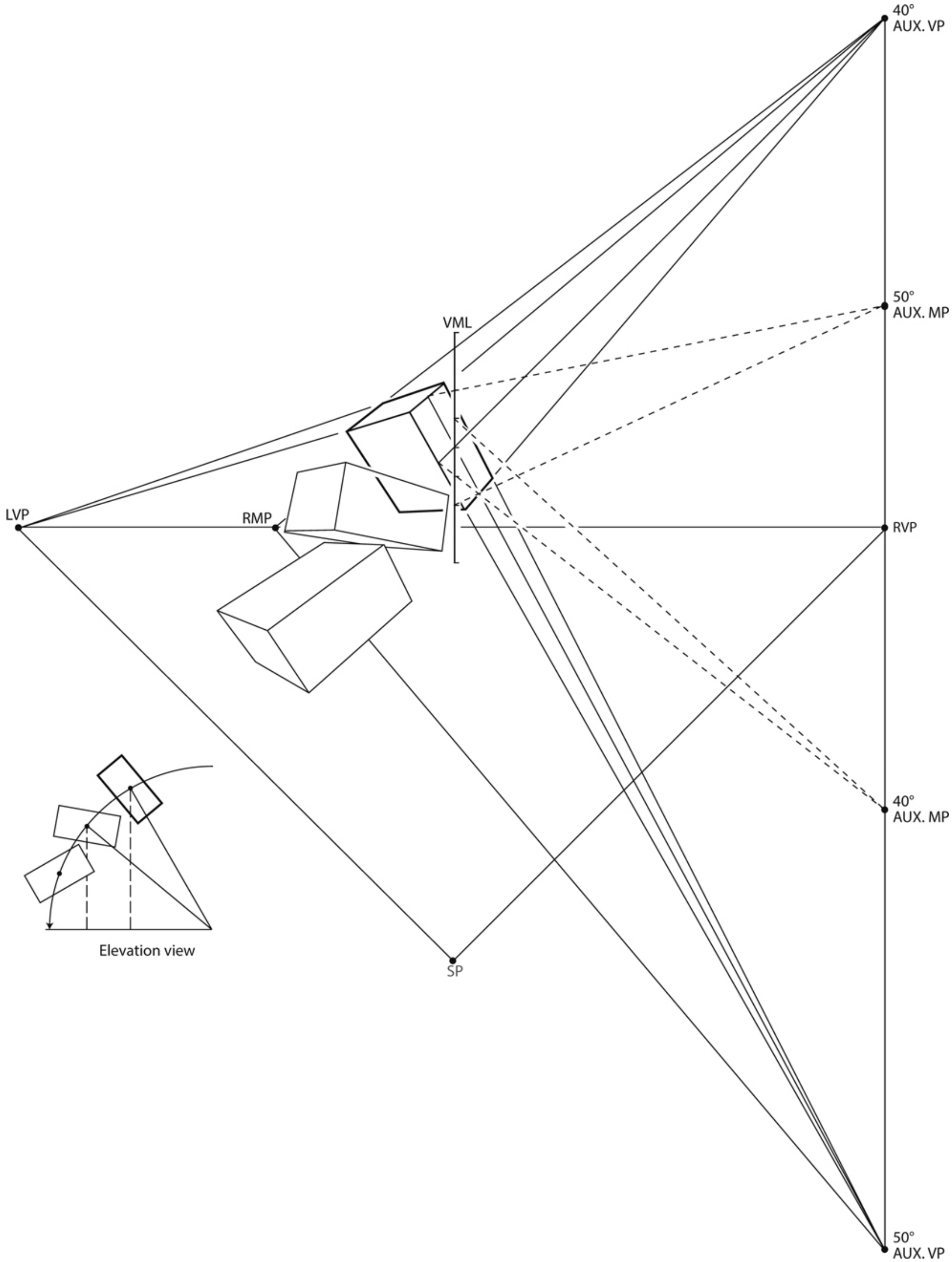


Figure 13.13 **The completed boxes.**

14
Tilted Tapered Forms

A tapered cup is simple when upright (Figure 8.16). When it tips over, it is not so simple. A tapered cup on its side creates some challenging angles, as well as an opportunity to practice ellipses.

The Cup

As an example, use the cup drawn in Figure 8.16. The cup is 1.5 units tall. The diameter of its base is 1 unit and the diameter of its top is 2 units. It may be worth reviewing Chapter 9, as this chapter builds on that information. Keep in mind that there are several ways to solve this problem. Understanding the geometry of perspective reveals a myriad of solutions. When several resolutions to the same problem can be conceived, the power of angles is beginning to be understood.

Elevation View

As pointed out previously, it is prudent to draw an elevation view of complicated inclines. An elevation view gives insight to the angles that will need to be drawn. The cup, when on its side, creates a 72° angle (rounded off) from the ground plane to its base (Figure 14.1).

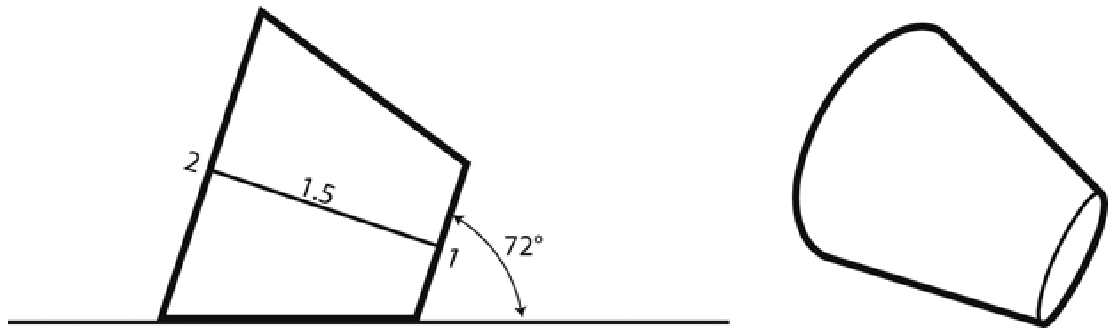


Figure 14.1 The angles and dimensions needed to draw the cup are best shown in an elevation view (left). The completed drawing is illustrated (right).

Base

The base's diameter is 1 unit. First, draw a 1 unit square, tilted 72°. Then draw an ellipse inside the tilted square (Figure 14.2). Review Chapter 8 for methods to draw an ellipse.

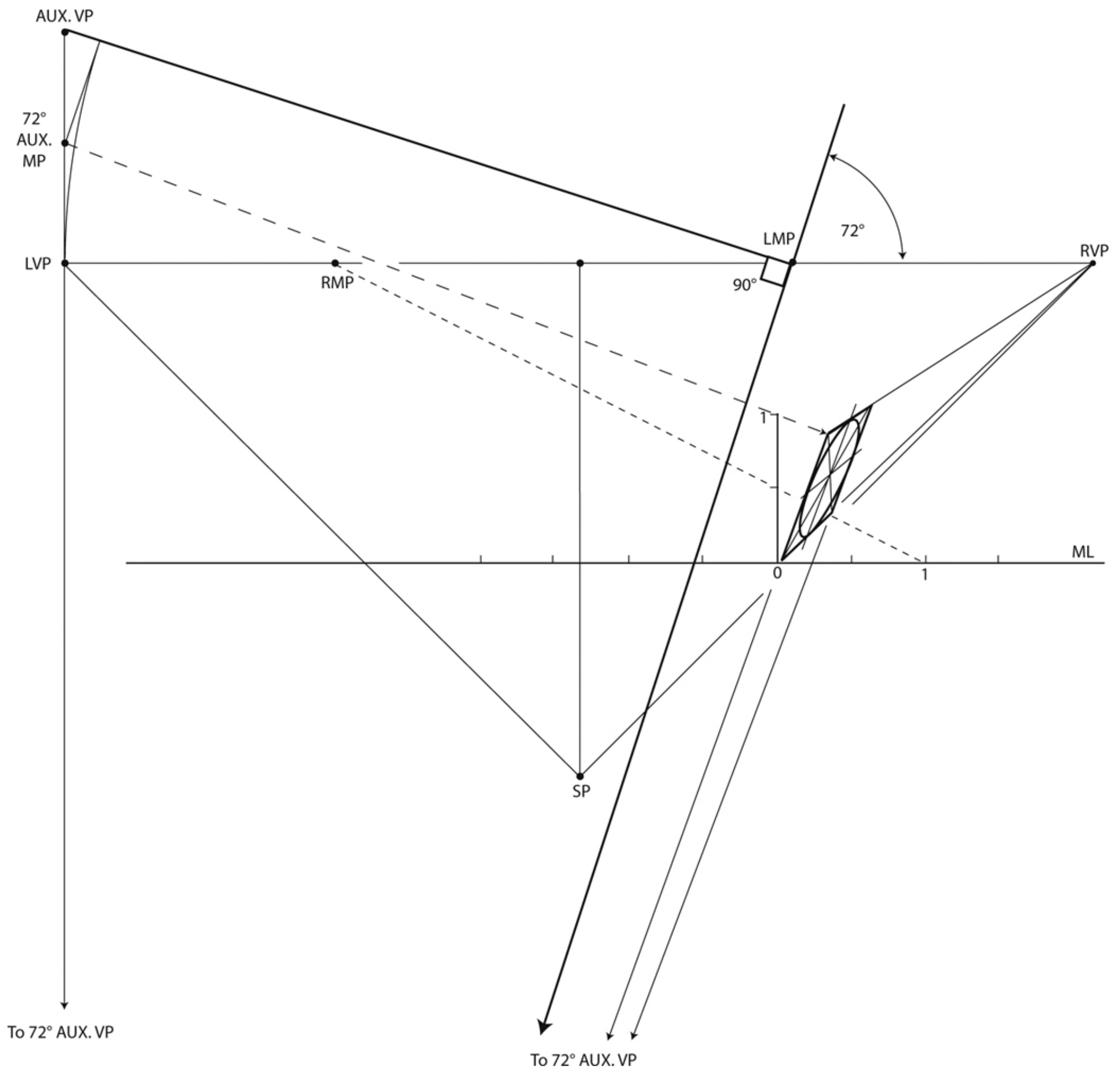


Figure 14.2 **Using the left measuring point, create a 72° incline.**

Height

The cup is 1.5 units tall. The centerline of the cup is 90° from the base. To draw the centerline, first plot an auxiliary vanishing point 90° from the vanishing point of the base (Figure 14.3). Use this auxiliary vanishing point to draw the centerline of the cup.

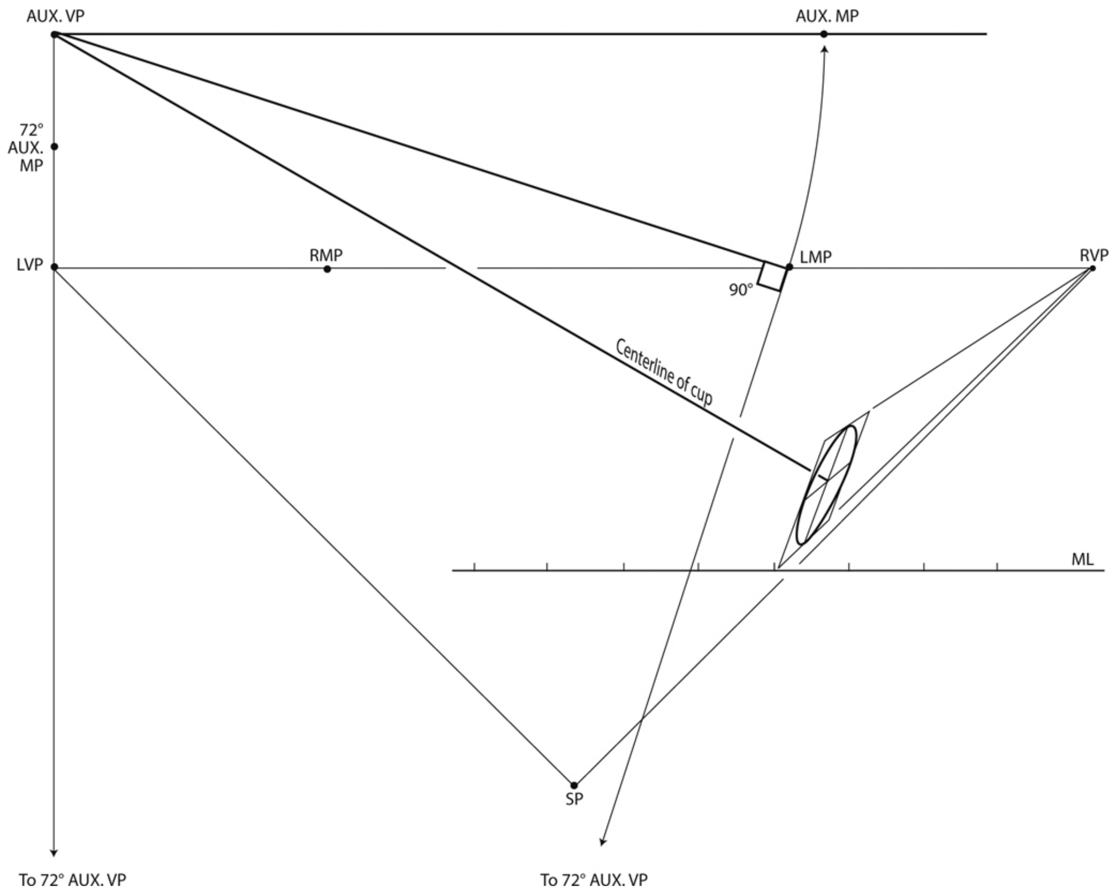


Figure 14.3 Draw a centerline 90° from the base.

To measure the height, establish an auxiliary measuring point and a measuring line parallel with the line the measuring point is on.

The measuring line must be moved so that it touches the centerline. There are many paths to do this. This example uses the 72° auxiliary vanishing point to project the measuring line up. Then, the right vanishing point is used to project the measuring line backward until it touches the centerline (Figure 14.4).

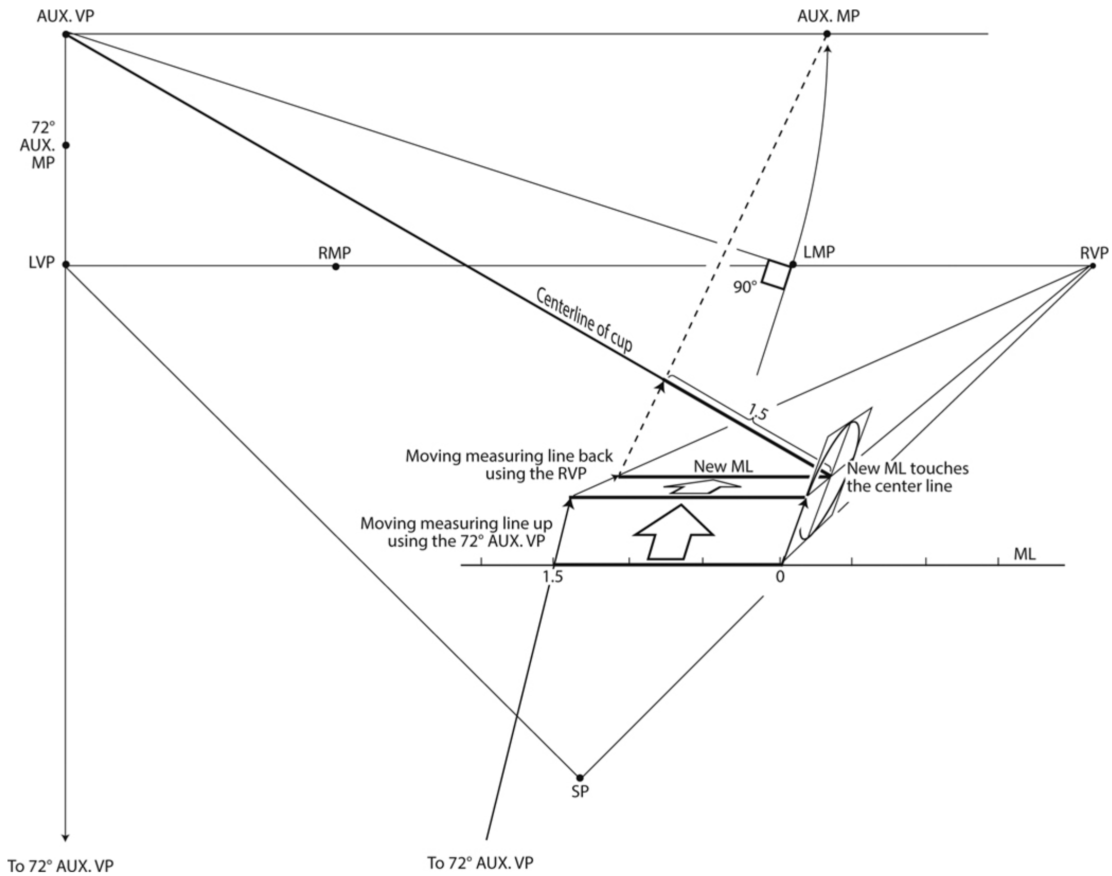


Figure 14.4 Measure the cup's height (1.5 units) by relocating the measuring line. It must touch the line being measured.

Top

The 1.5 unit centerline establishes the middle of the cup's top. The cup's top has a radius of 1 unit. Measure out from the center, 1 unit on each side, to create a 2 unit square, then draw an ellipse (Figure 14.5).

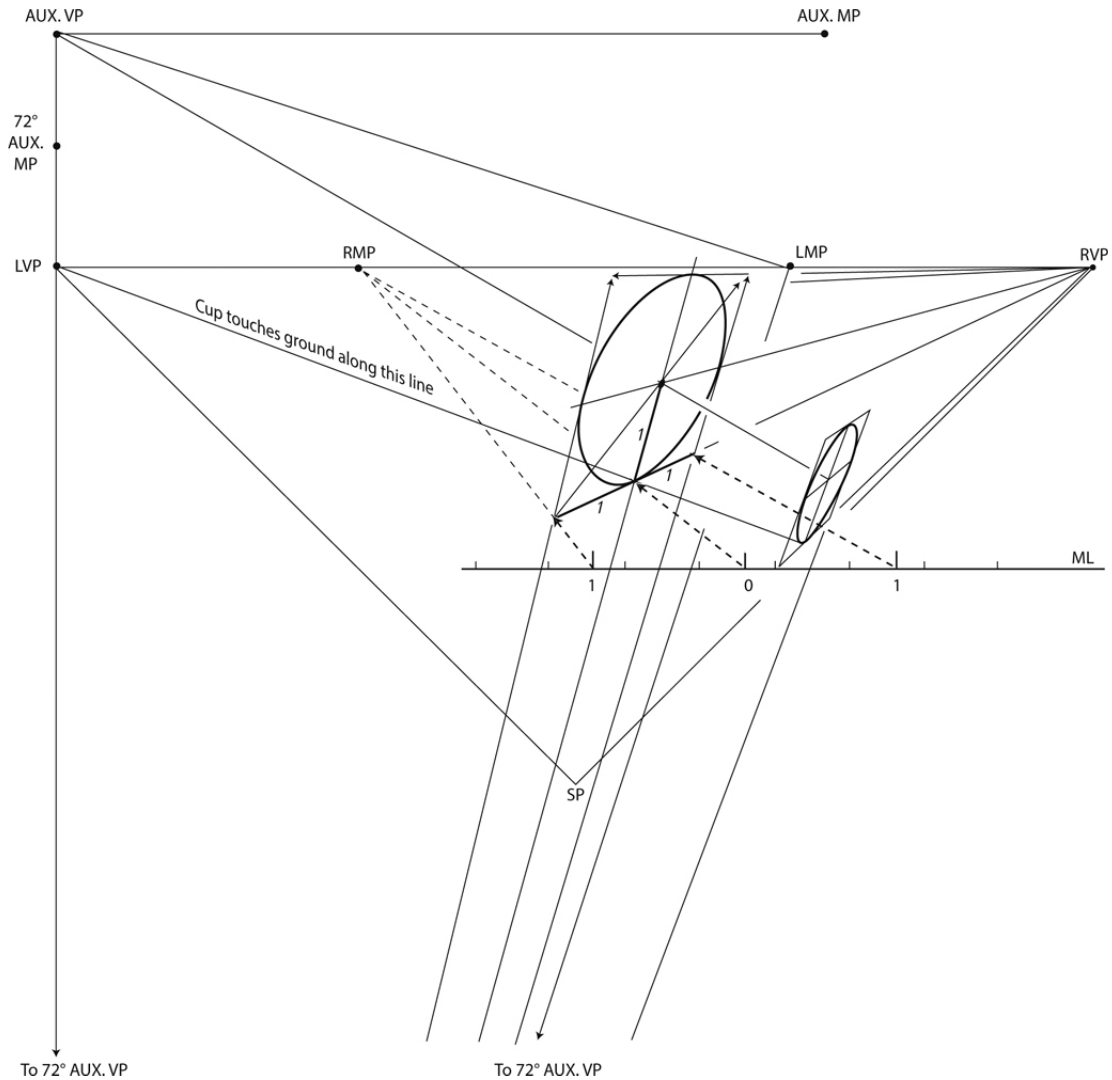


Figure 14.5 There are several steps in this illustration. First, find the point where the top of the cup touches the ground. Measure 1 unit on each side of this point. Project a diagonal line from the bottom left corner through the center point, until it intersects the top right corner of the square. Draw an ellipse.

Finish

Connect the ends of the two ellipses to finish the cup (Figure 14.6).

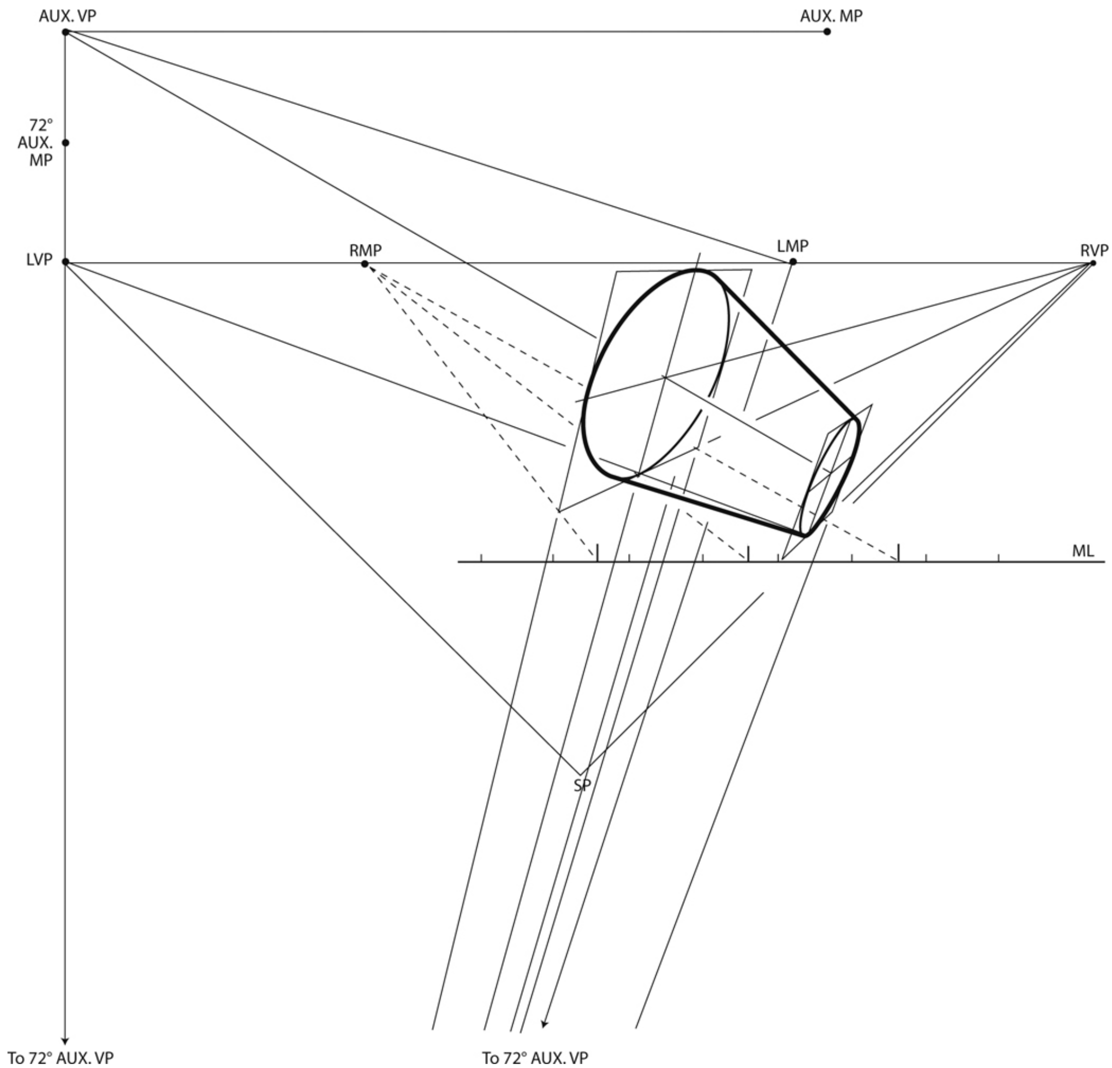


Figure 14.6 **Connect the ellipses to complete the cup.**

15
Perspective in the 1400s

The measurement system that has been used thus far is not the only way to draw in perspective: there is an alternative to drawing isosceles triangles. This method does not use measuring lines and measuring points. The Quattrocento artists approached perspective by projecting the object being drawn to the picture plane using the visual pyramid (lines drawn to the viewer's eye). Today this method is called "plan/elevation view perspective." It utilizes techniques that can be traced directly to Leon Battista Alberti. The plan/elevation view diagram used today is arranged differently, but the procedure has not changed in 600 years. As a direct descendant of the first perspective diagram, plan/elevation view perspective presents an opportunity to discuss some history, and explain perspective's evolution. Exploring this 600-year-old procedure segues seamlessly into the modern measurement system. So, this chapter begins by outlining how perspective originated.

Fillipo Brunelleschi is the founder of perspective. In 1413 he painted the first image to fully adhere to its rules. Leon Battista Alberti was the first to diagram this approach, publishing his book *On Painting* in 1435, and introducing perspective to the world. The creative milieu did not immediately adopt this new procedure. Artists had been painting for hundreds of years without perspective techniques. Change is difficult, as is perspective. Many ignored the arduous guidelines. Others, however, embraced the new technology.

To understand the artists' emotional response to perspective's arrival, look no further than a few decades ago. In many ways, the history of the computer mirrors the history of perspective. When these electronic instruments arrived in the art world of the 1980s, they were slow and foreign to artists accustomed to traditional tools. Many rejected these glowing boxes, considering them a temporary irritation, a passing fad. They forecasted the day consumers would tire of the digital look, and long for the return of the personal touch. This novelty would soon fade, they thought. Others, however, were excited—even giddy—about the possibilities of these new contraptions. They saw their potential.

Computers prevailed and are now the standard in commercial art production. Eventually, perspective theory also prevailed. Understanding and properly executing perspective soon became orthodox. That is, until modern art dethroned the depiction of representational space. Perspective, however, remains a valuable skill. Computers have not eliminated the need for understanding the geometry of vision.

Perspective, from the beginning, was rooted in geometry. Quattrocento artists did not rely on angles. They did, however, have a solid understanding of how the picture plane worked— how lines, projected from the object to the viewer's eye, would create an accurate image of reality at the intersection of the picture plane. They cleverly used this knowledge to construct a diagram that took advantage of this insight. This diagram gave artists something they never had: a way to draw with accuracy.

Alberti's measuring methods

Width

Once artists discovered that parallel lines connect to a vanishing point, measuring horizontal dimensions was relatively easy. Horizontal lines are not foreshortened, they are parallel with the picture plane. They can be measured with a ruler (Figure 15.1). Measuring depth is trickier.

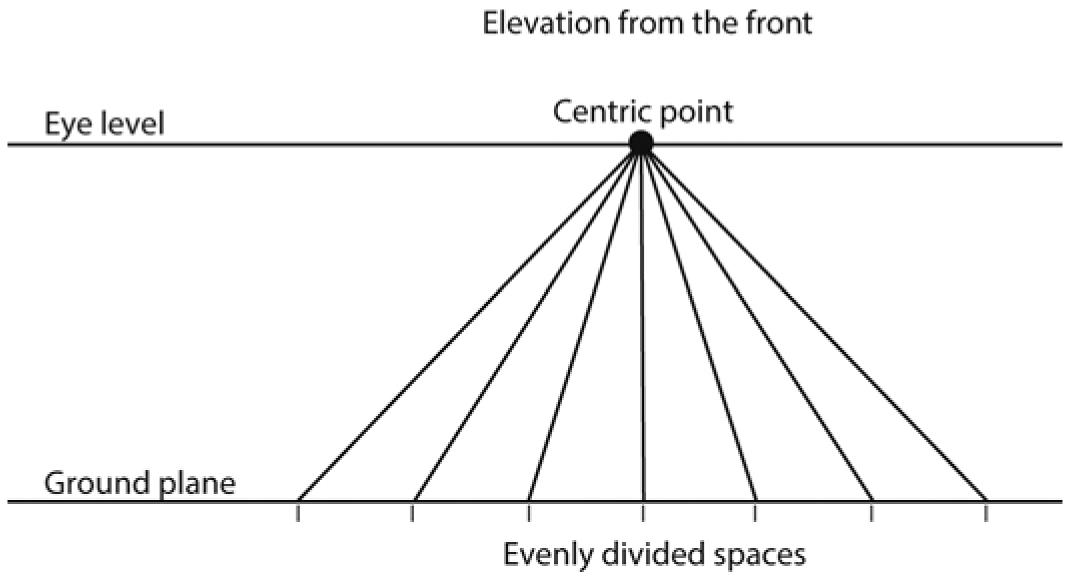


Figure 15.1 **Evenly spaced horizontal lines receding in space**.

Depth

Depth is foreshortened. It can't be measured directly with a ruler. In the previous chapters, an isosceles triangle was used to measure foreshortened lines. The Quattrocento artists approached it differently—they used an elevation view to plot depth. Drawing the intersection of the visual pyramid on the picture plane projected the foreshortened line to a flat surface (Figure 15.2).

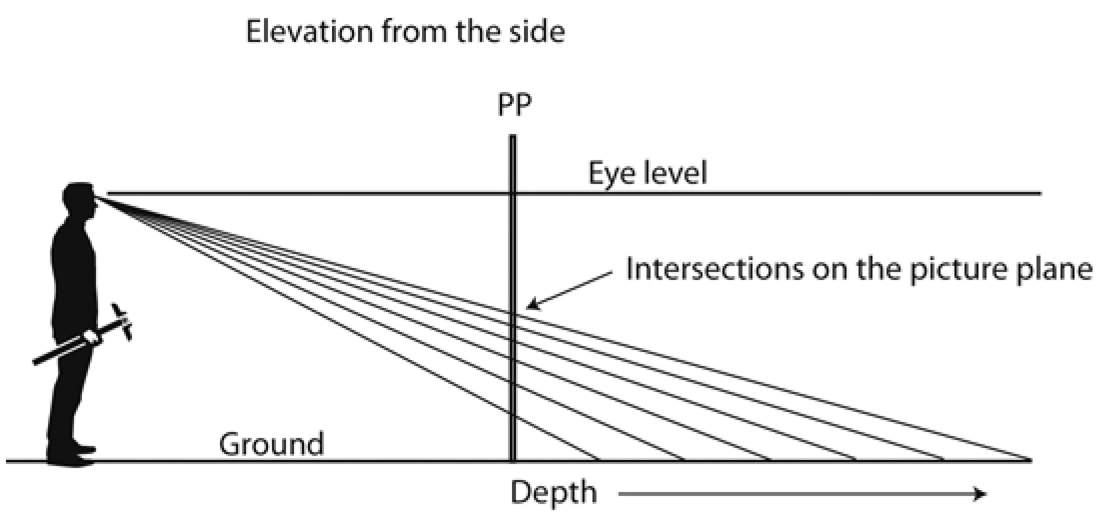


Figure 15.2 **Using an elevation view to define depth**.

One-Point Diagram

So, now that width can be defined using an elevation view from the front, and depth defined using an elevation view from the side (Figure 15.1 and Figure 15.2), combining the two views gives the foundation of Alberti's 600-year-old perspective diagram (Figure 15.3). A perspective view of this diagram may better explain the relationships (Figure 15.4). Alberti's diagram creates a one-point perspective grid.

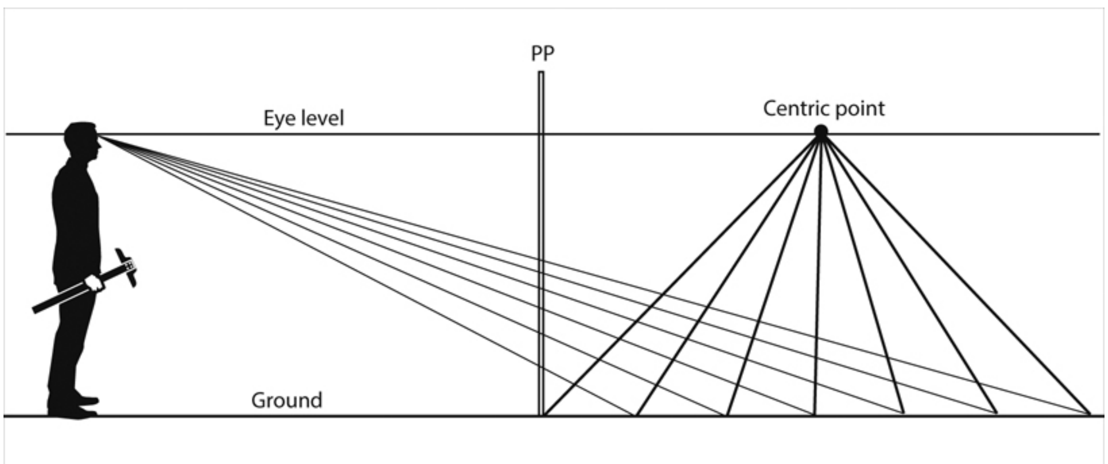


Figure 15.3 **Combining the two elevation views into one diagram**.

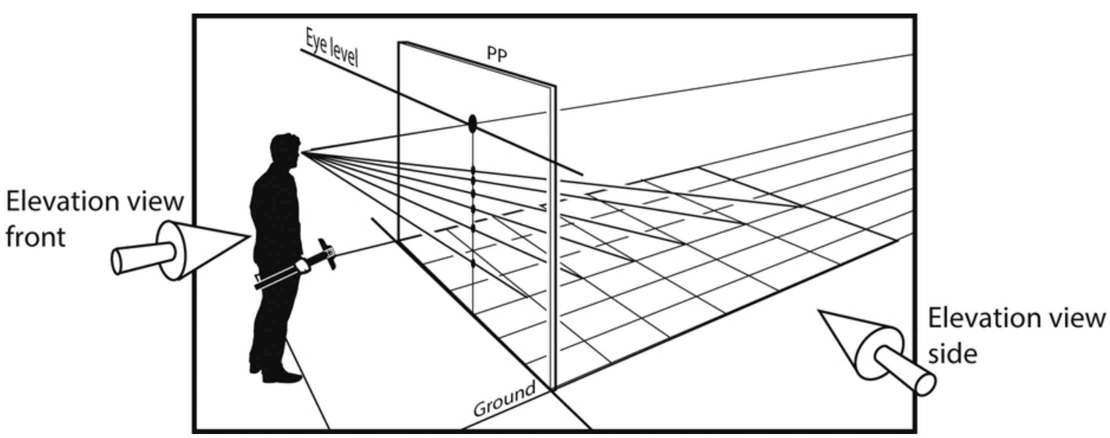


Figure 15.4 **A three-dimensional view of** Figure 15.3.

One-Point Grid

The intersections at the picture plane define depth. Project the intersections horizontally to create a grid (Figure 15.5).

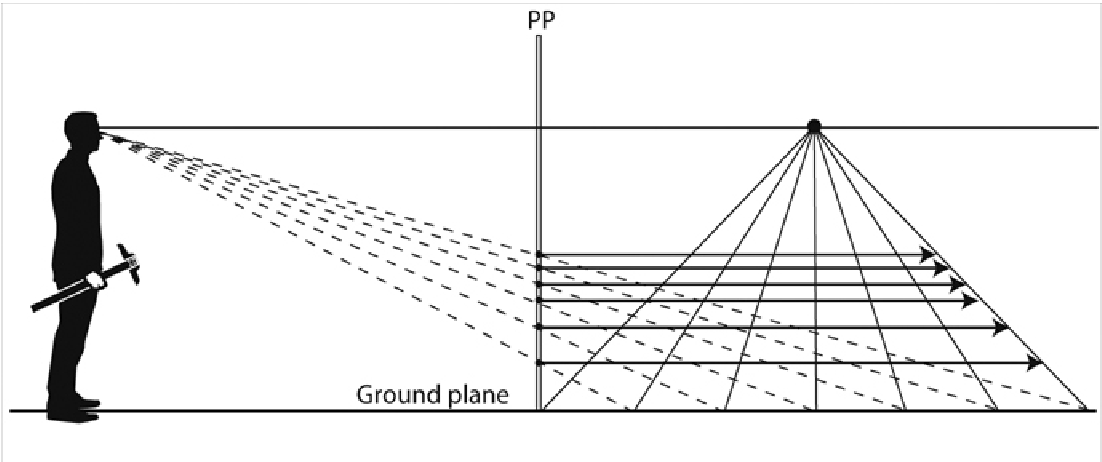


Figure 15.5 A one-point perspective grid using Alberti's 1435 diagram.

16
Plan/Elevation View Perspective

The Diagram

Alberti used two elevation views: an elevation view from the front (for width), and an elevation view from the side (for depth). A more contemporary approach is to use a plan and an elevation view. The theory is the same; the configuration is different.

A plan view is a view from above. It shows the station point and its distance to the picture plane. The elevation view is a view from the front. It shows the horizon line, the center of vision, and the **ground line** (because there is no measuring, the measuring line is now called a ground line) (Figure 16.1).

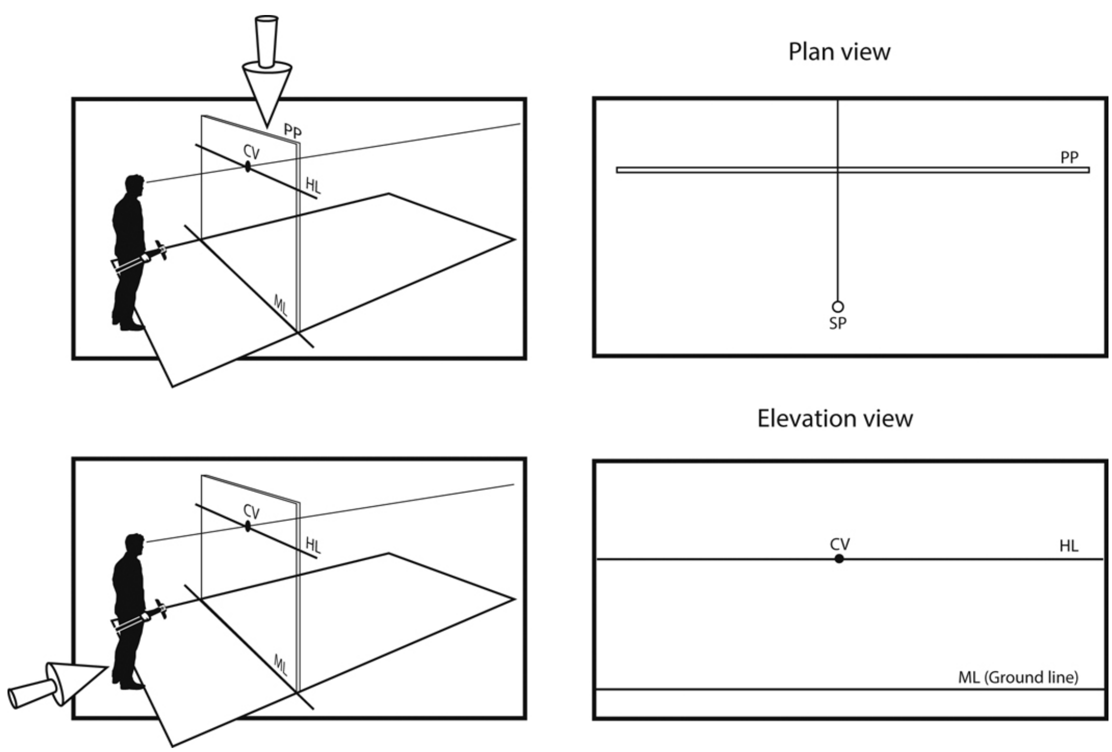


Figure 16.1 Plan and elevation views. On the left is a three-dimensional view. On the right is how this view is represented on paper.

Station Point

The distance from the station point to the picture plane determines how large the cone of vision will be. The farther away the viewer is from the picture plane, the larger the cone of vision (the station point is often placed below the elevation view to allow for a large image area).

Ground Line

What was called the measuring line is now called a ground line. The ground line is located at the picture plane and determines the viewer's eye level. The closer the ground line is to the horizon line, the lower the eye level.

The Object

The plan view displays the top of the object (this gives the width and depth); the elevation view displays the front of the object (this gives the height) (Figure 16.2).

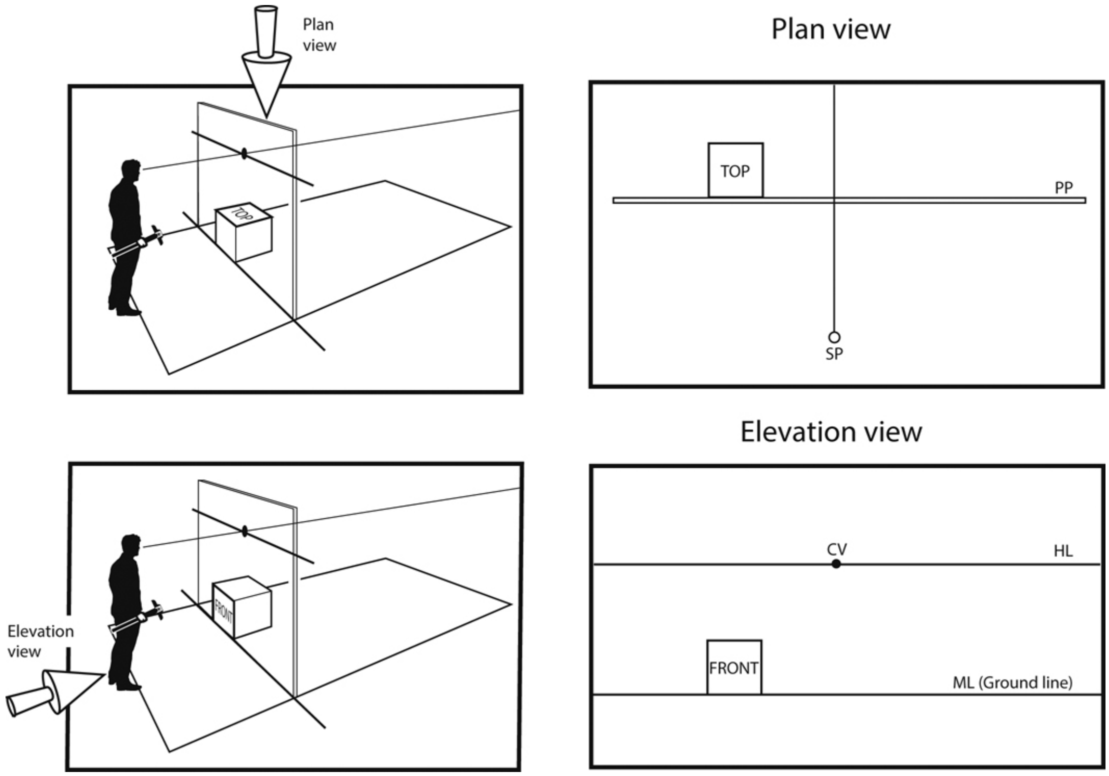
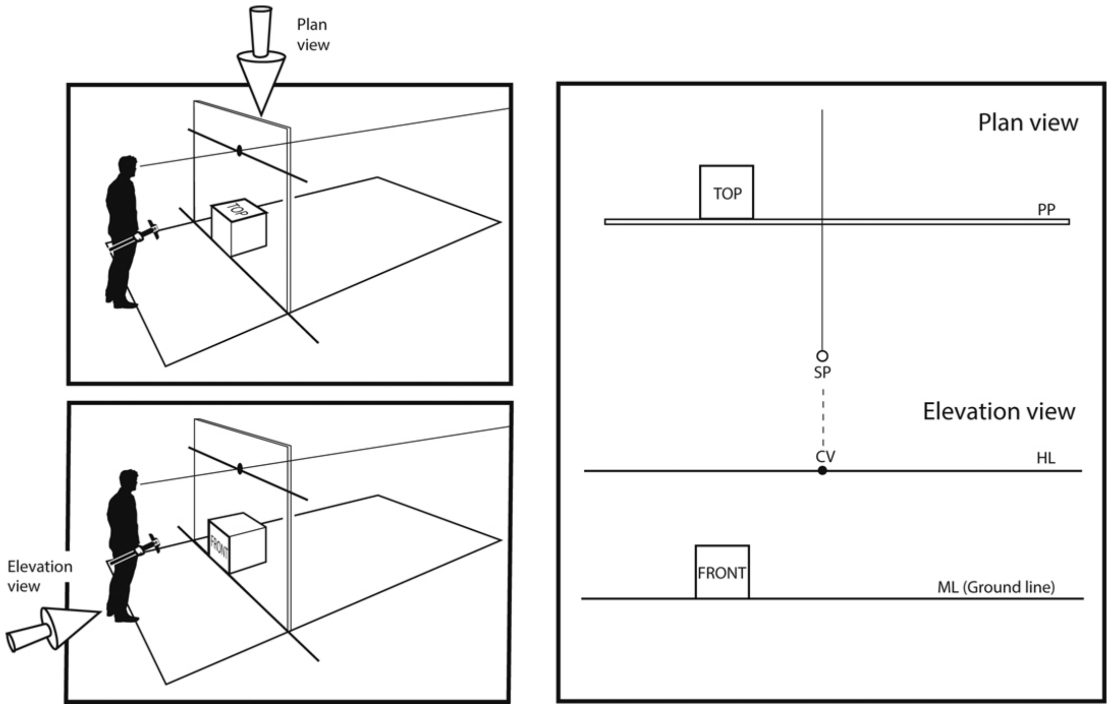


Figure 16.2 A cube in plan and elevation view, in three- and two-dimensional views.

The Drawing

With the elevation and a plan view in place, it is time to create the drawing. Alberti superimposed the two elevation diagrams, putting one on top of the other. The modern approach is a little different: the plan view is separated from the elevation view. The plan view is traditionally placed at the top of the paper, with the elevation view placed below. It does not matter how far apart the diagrams are, but the center of vision in the elevation view must be aligned with the station point below (Figure 16.3).



[Figure 16.3](#) **The plan view is placed above the elevation view. The station point and center of vision must be aligned.**

The box in this example is in one-point perspective. This is shown by its position relative to the picture plane. The box in the plan view and the box in the elevation view are the same object, therefore the plan and elevation view must be drawn to the same scale. If there are several objects to draw, the size and positioning of each object must also be to scale.

Elevation View

Foreshortened lines in one-point perspective connect to the center of vision ([Figure 16.4](#)).

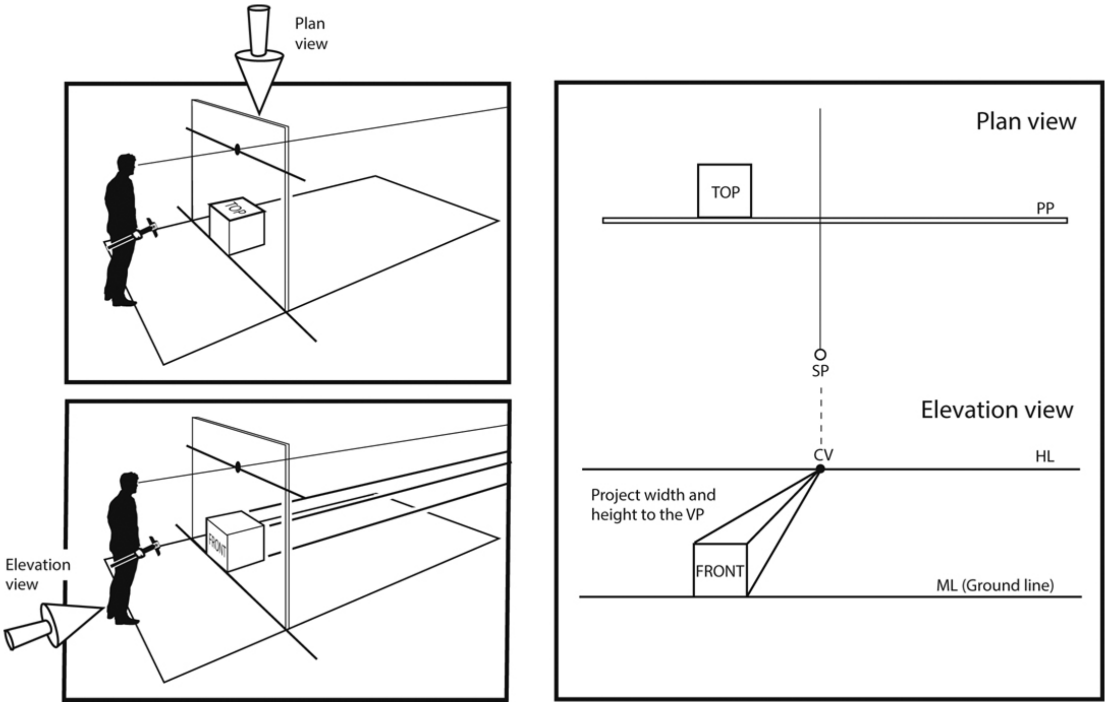


Figure 16.4 Connect foreshortened lines to the center of vision.

Plan View

To measure the depth, draw a line from the back of the box to the station point. Where this line intersects the picture plane, project downward to the ground plane, defining the back of the box (Figure 16.5).

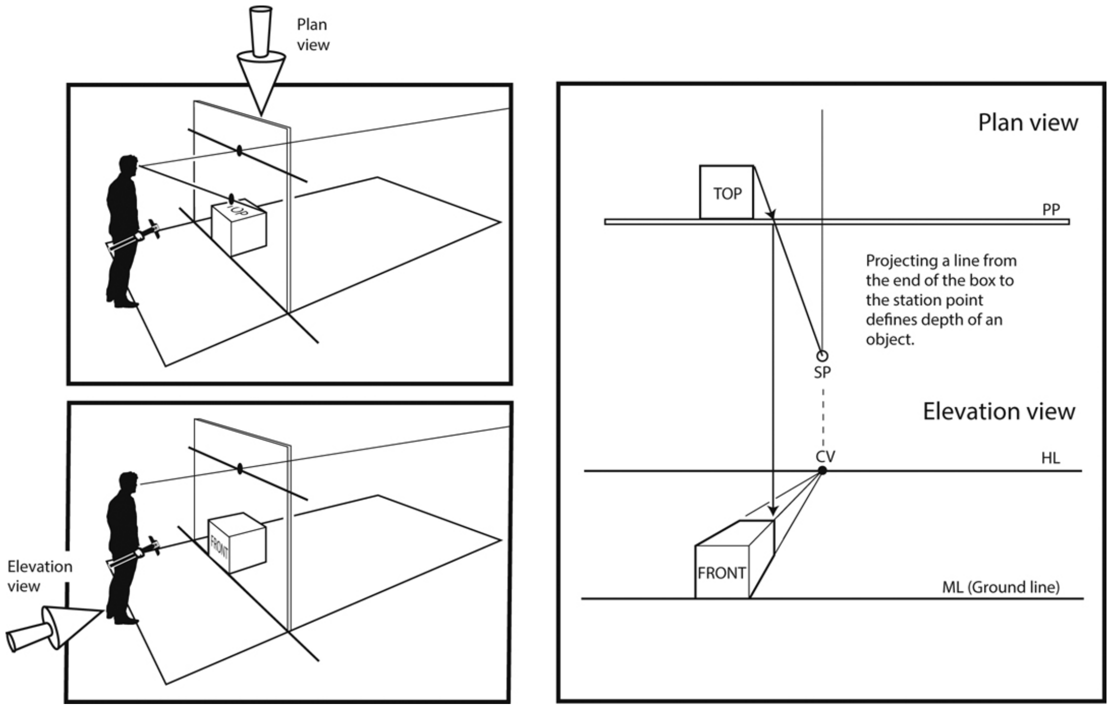


Figure 16.5 Measure depth by plotting its intersection on the picture plane.

This is how to draw a box using the plan and elevation method. No measuring points or measuring line was used, and all dimensions were projected to the picture plane. Now try to do another one-point view with a different shape, in a different position.

Objects Not Touching the Picture Plane

When a one-point box touches the picture plane, height and width are actual size. In this next example, the box is behind the picture plane. Begin by drawing the diagram. Place the elevation view of the box to the right of the image area. It is usually more convenient— especially when drawing several objects—to place the elevation view on the right side of the paper. This keeps the diagram less cluttered (Figure 16.6, bottom).

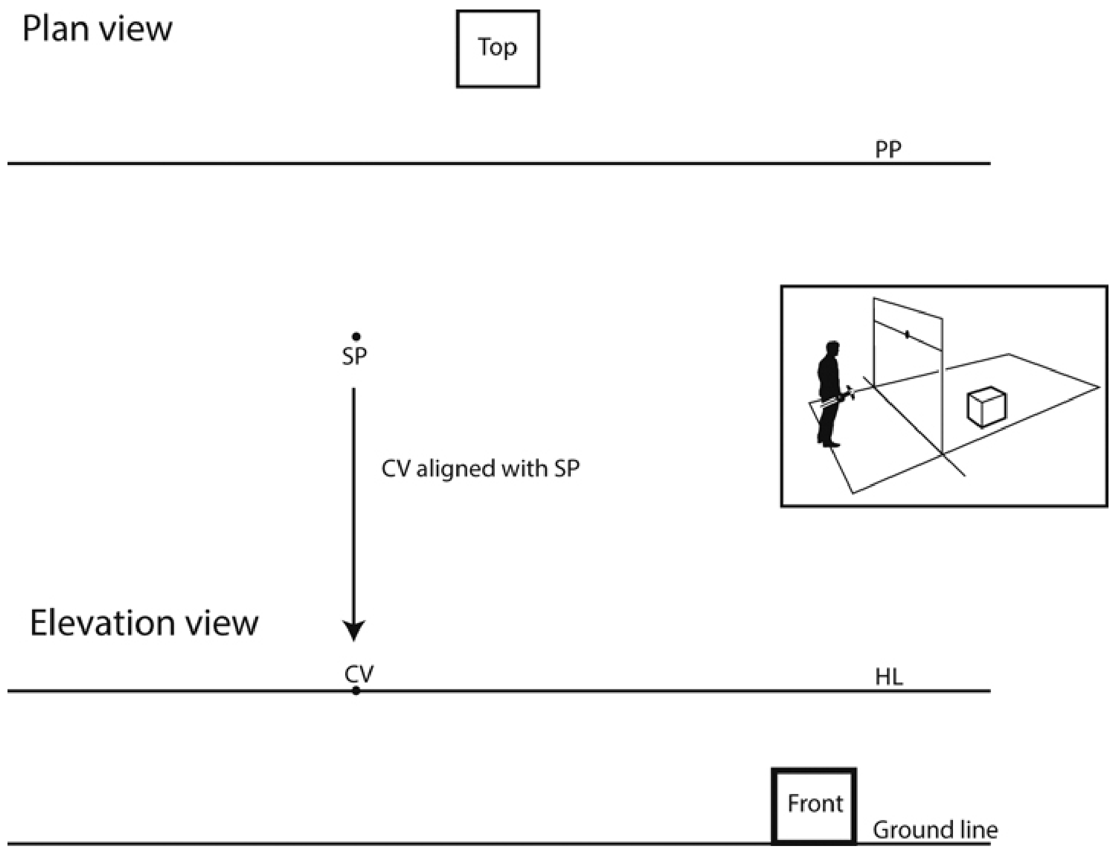
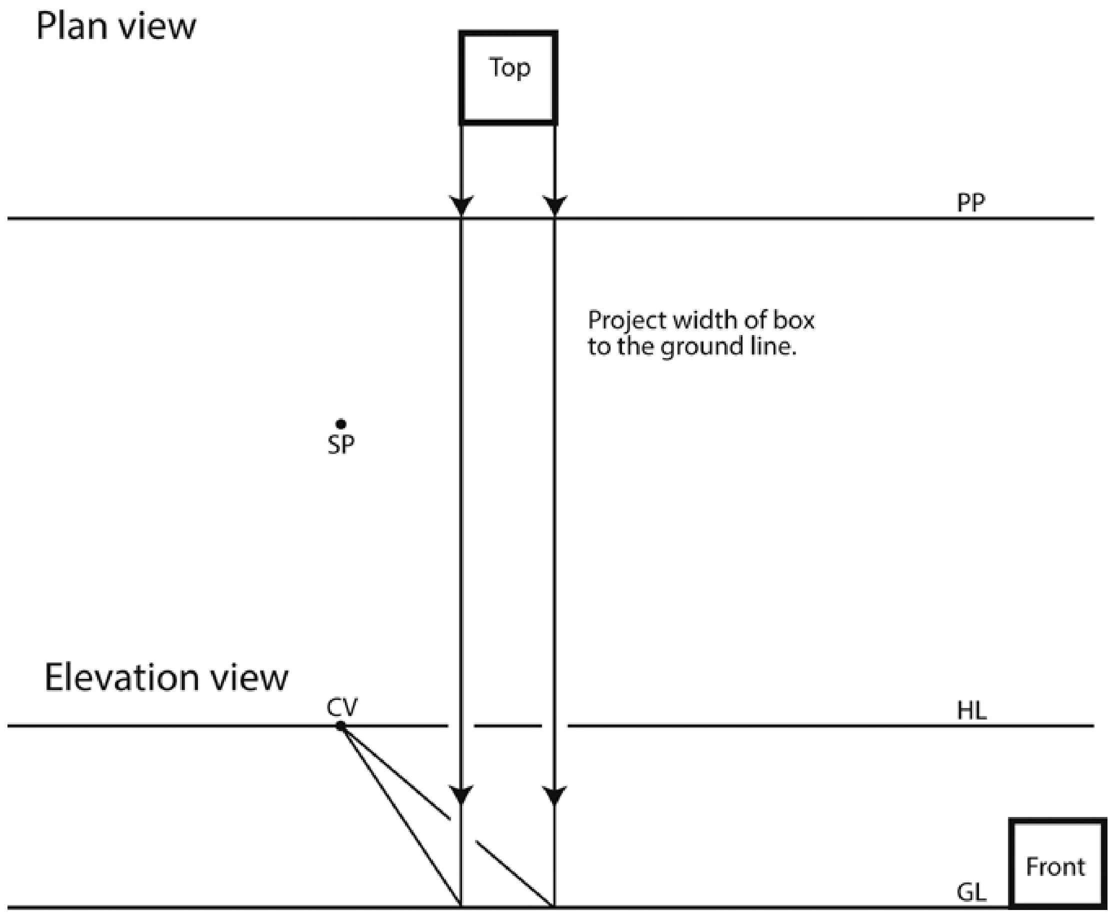


Figure 16.6 **Plan and elevation diagram showing a cube in one-point perspective. In the elevation view, the cube is placed to the right of the image area.**

Width

Project the width to the picture plane, then drop that distance to the ground line. Any point not touching the picture plane must first be projected to the picture plane, before being drawn to the ground (Figure 16.7).



[Figure 16.7](#) **Plot the width of the cube by projecting it to the picture plane and then to the ground line.**

Depth

Determine depth by drawing a line from the object's corners to the station point. Plot the intersection at the picture plane, then project the intersection to the ground (Figure 16.8).

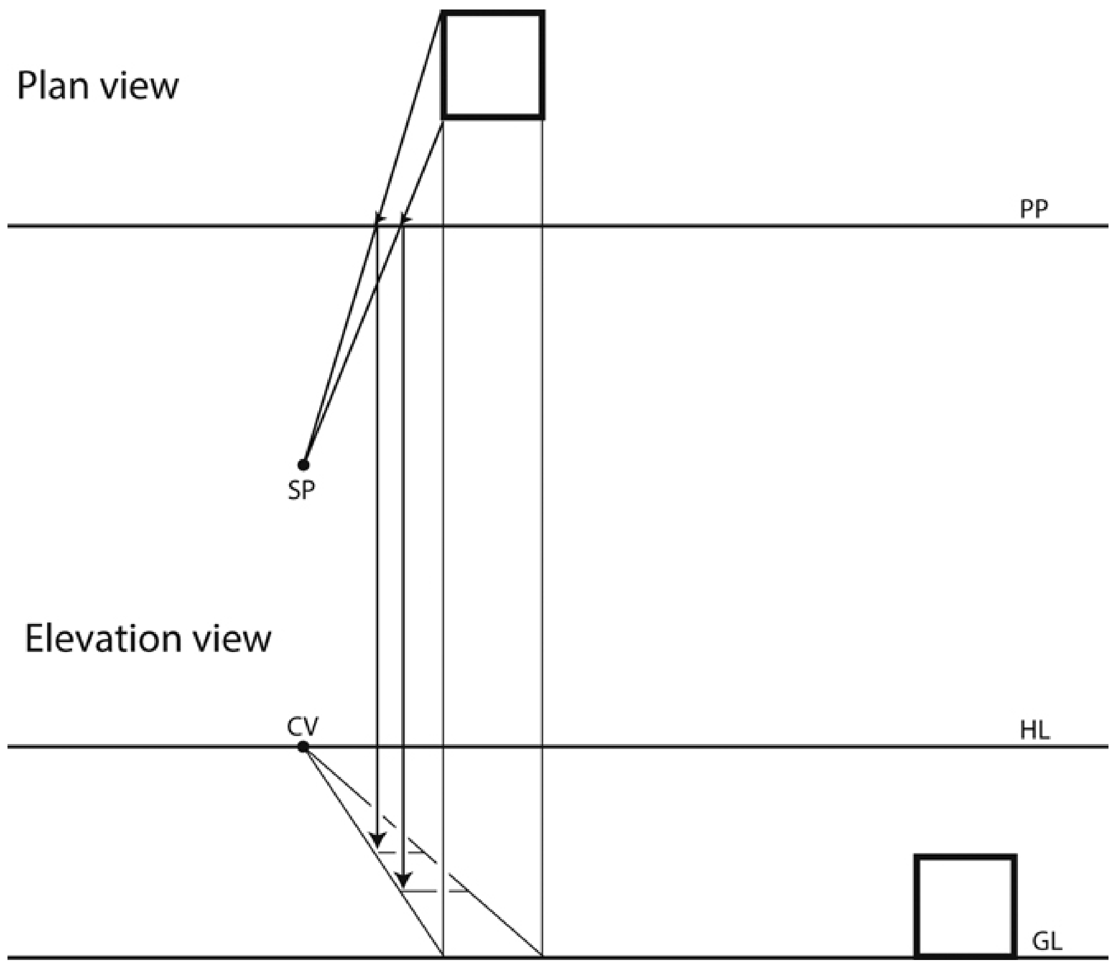


Figure 16.8 **Use the station point to plot the depth of the cube.**

Height

The elevation view displays the true height at the picture plane. This dimension must be projected back in space if the object being drawn does not touch the picture plane (Figure 16.9).

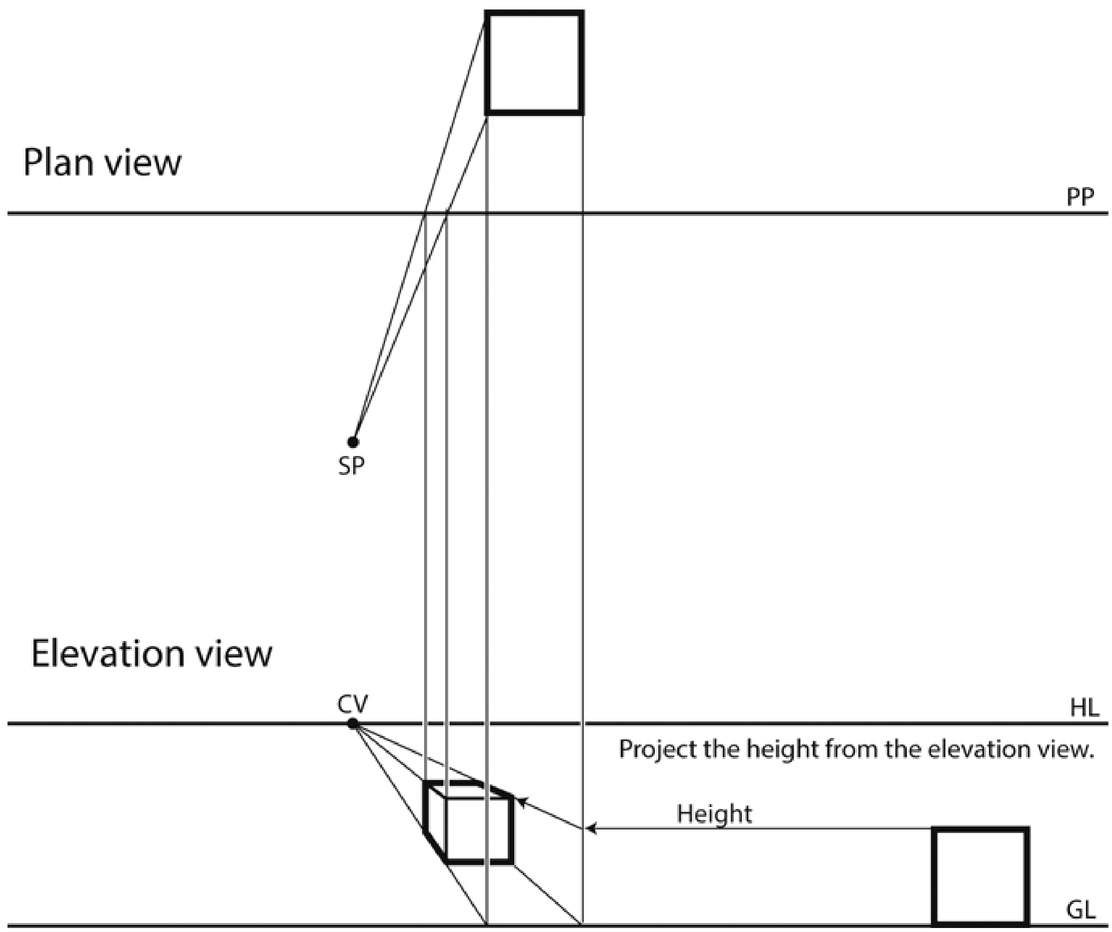


Figure 16.9 **Plot the height of the cube using the elevation view.**

Two-Point Plan/Elevation

Finding Vanishing Points

Objects in two-point perspective are angled to the picture plane. These angles can be seen in the plan view. The placement of the right and left vanishing points must reflect the angle of the object being drawn. Since angles at the station point are true angles, the lines projected from the station point must be parallel with the angles of the object. Where the lines intersect the picture plane, drop them to the horizon, creating a left and right vanishing point (Figure 16.10).

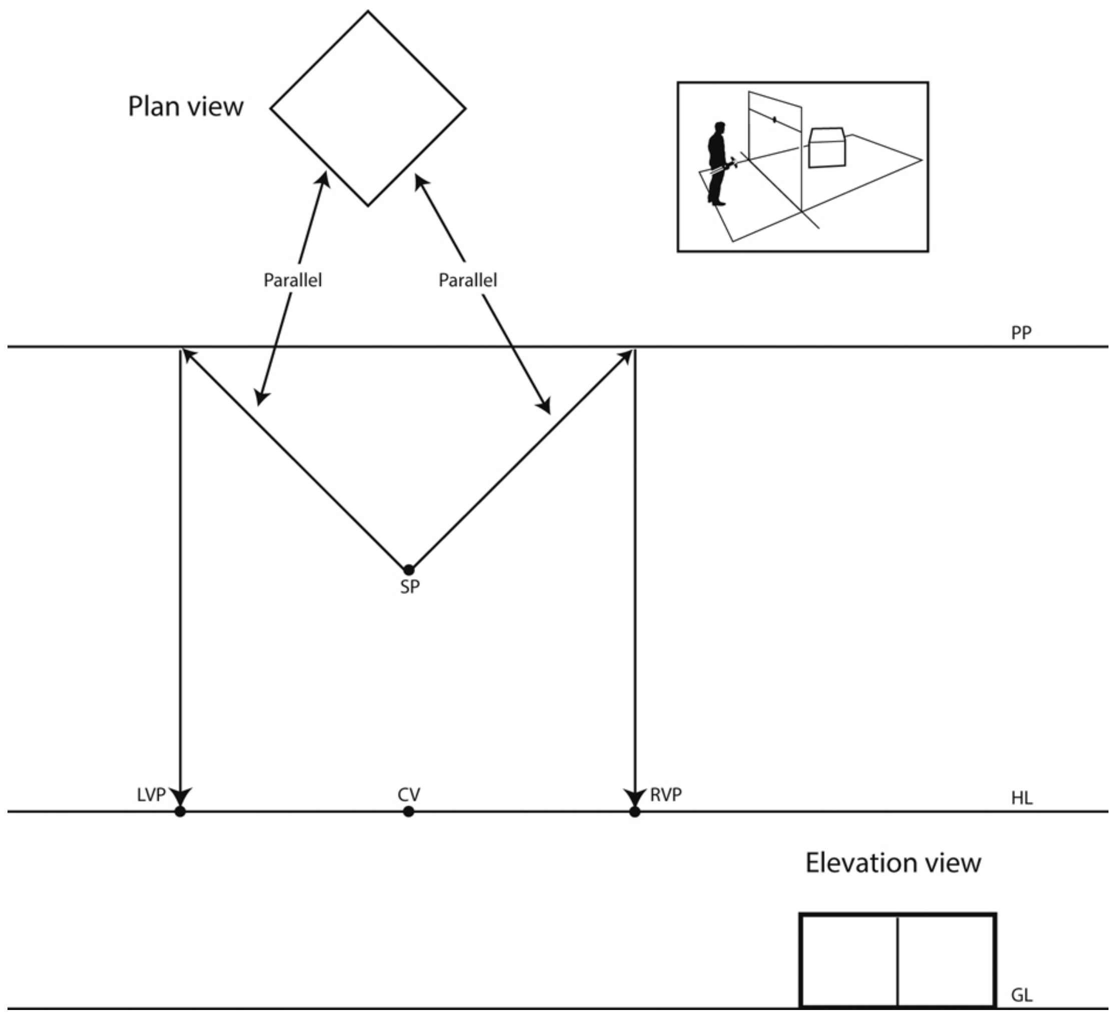


Figure 16.10 **Establish vanishing points by drawing lines parallel with the object in the plan view**.

The Drawing

As in one-point perspective, lines that do not touch the picture plane must be projected to the picture plane. Once the line touches the picture plane, it is then dropped down to the ground line. From the ground line, connect the lines to their respective vanishing points. The intersection of the two angles indicates the front corner of the box (Figure 16.11).

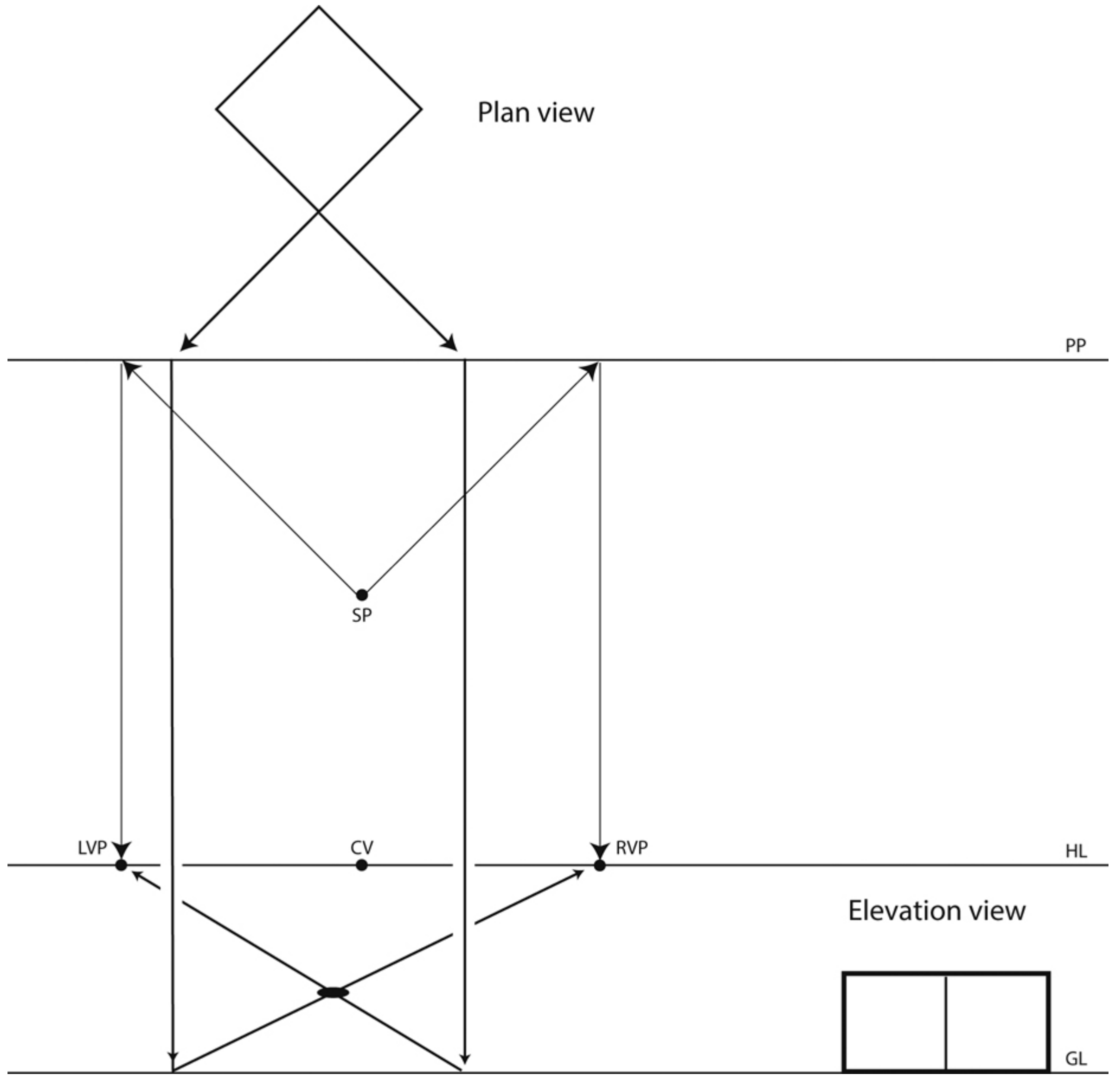


Figure 16.11 **The intersection of the two lines defines the location of the box's front corner.**

Depth

In the plan view, draw a line from the left and the right corner of the box to the station point. Use the intersection at the picture plane to define the depth of the box (Figure 16.12).

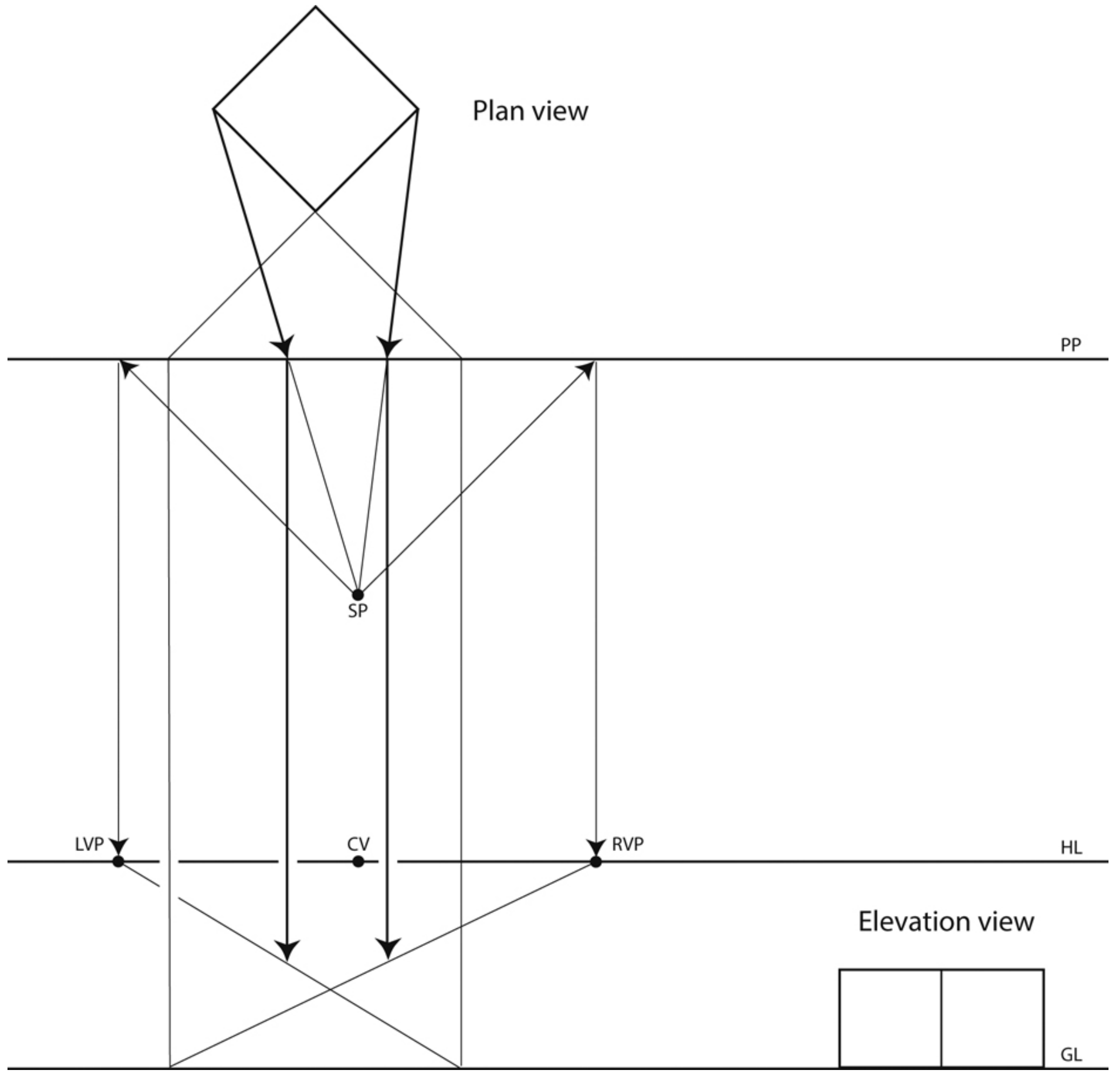


Figure 16.12 **To determine depth, draw a line to the station point intersecting the picture plane, then drop these lines to the box.**

Height

Height is determined from the elevation view. Project the height across the picture plane, then backward using a vanishing or reference point (Figure 16.13).

Connect lines to vanishing points to finish the box.

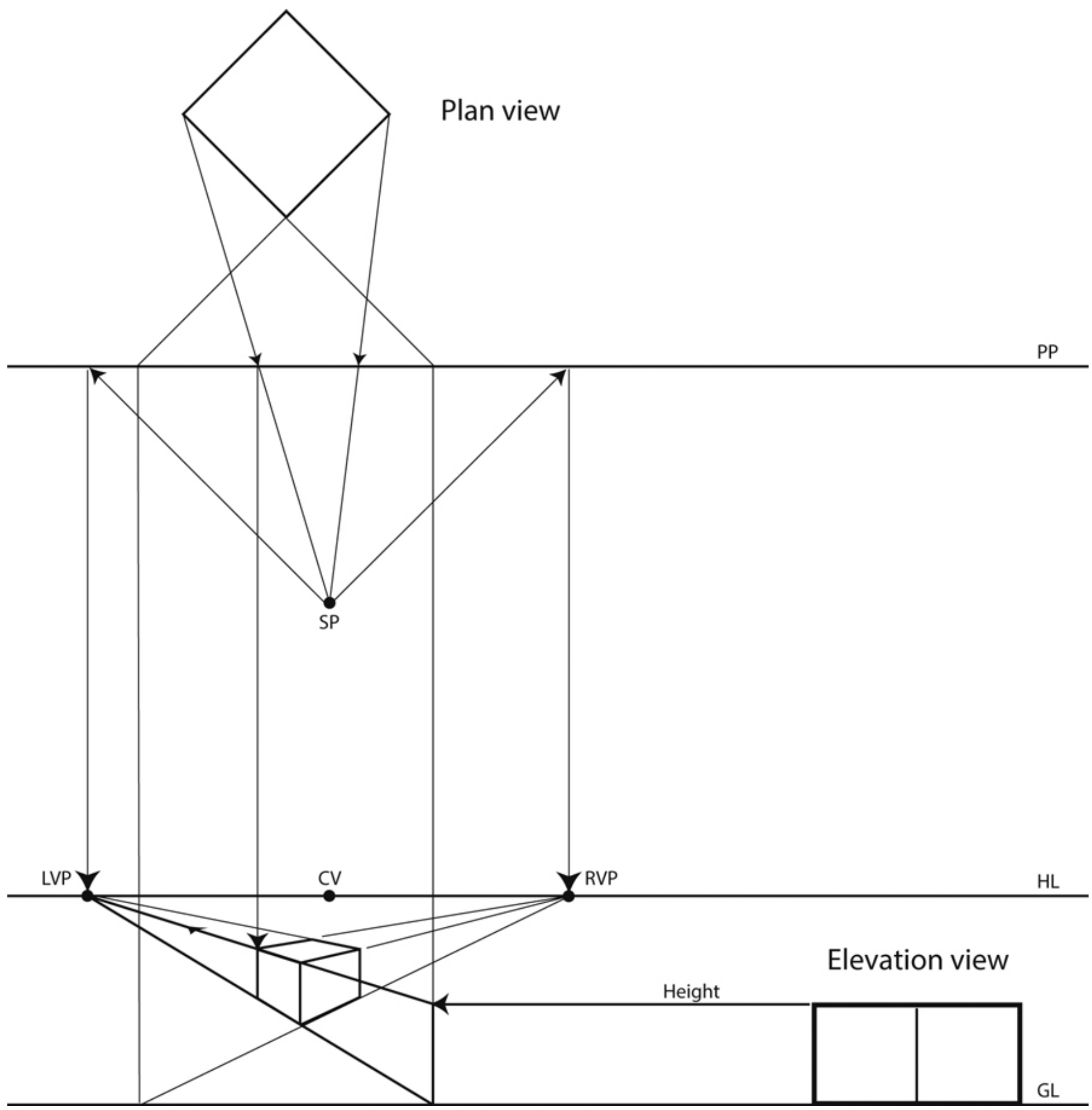


Figure 16.13 Use the elevation view to establish the height. Connect lines to vanishing points to finish the box.

The Problem of Distant Vanishing Points

If a right or left vanishing point is far off the page (Figure 16.14), the two-point object can be drawn using one-point techniques. Any single point may be found using one-point perspective. Once the distant corner is found, connect the dots (Figures 16.15–16.17).

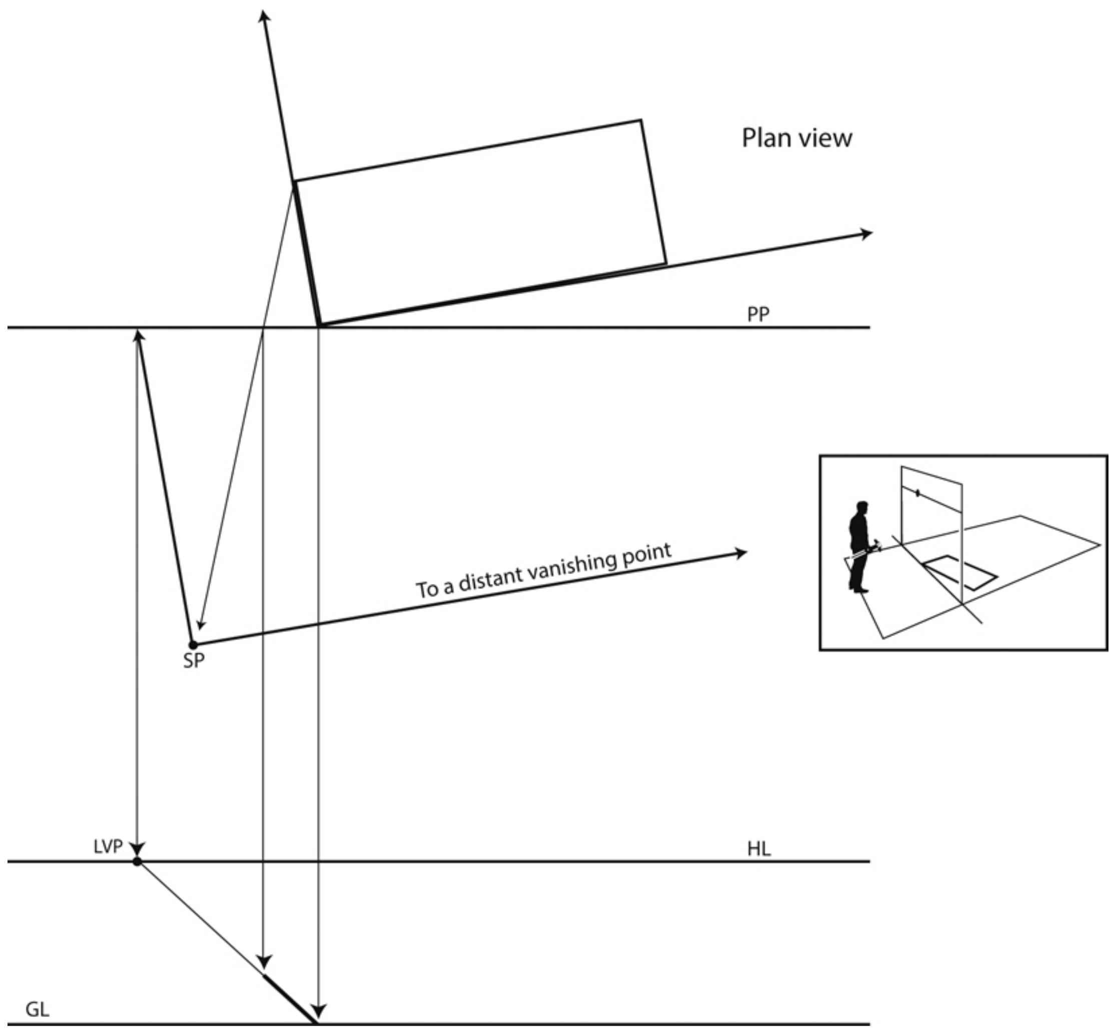
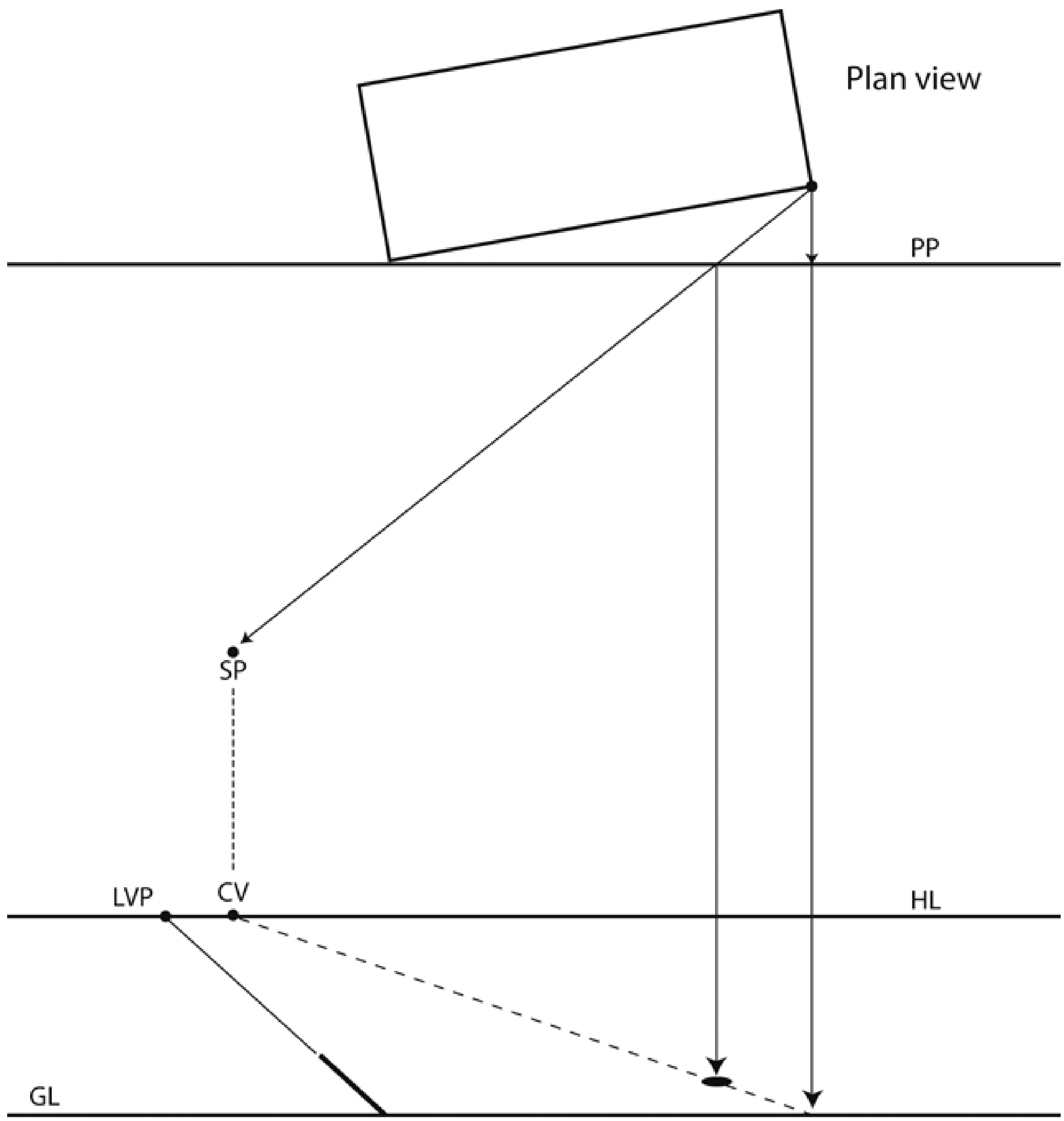


Figure 16.14 The right vanishing point is too far away to plot comfortably.



[Figure 16.15](#) Use one-point perspective techniques to draw the right front corner of the box. Any point in space can be found using one-point perspective.

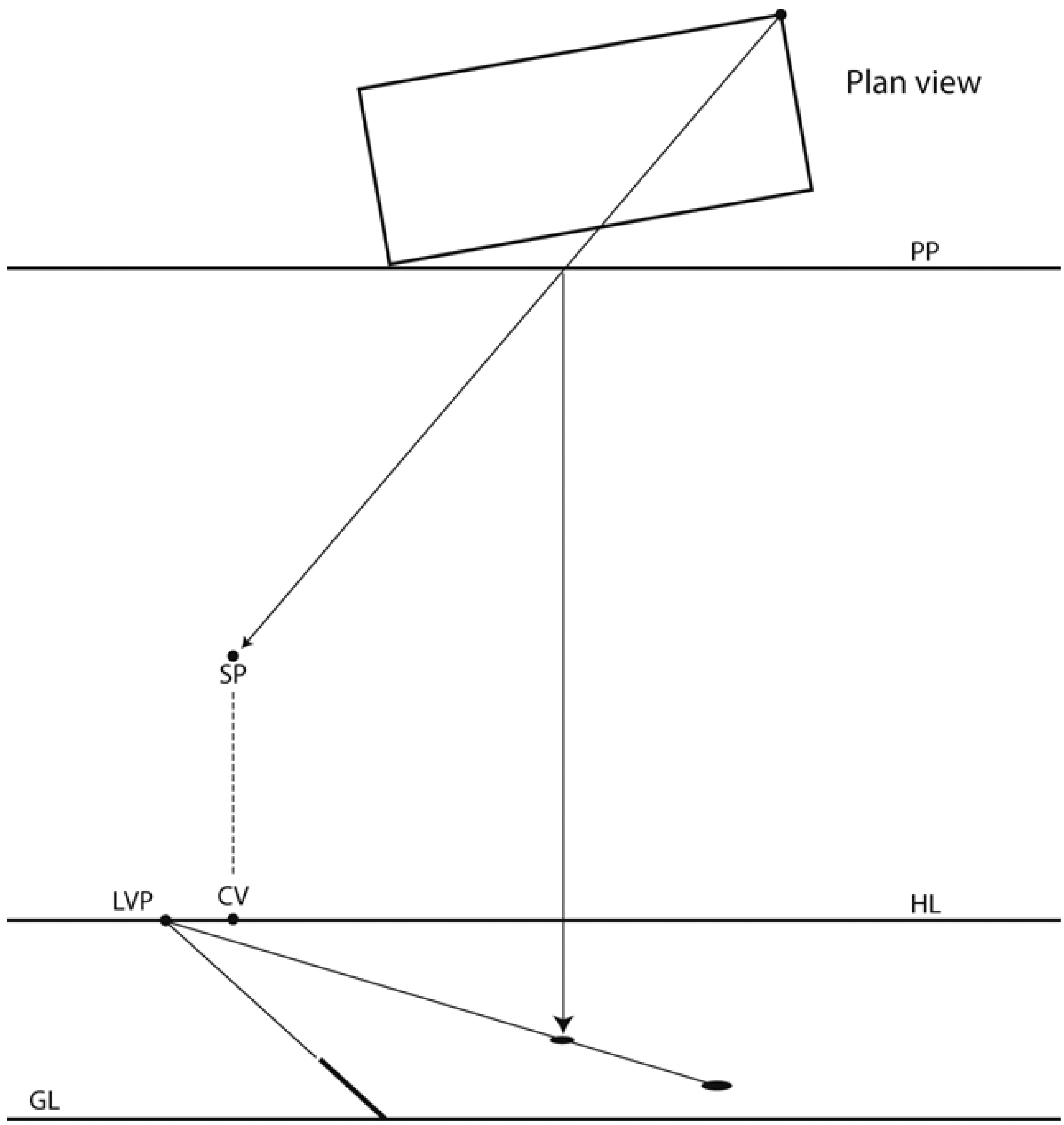


Figure 16.16 Use the same technique to draw the back corner of the box.

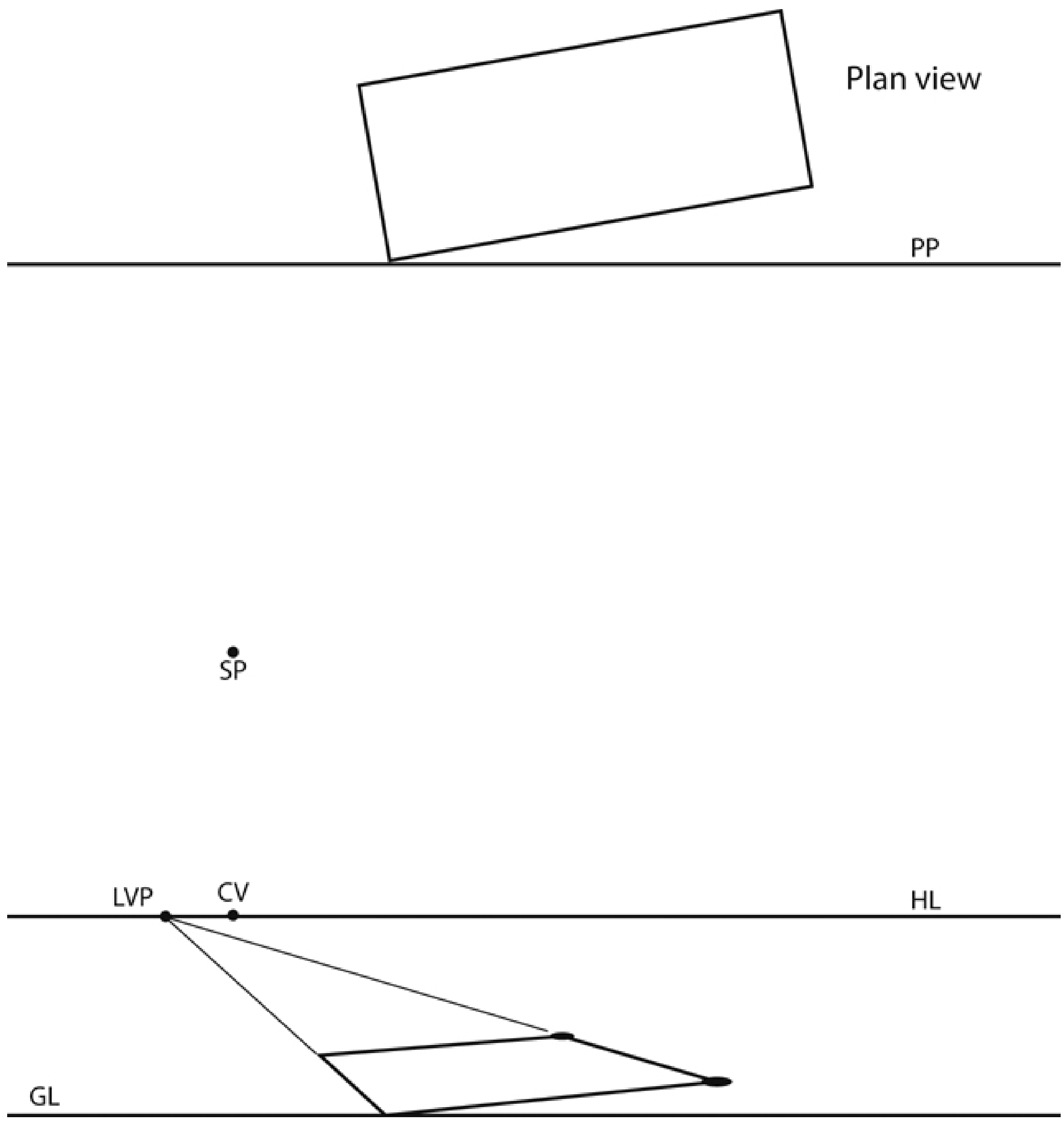


Figure 16.17 **Connect the dots to complete the box.**

17
Three-Point Perspective

Three-point perspective is defined by the center of vision. In one- and two-point perspective, the line of sight is parallel with the ground plane; the center of vision is focused on the horizon line. In three-point perspective, the line of sight is angled to the ground plane; the center of vision is above or below the horizon line. To put it simply, the viewer is looking up or looking down in three-point perspective. If the viewer is looking up (a "worm's-eye" view), the center of vision is above the horizon line. If the viewer is looking down (a **"bird's-eye" view**), the center of vision is below the horizon line (Figure 17.1). All lines are foreshortened in three-point perspective, and none are parallel with the picture plane.

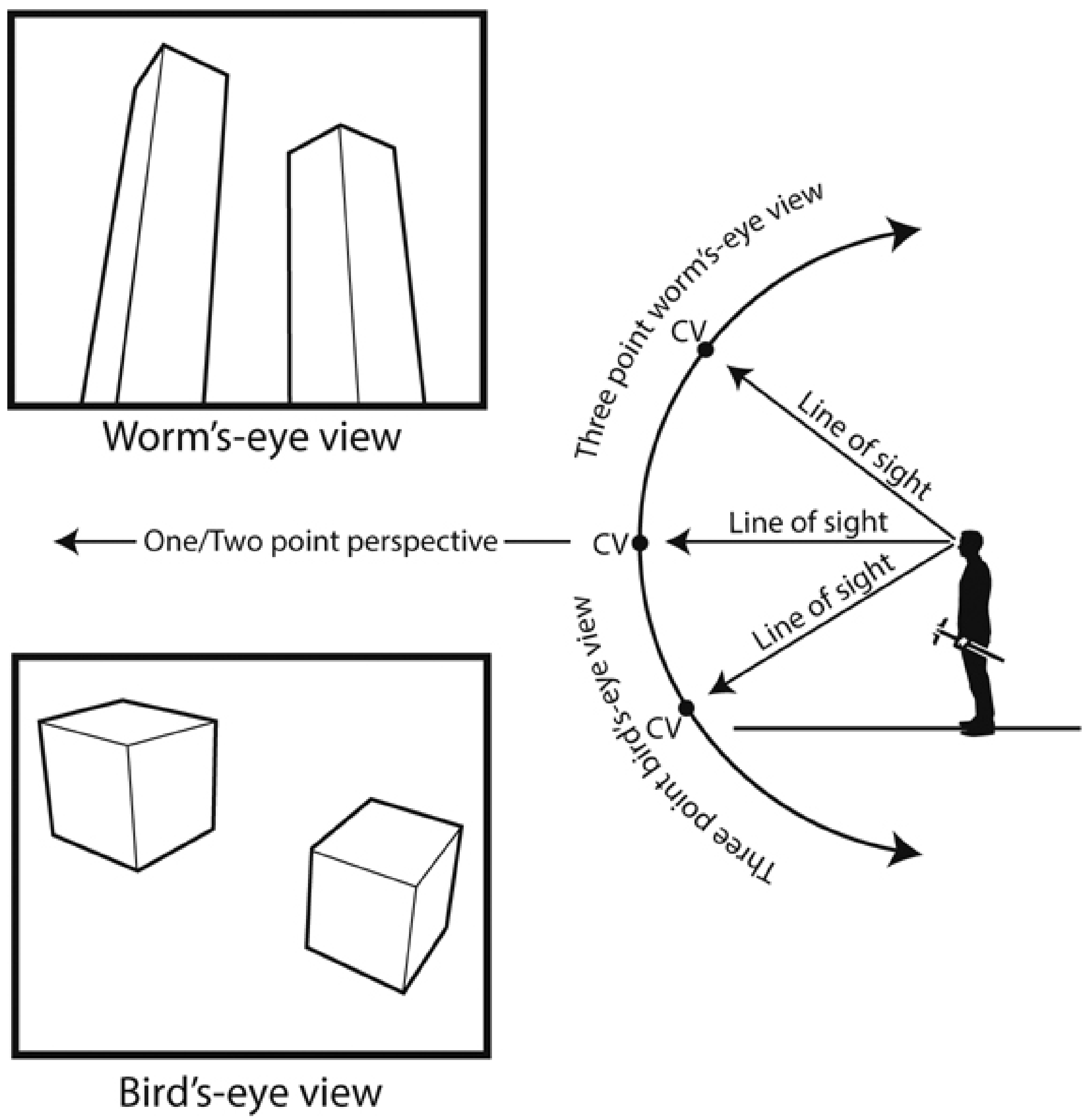


Figure 17.1 The viewer is looking up or down in three-point perspective.

In three-point perspective, there are a myriad of variables. When considering the relationship of the viewer to the vanishing points and the object being drawn, the possibilities seem endless. It is not possible to address all the situations that might be encountered. However, the following chapters give a foundation that enables the artist to arrive at solutions to problems not covered in this book. Before addressing the more obscure and challenging aspects of three-point perspective, begin by setting up a basic three-point diagram. This is a generic set-up. Variations will be discussed later, as well as how to tailor the diagram to specific circumstances.

Three-Point Perspective Components

Picture Plane

The picture plane must be at a right angle to the line of sight. Therefore, in three-point perspective, the picture plane is angled to the ground plane (Figure 17.2).

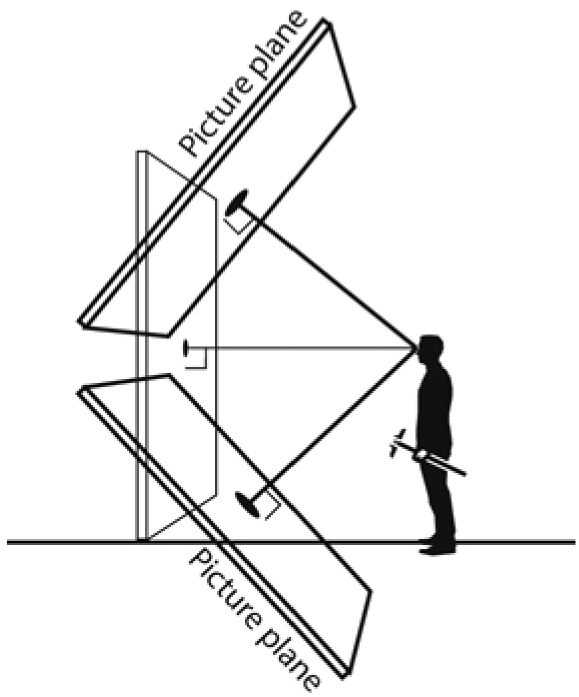
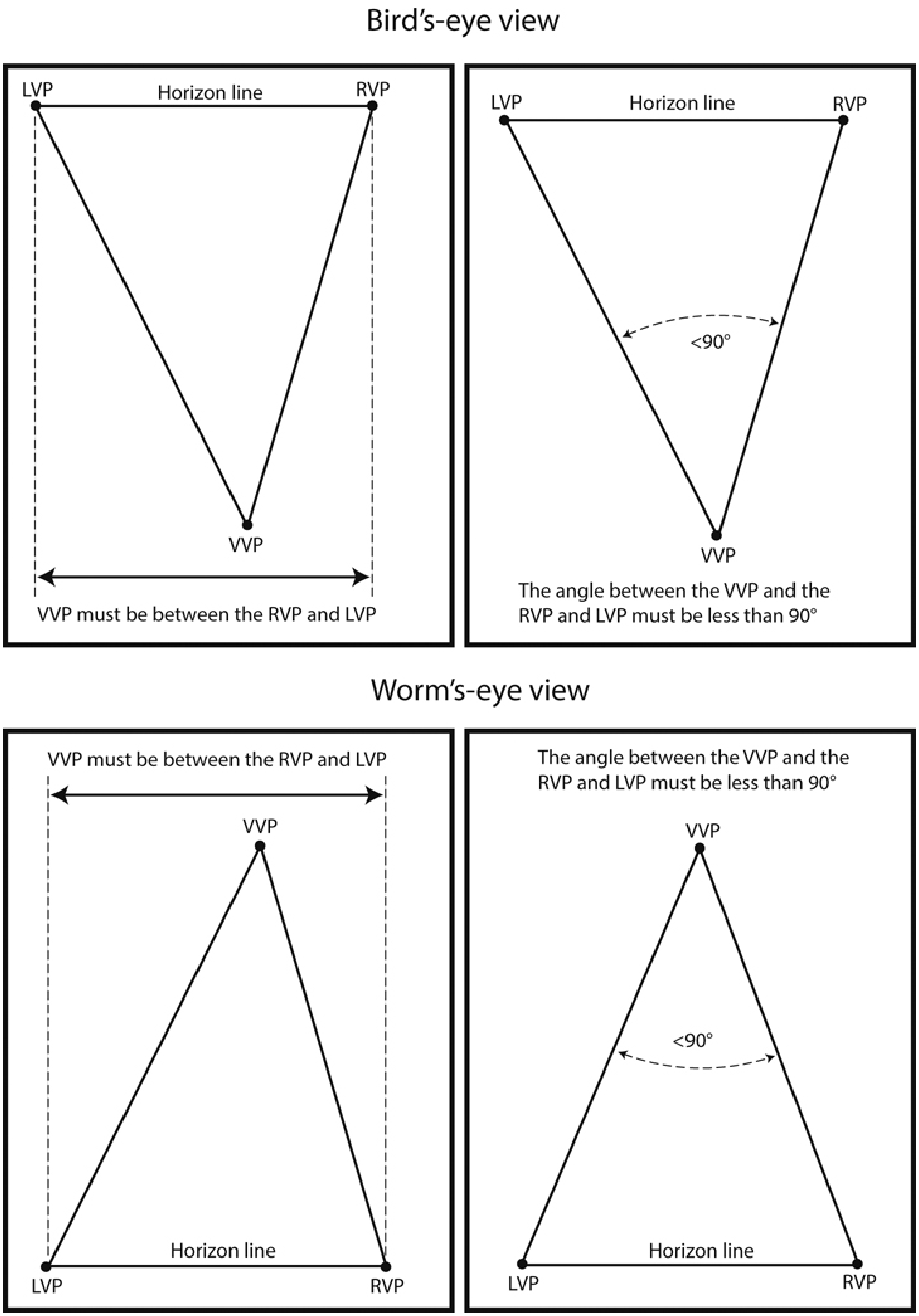


Figure 17.2 The picture plane is at a right angle to the line of sight.

Vanishing Points

Predictably, there are three of them: a right vanishing point, a left vanishing point, and a vertical vanishing point (VVP). If the object's angle to the picture plane is of no concern or is unknown, or the orientation of the line of sight to the ground plane is unknown, the three vanishing points can be located anywhere—well, almost anywhere. There are three caveats to keep in mind: the vertical vanishing point must be between the right and left vanishing point; the horizon line must be horizontal; and the angle at the vertical vanishing point must be less than 90° (Figure 17.3). Within these guidelines, there are many different configurations possible. How to choose the right configuration for specific needs will be covered later in this chapter. Until then, construct a generic three-point diagram with randomly placed vanishing points.



[Figure 17.3](#) **Follow these guidelines for the proper placement of vanishing points.**

The Diagram

The following steps describe the construction of a bird's-eye view. The steps for a worm's-eye view are the same, only upside-down. A worm's-eye view diagram is a bird's-eye view diagram turned 180°.

Center of Vision

Once the three vanishing points have been established, the location of the center of vision is predetermined. Its position is dictated by the geometry. First, draw three lines (reference lines) at right angles from the lines connecting each vanishing point: a left reference line (LRL), a right reference line (RRL), and a vertical reference line (VRL). The reference lines connect to the left vanishing point, right vanishing point, and the vertical vanishing point respectively. These three reference lines intersect at the same spot, at the center of vision (Figure 17.4).

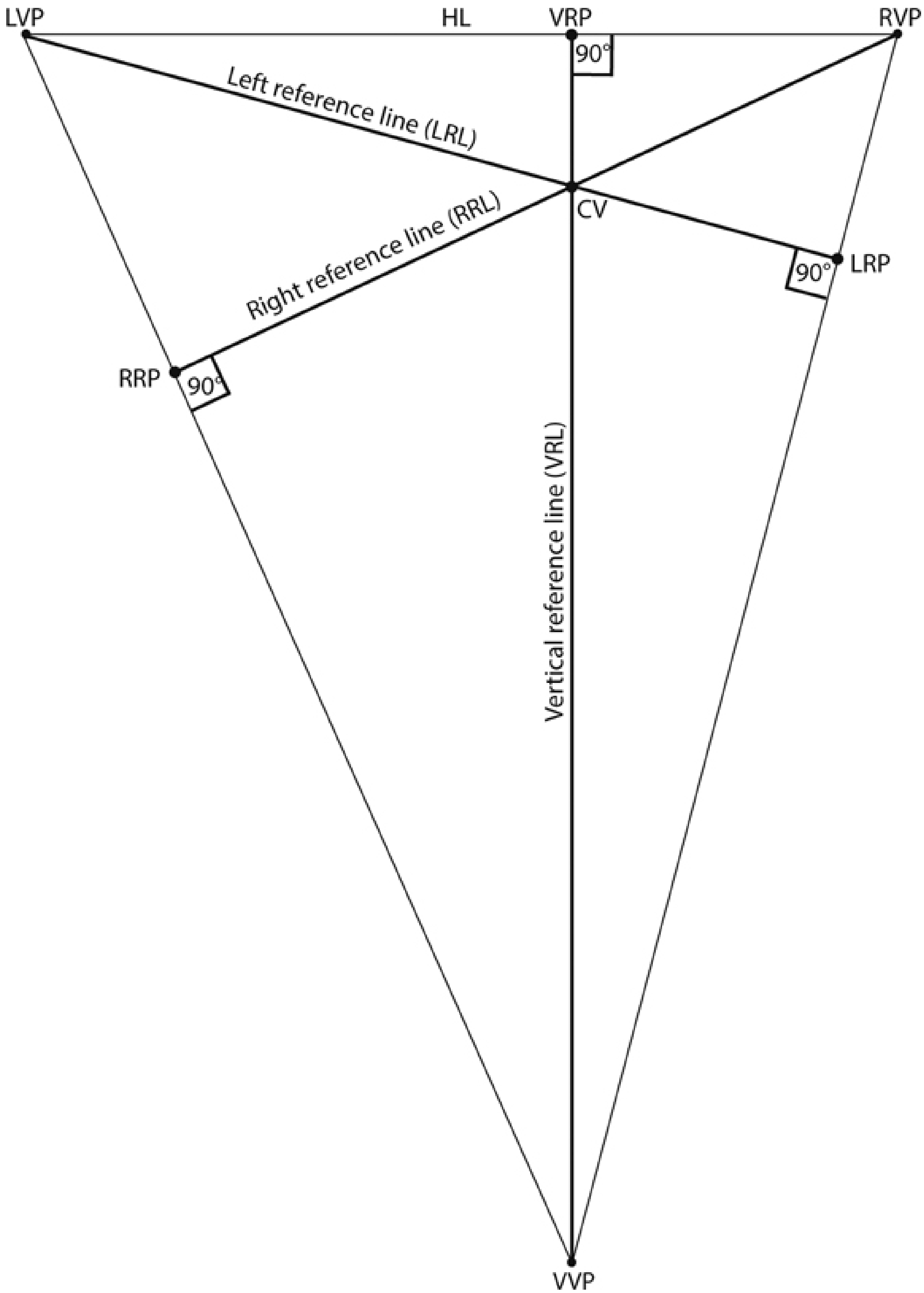


Figure 17.4 **The location of the center of vision is at the intersection of the three reference lines.**

The intersection of each reference line with their orthogonal creates a reference point: a left reference point (LRP), right reference point (RRP), and a vertical reference point (VRP). These reference points are needed to locate the station points.

Station Points

Each vanishing point has a dedicated station point. The left station point (LSP), right station point (RSP), and vertical station point (VSP) are necessary to locate measuring points and define the cone of vision.

Left Station Point

Like the center of vision, the location of the station points is predetermined. Finding the station points can be a little tricky. First, from the center of vision, draw a line 90° from the left reference line (Figure 17.5).

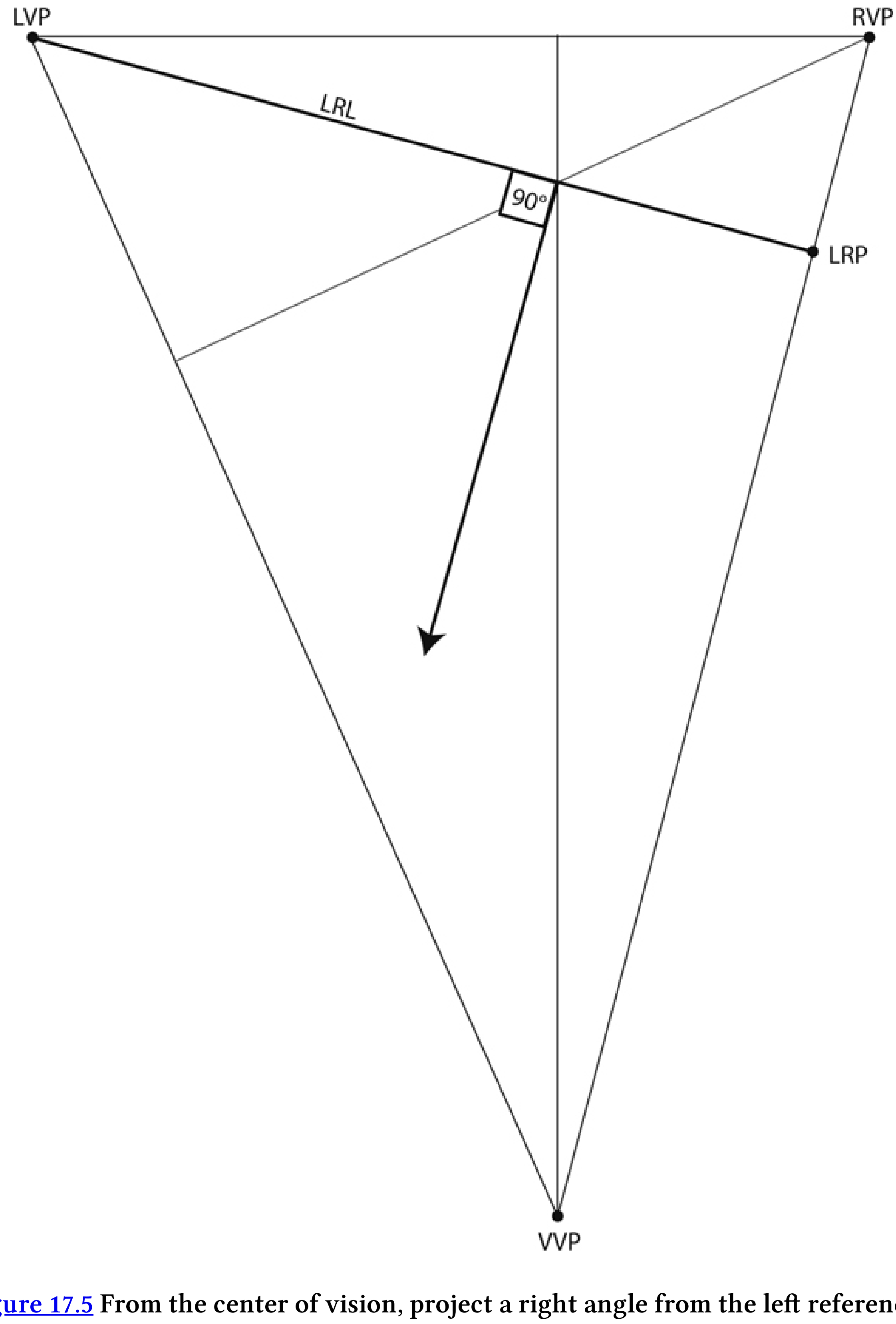


Figure 17.5 From the center of vision, project a right angle from the left reference line.

Here is the tricky part. From the center of vision, a line needs to be triangulated between the left vanishing point, the left reference point, and the line drawn in Figure 17.5. A triangle is the best tool for this job. The left reference point is at the intersection of the left reference line and the orthogonal. The station point's location can vary depending on the diagram. Its placement is determined by the angles created in this procedure. Follow them carefully (Figure 17.6).

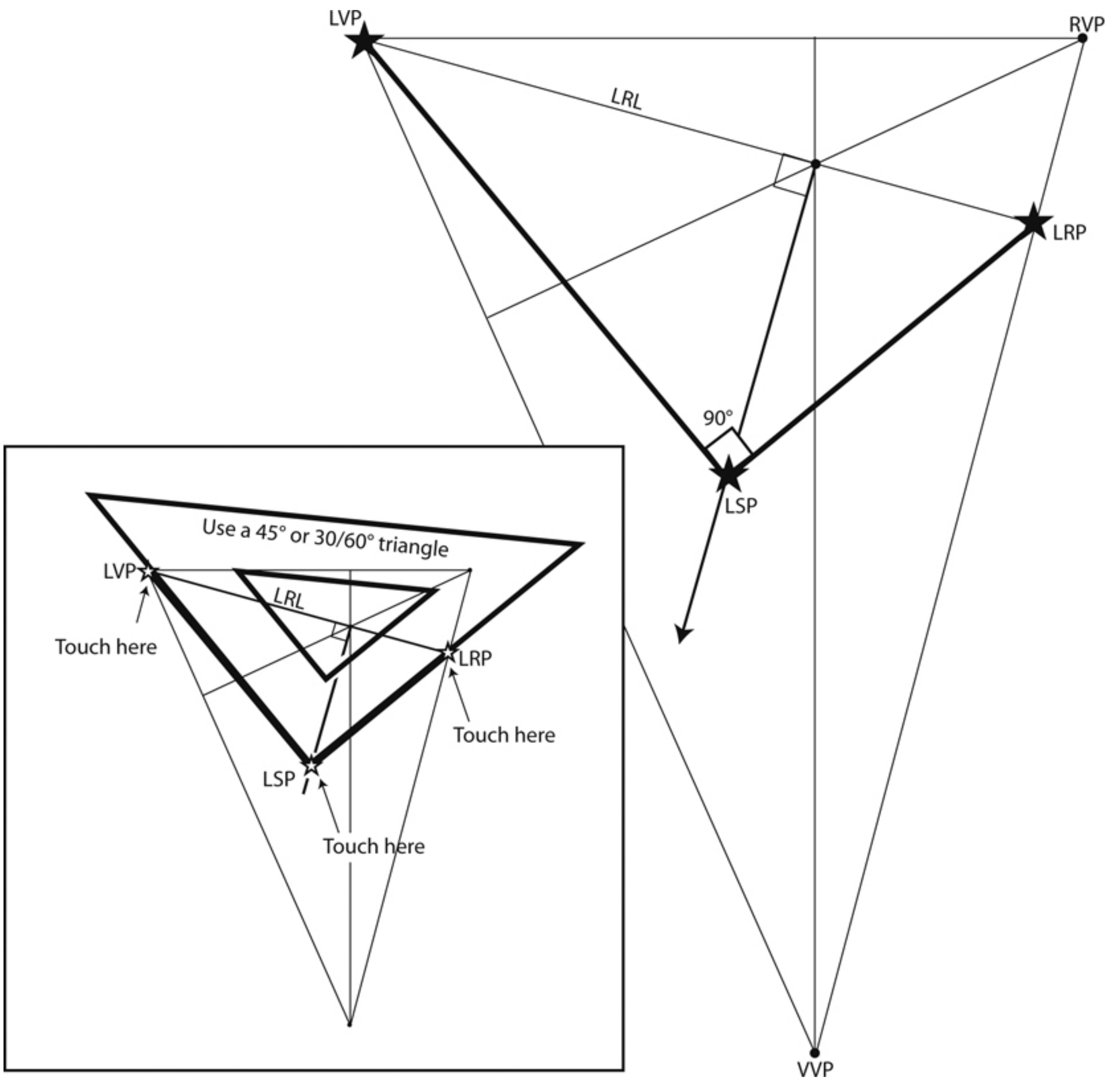


Figure 17.6 Use a triangle to locate the left station point.

Left Measuring Point

Measure the distance from the left vanishing point to the station point, and transfer that distance to the horizon line using a compass or a ruler. The distance from the vanishing point to the station point is the same as the distance from the vanishing point to the measuring point (Figure 17.7).

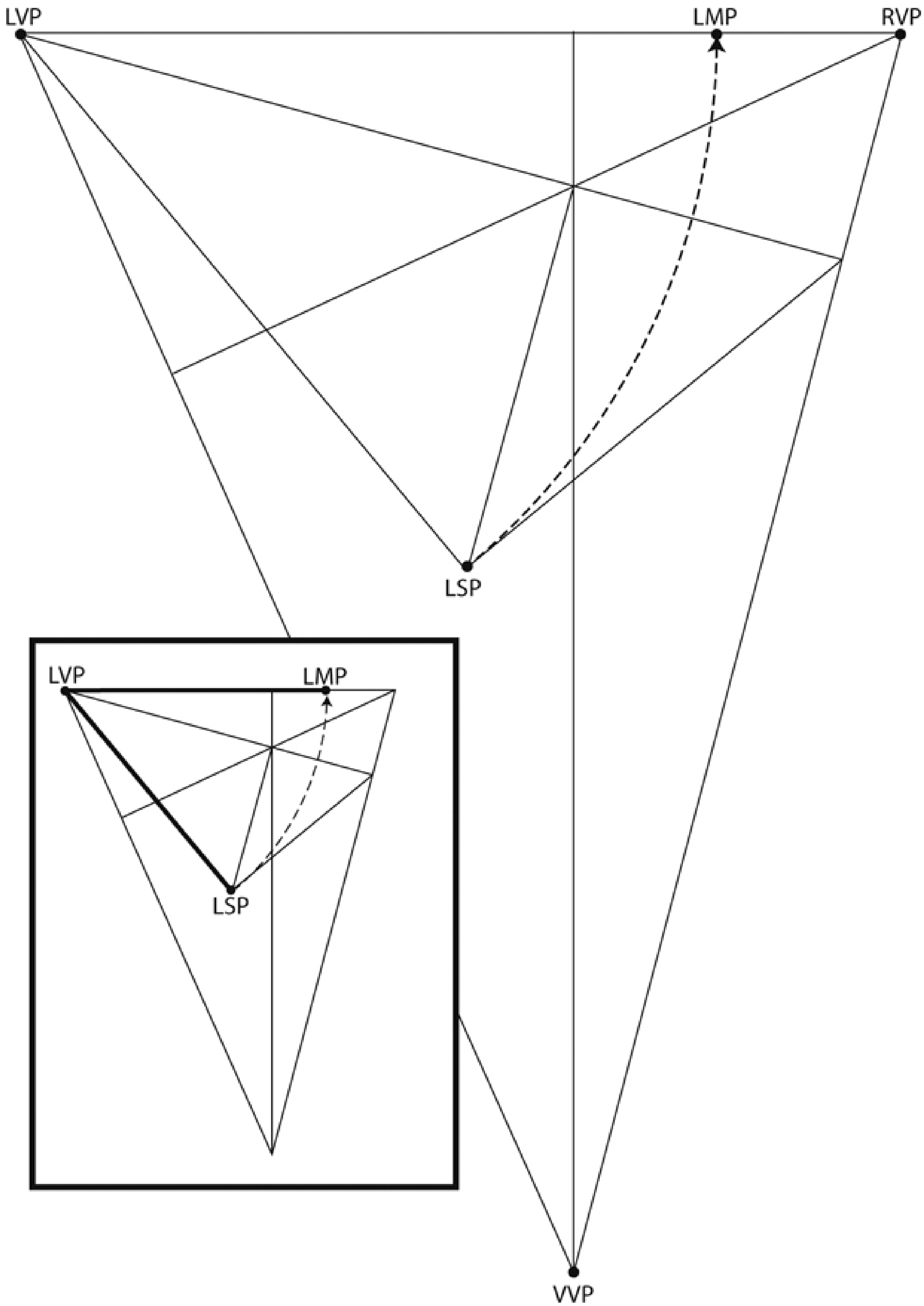


Figure 17.7 Measure the distance from the left vanishing point to the left station point. Transfer that distance to the horizon line.

Right Measuring Point

Use the right reference line to locate the right station point (Figures 17.8–17.9). Transfer the distance from the station point to the horizon line (Figure 17.10).

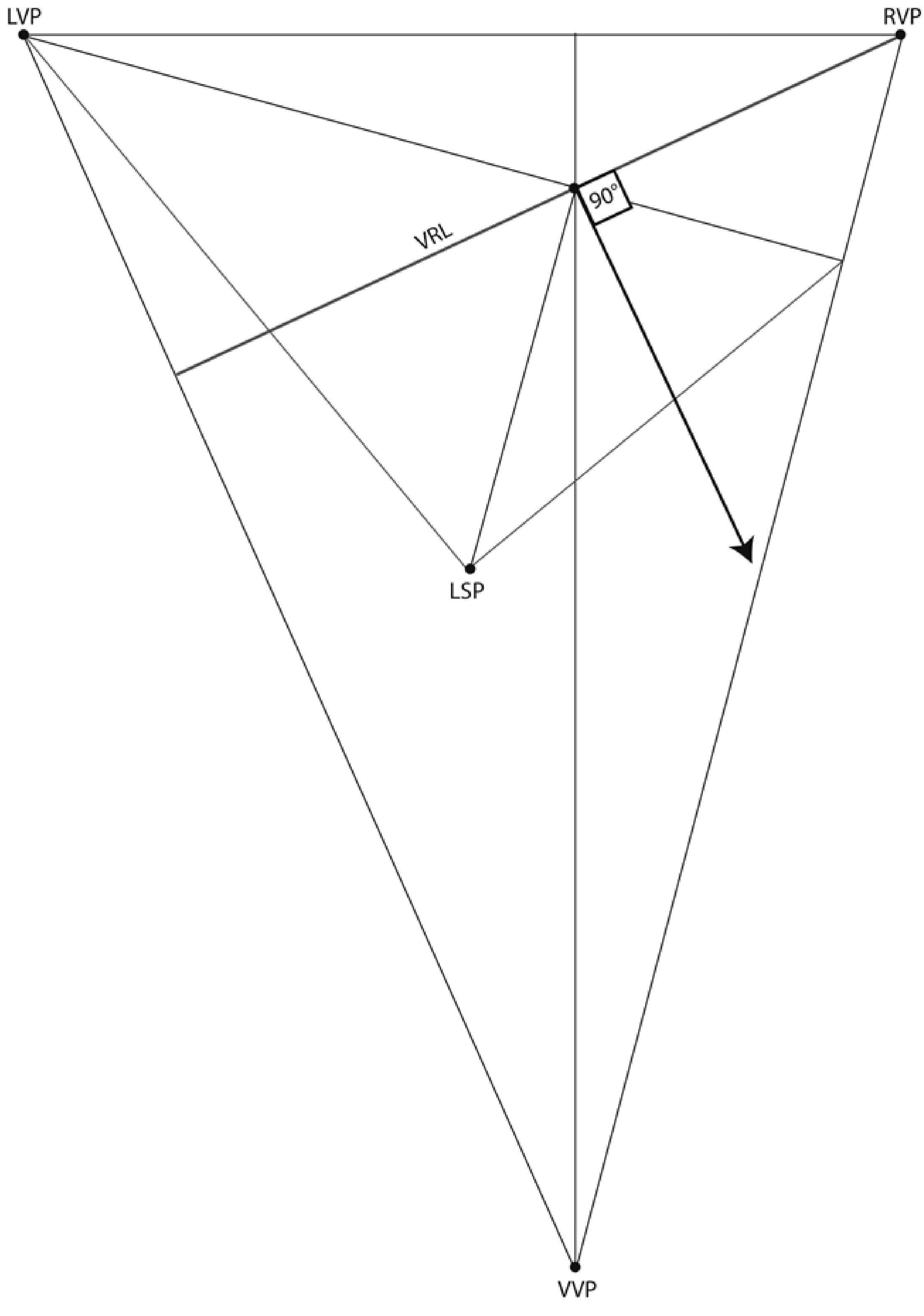
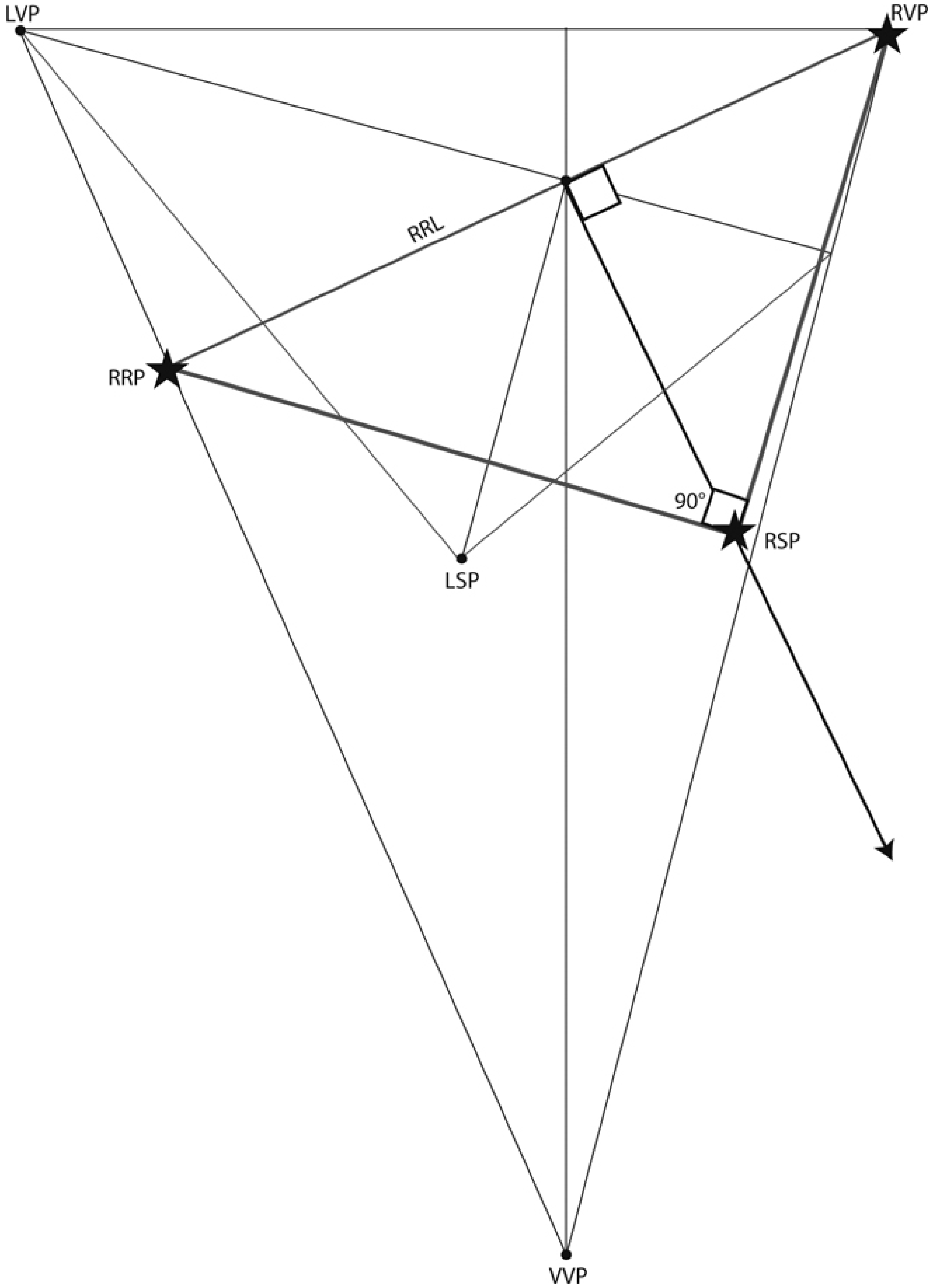


Figure 17.8 At the center of vision, extend a line 90° from the right reference line.



[Figure 17.9](#) **Triangulate the right vanishing point, the right reference point, and the right station point.**

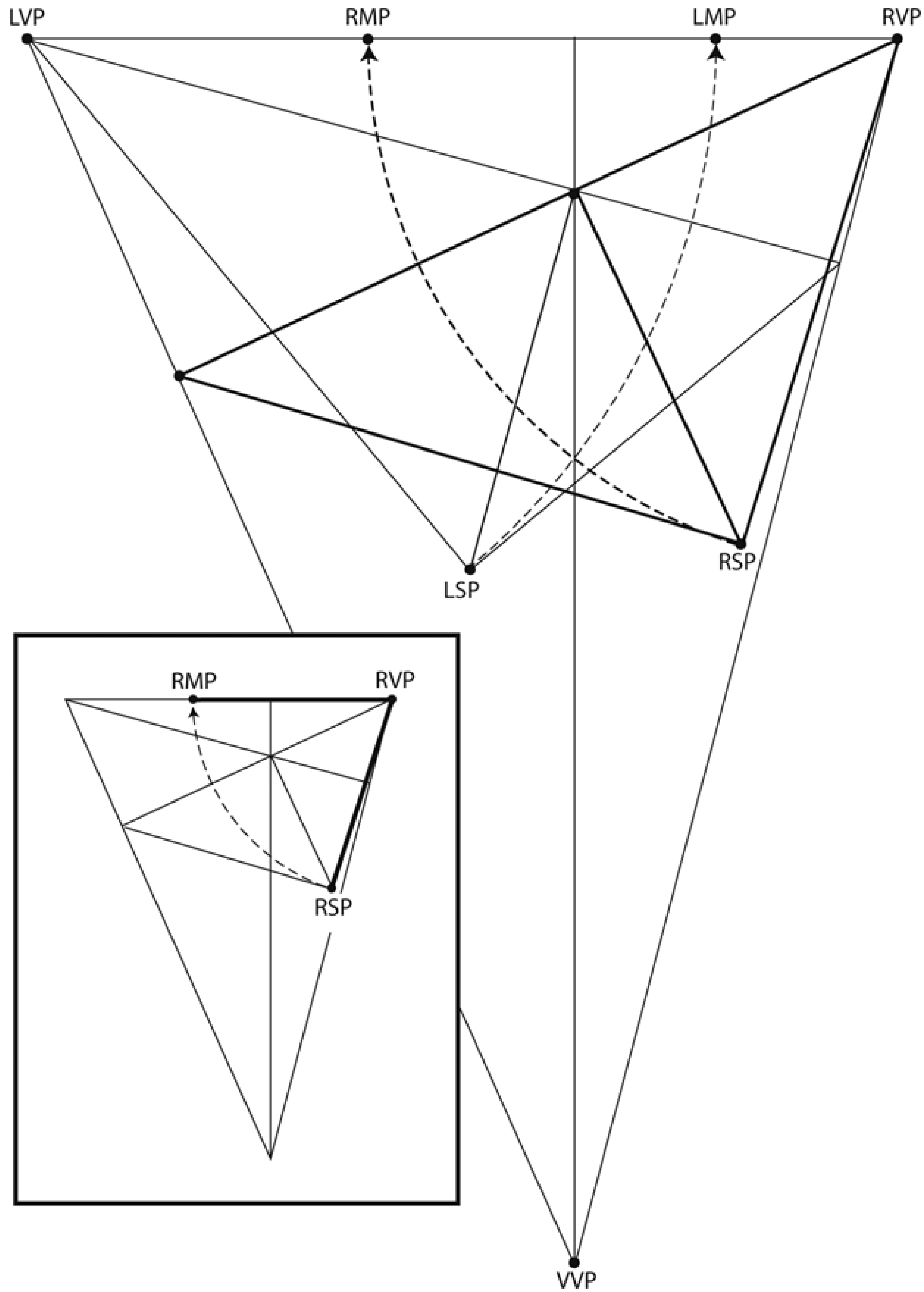
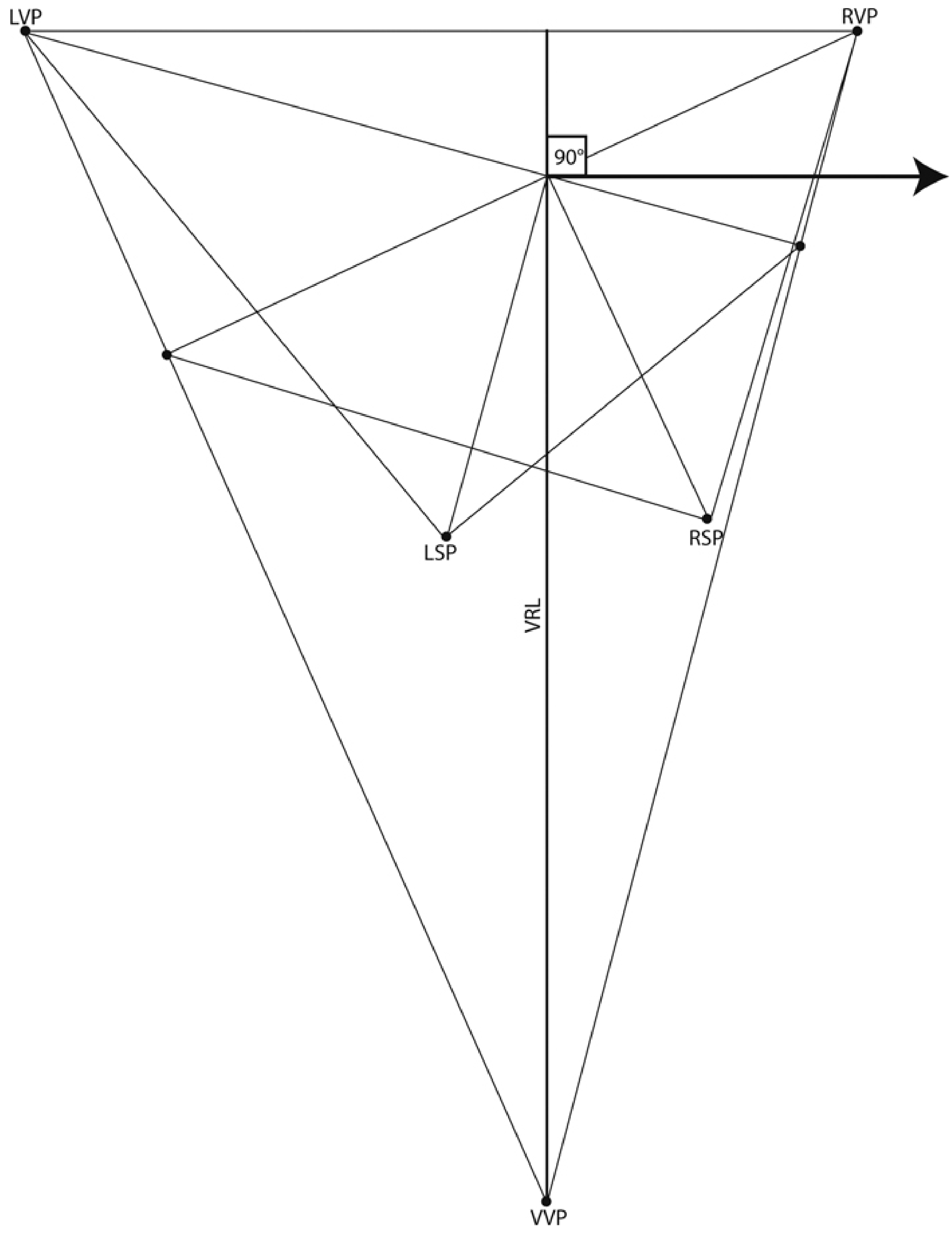


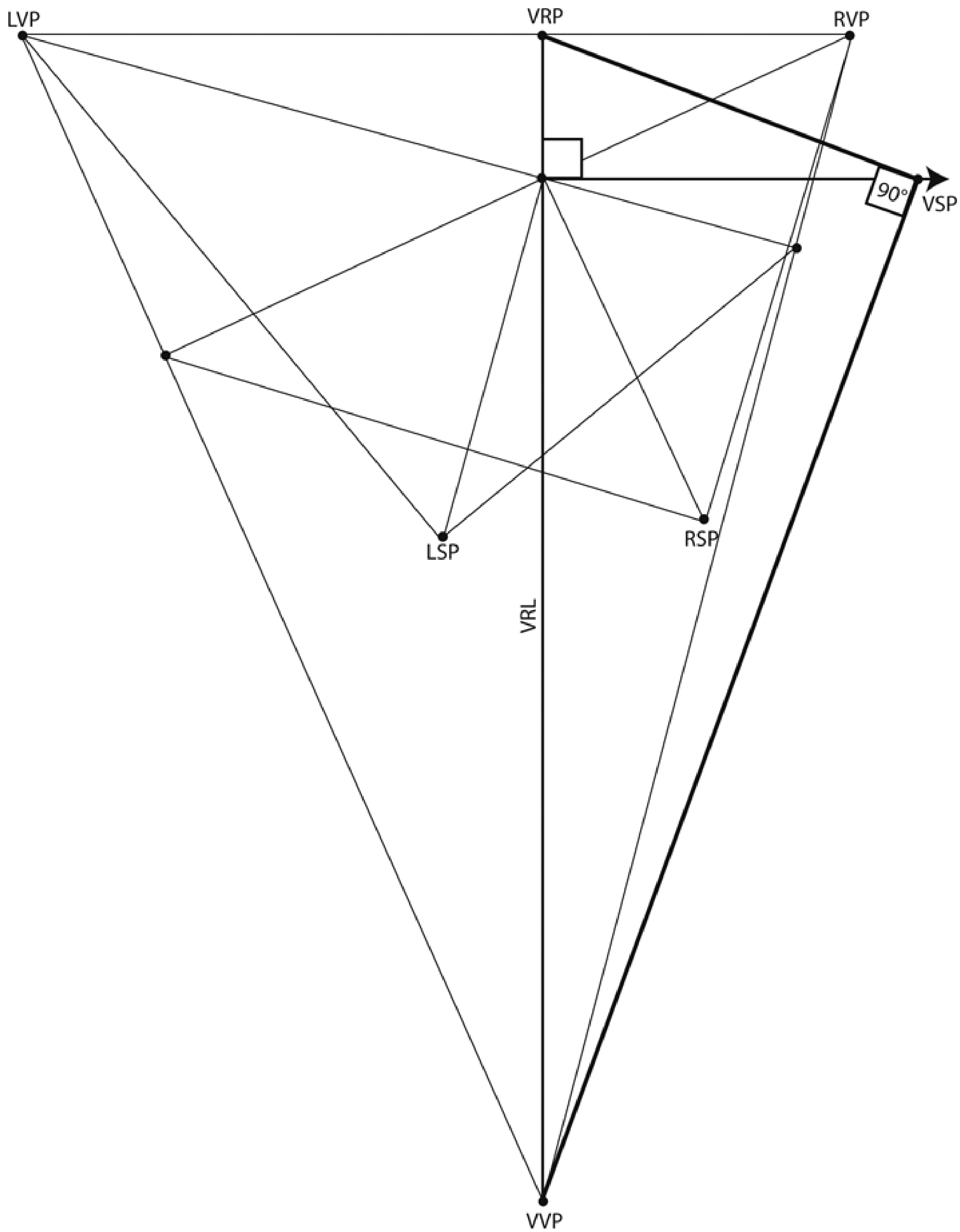
Figure 17.10 **Measure the distance from the right vanishing point to the right station point and transfer that distance to the horizon line.**

Vertical Measuring Point

Use the vertical reference line to locate the vertical station point (Figures 17.11–17.12). Measure the distance from the vertical vanishing point to the vertical station point. Transfer that distance to the line that connects the vertical vanishing point to the left vanishing point. The vertical measuring point is not on the horizon line (Figure 17.13).



[Figure 17.11](#) **Extend a line 90° from the vertical reference line.**



[Figure 17.12](#) **Triangulate the vertical vanishing point, the vertical reference point, and the vertical station point.**

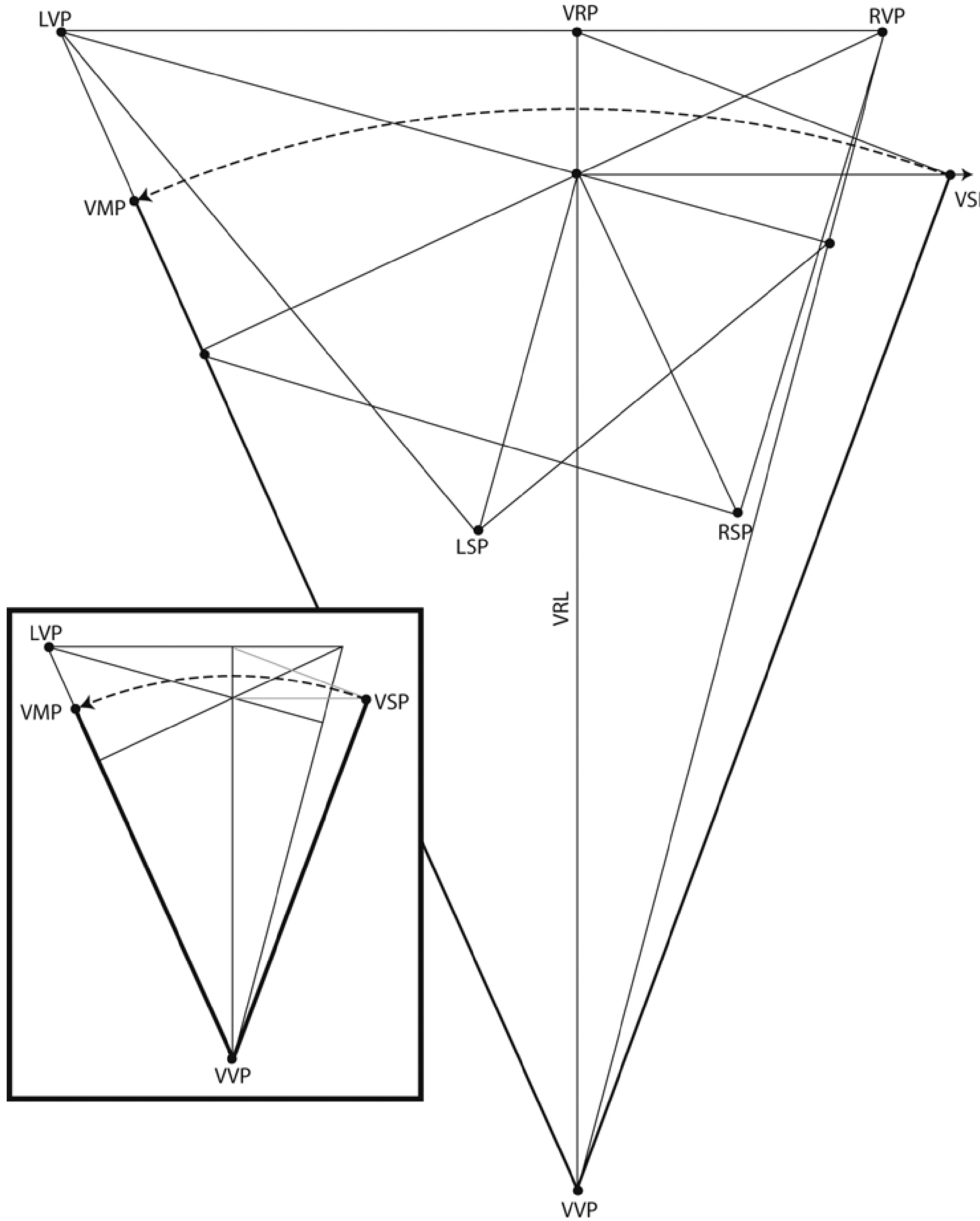
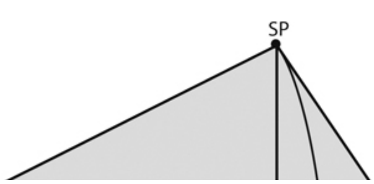


Figure 17.13 Measure the distance from the vertical vanishing point to the vertical station point. Transfer that distance to the line connecting the left vanishing point to the vertical vanishing point.



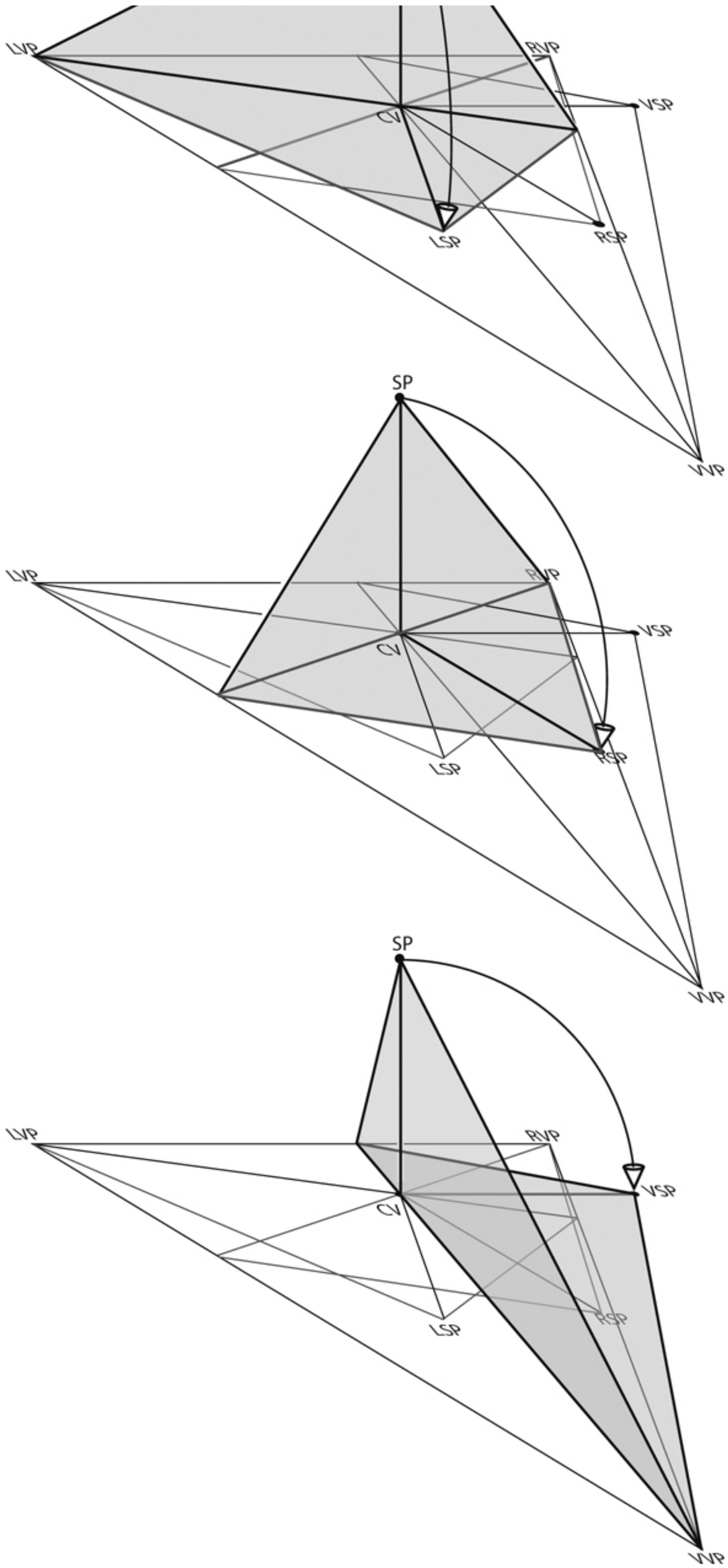


Figure 17.14 **The station point is positioned in three different locations.**

The Shortcut

It may seem strange that there are three station points—and it should. The station point represents the viewer, and there is only one viewer. So why are there three station points? The answer is simple: there are not three station points. Each station point is the same point. The left, right, and vertical station points are all equal distance from the picture plane. Each represent the same point in space. The same station point has been placed in three different locations, enabling it to find three different measuring points (Figure 17.14).

This shortcut is based on all three of the station points being equal distance from the picture plane. Locate one station point, and then repeat that distance to find the others. After finding the first station point, use a compass to find the other two (Figure 17.15). Then add the measuring points using the previously described method (Figure 17.16).

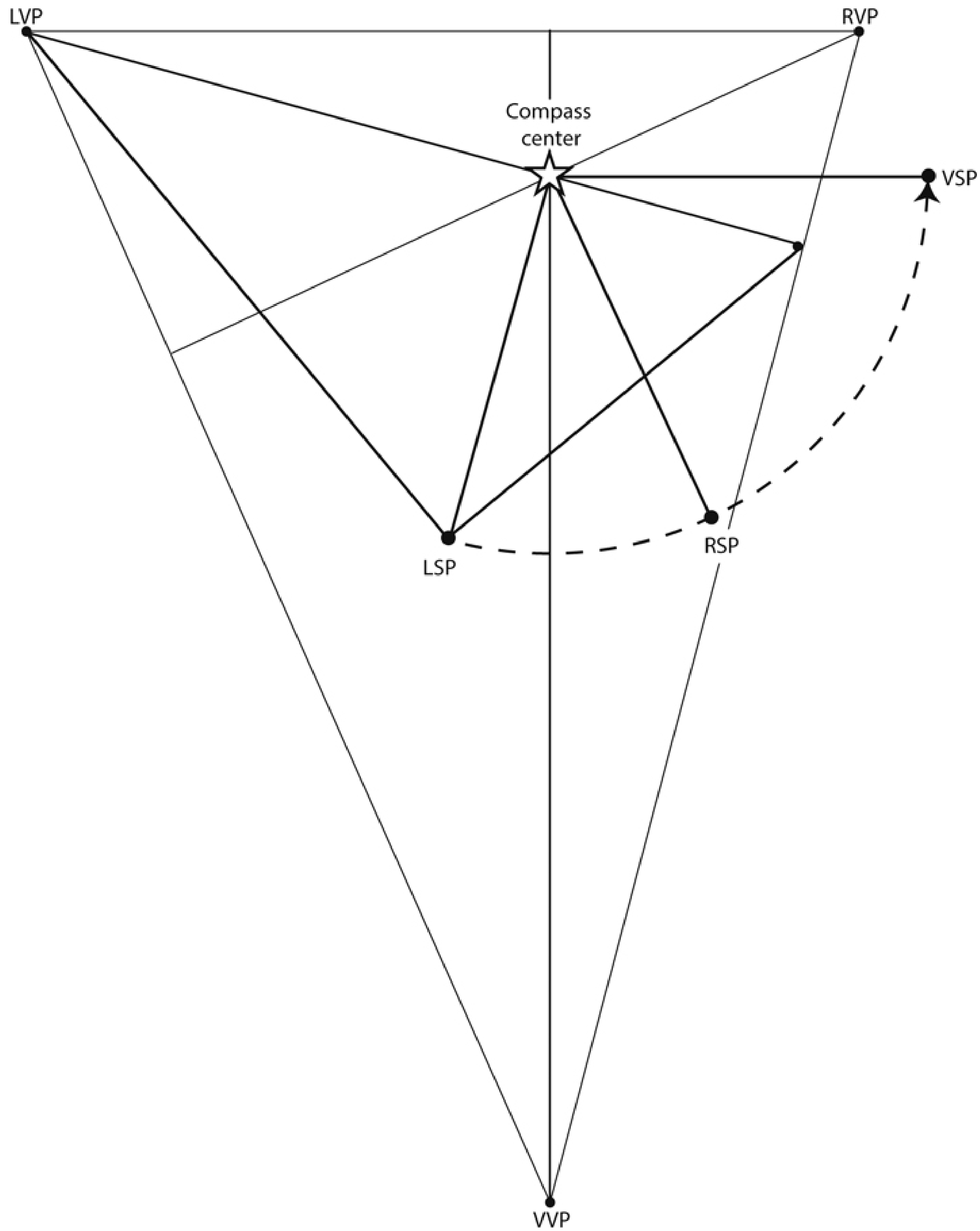
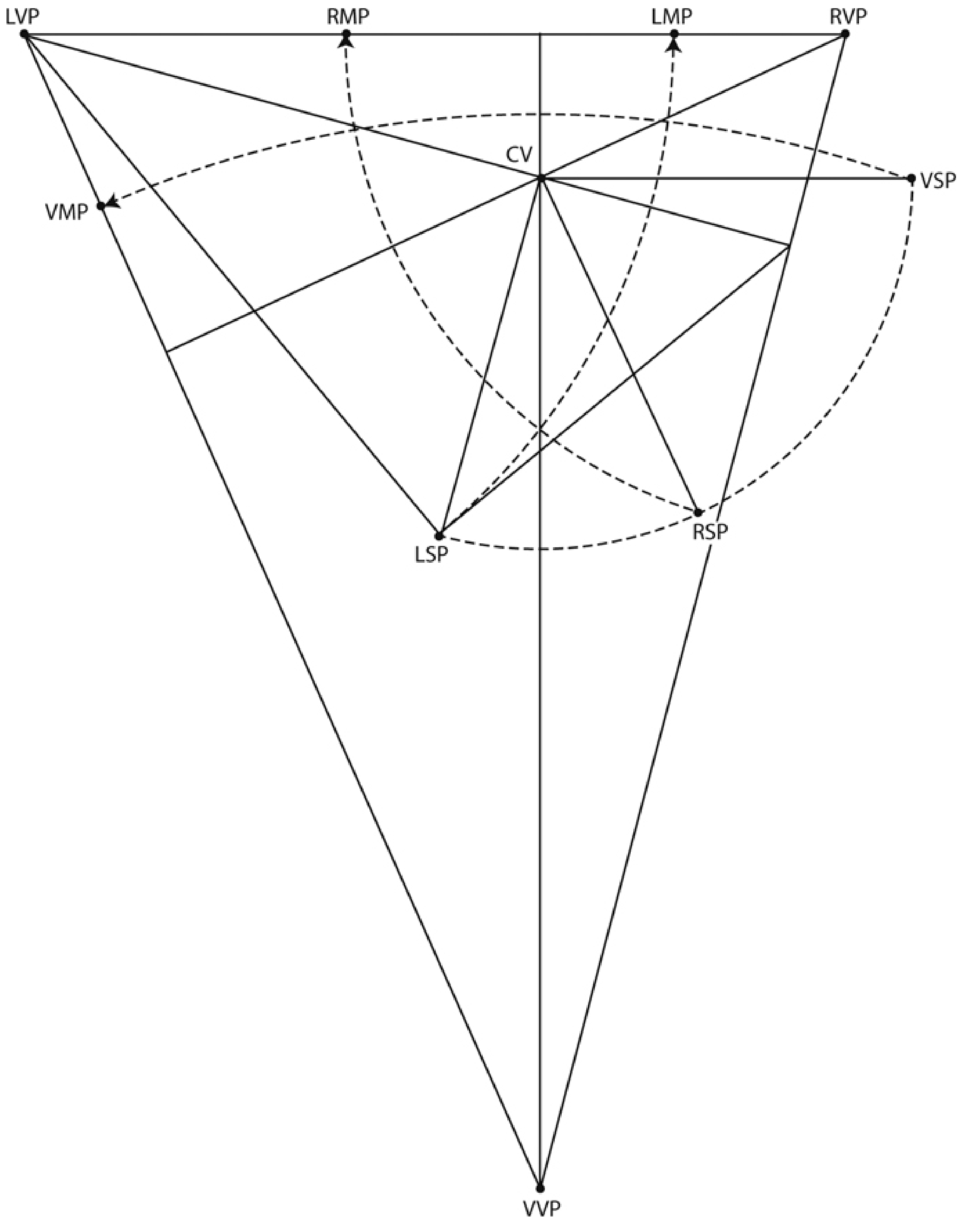


Figure 17.15 **This shortcut for locating station points saves time and uses fewer lines.**



[Figure 17.16](#) The distance from the station point to the vanishing point is the same as the distance from the vanishing point to the measuring point.

Cone of Vision

The cone of vision is, as always, 60°. Any station point can be used to determine the cone of vision (Figure 17.17). The left, right, or vertical station point will all give the same result.

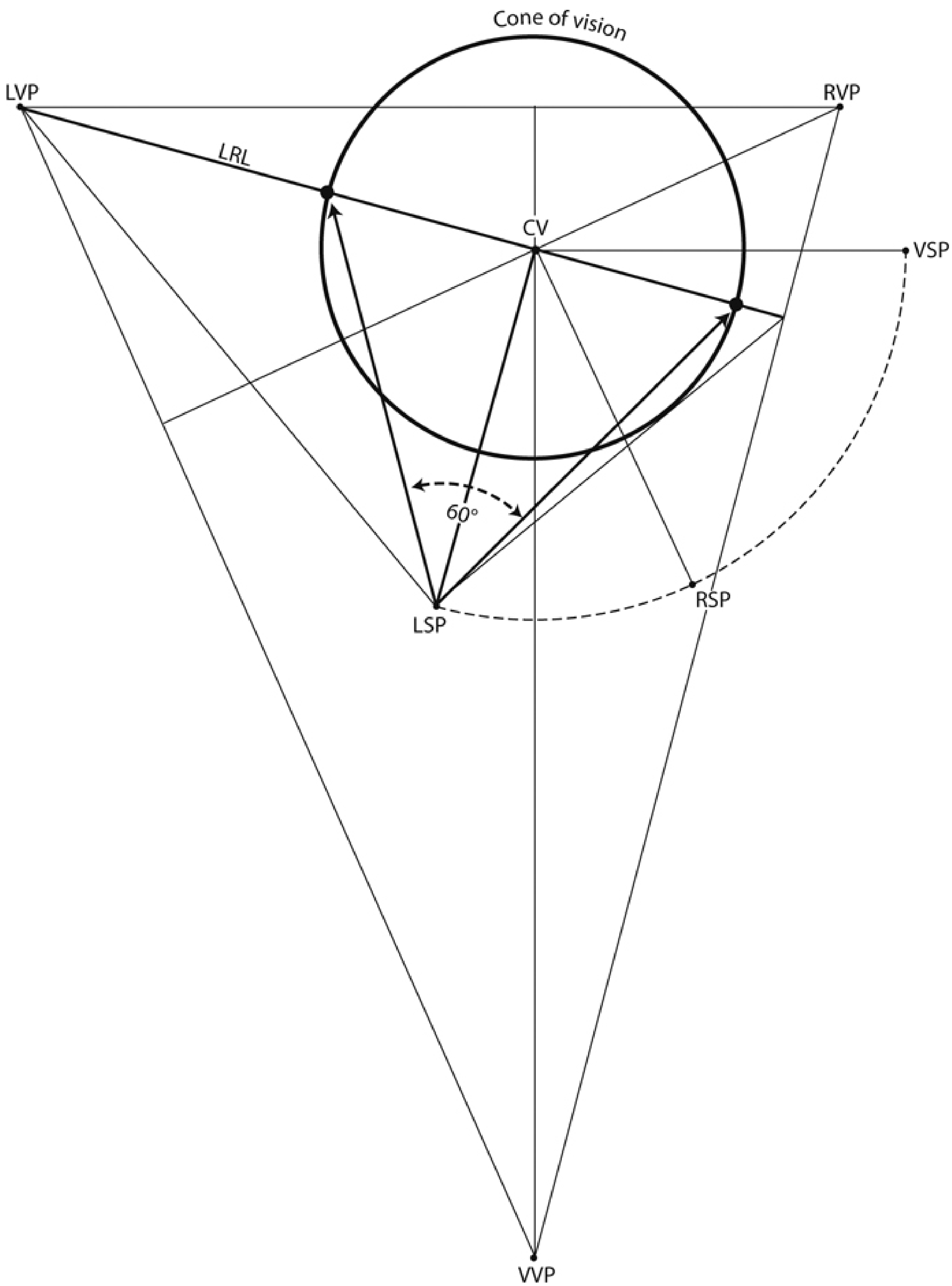


Figure 17.17 **To establish the cone of vision, draw a 60° angle from the station point**.

Measuring Line

There are two measuring lines in three-point perspective: a horizontal and a vertical measuring line. The horizontal measuring line (HML) is used for measuring lines parallel with the ground plane. The vertical measuring line (VML) is used for measuring lines perpendicular to the ground plane. For the geometry to be correct, it is critical that the measuring line is parallel with the line the measuring point is on. The right and left measuring points are on the horizon line, so the measuring line must be horizontal. The vertical measuring point is on an angled line so the measuring line must be at the same angle. A line perpendicular to the right reference line will be parallel with the line the measuring point is on. The line the right station point is on becomes the vertical measuring line (Figure 17.18).

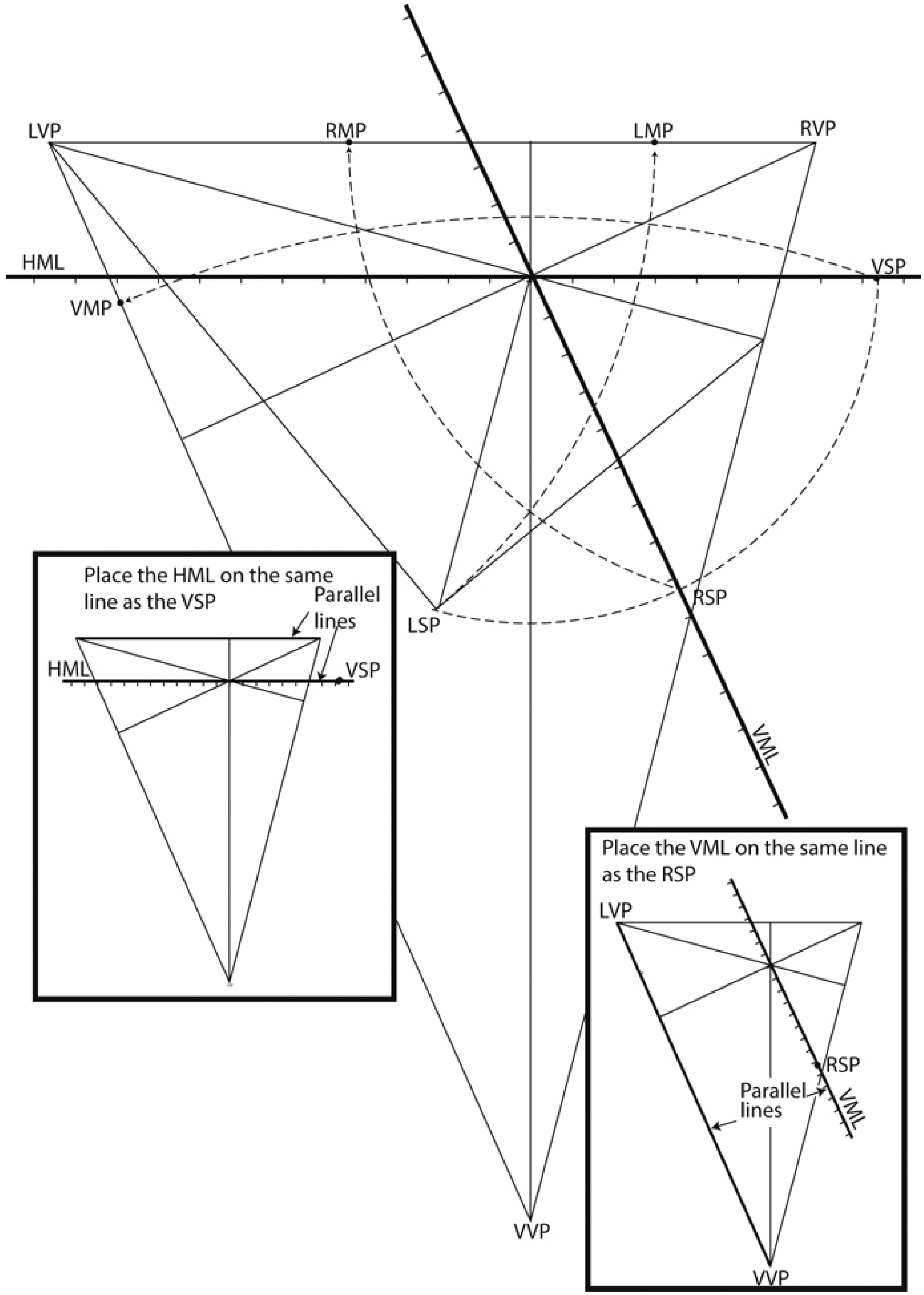


Figure 17.18 The placement of measuring lines completes the diagram.

Congratulations—the three-point perspective diagram is finished. Now it is time to draw something.

Drawing a Cube

For the first three-point example, draw a 5 unit cube, its front corner touching the picture plane and tangent to the center of vision.

Horizontal Dimensions

Measuring horizontal lines (the top of the box) in three-point perspective is done exactly the same as in two-point perspective (Figure 17.19). The same rules apply. Confirm that what is being measured and the measuring line are on the same plane. This can be tricky in three-point perspective, as the measuring line is not always on the ground.

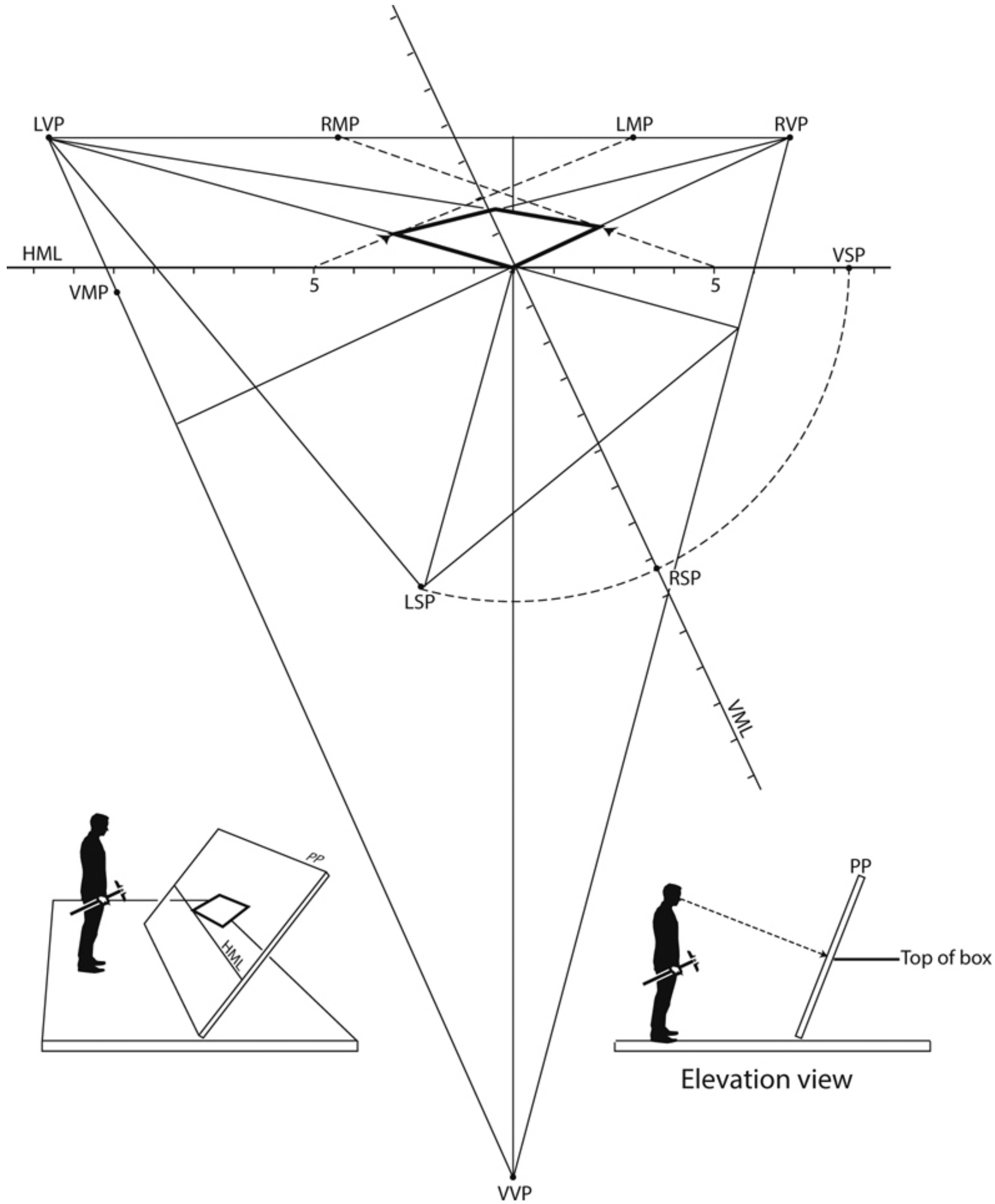


Figure 17.19 **Measuring horizontal dimensions in three-point perspective is no different than measuring horizontal dimensions in two-point perspective.**

Vertical Dimensions

All vertical lines connect to the vertical vanishing point. To measure these lines, use the vertical measuring line and the vertical measuring point. In this example, the line being measured is the vertical reference line.

From the center of vision, count 5 units along the vertical measuring line. Then, using the vertical measuring point, project that dimension to the vertical reference line. Connect to the vertical measuring point, intersecting the vertical reference line. This intersection represents 5 units in perspective (Figure 17.20).

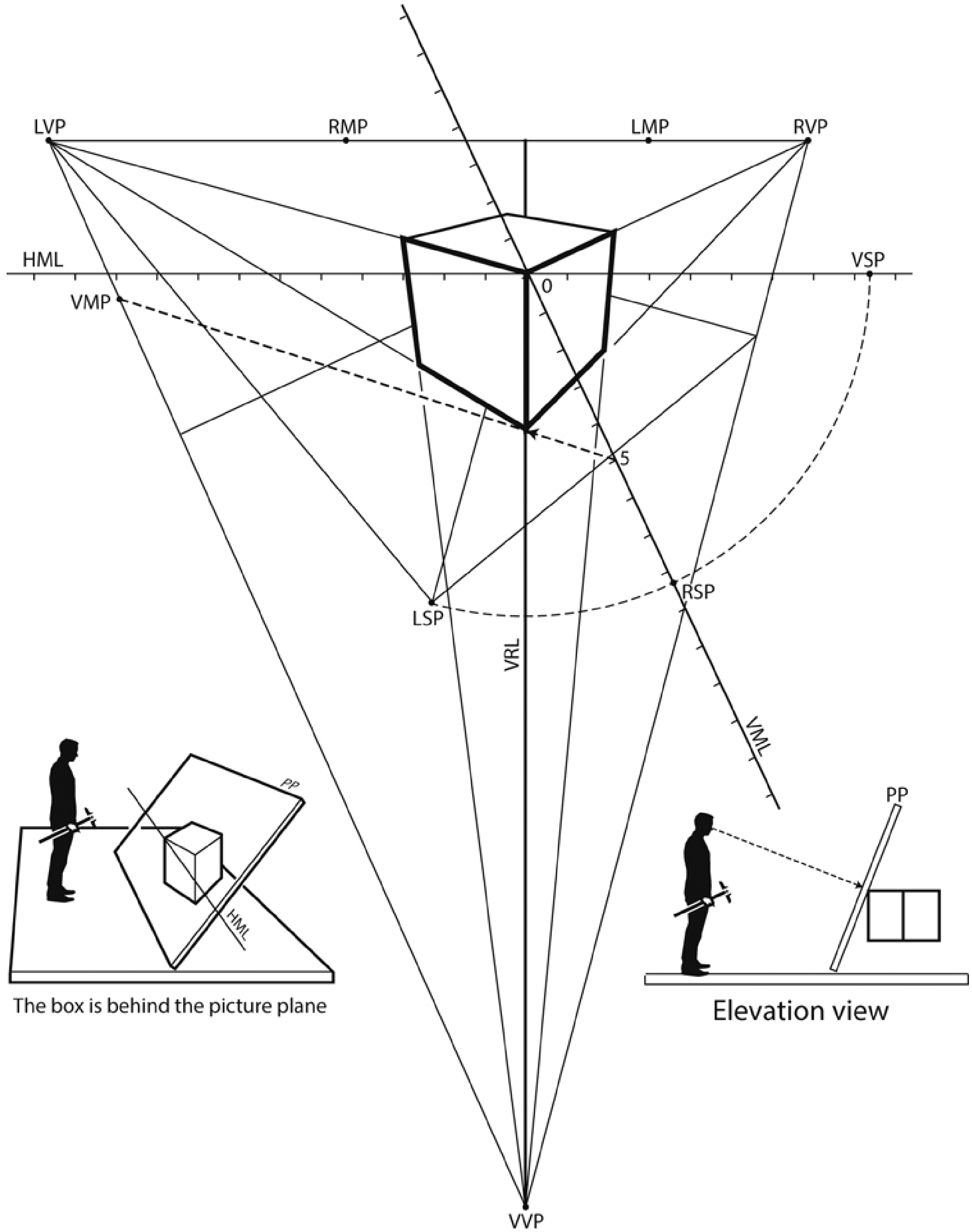


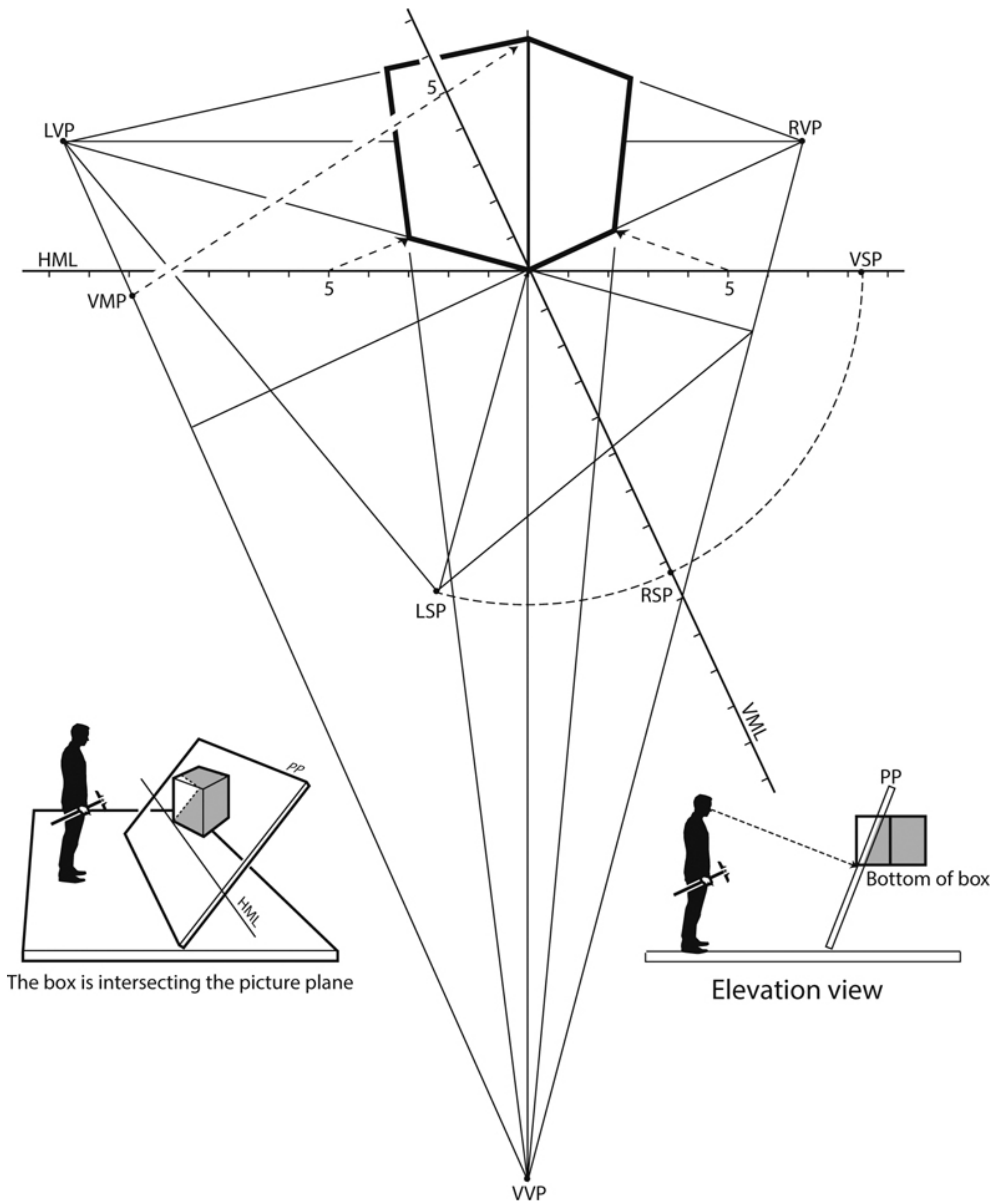
Figure 17.20 **Use the vertical measuring point to measure vertical dimensions. This is a 5 unit cube.**

The vertical measuring point is used for all vertical measurements. It is critical that the object being measured is on the same plane as the measuring line. When measuring vertical dimensions, this can be difficult to determine.

A safe bet is to use the vertical reference line for all vertical measurements. The vertical reference line is always on the same plane as the vertical measuring line. Measure the height along the vertical reference line, then project that distance (using a vanishing point or reference point) to its desired location. This takes some practice.

Ground Plane

In Figure 17.20, the top of the cube is at the level of the horizontal measuring line. It is 5 units above the ground plane. If the horizontal measuring line is on the ground plane, measure vertical dimensions up instead of down. The measurements are still along the vertical measuring line, still projected to the vertical measuring point, and still placed along the vertical reference line. Now the *bottom* of the box is on the same plane as the horizontal measuring line, and the *top* of the box is 5 units above (Figure 17.21).



[Figure 17.21](#) Measuring above the horizontal measuring line follows the same procedures as measuring below the horizontal measuring line. The shaded area represents the part of the box that is behind the picture plane.

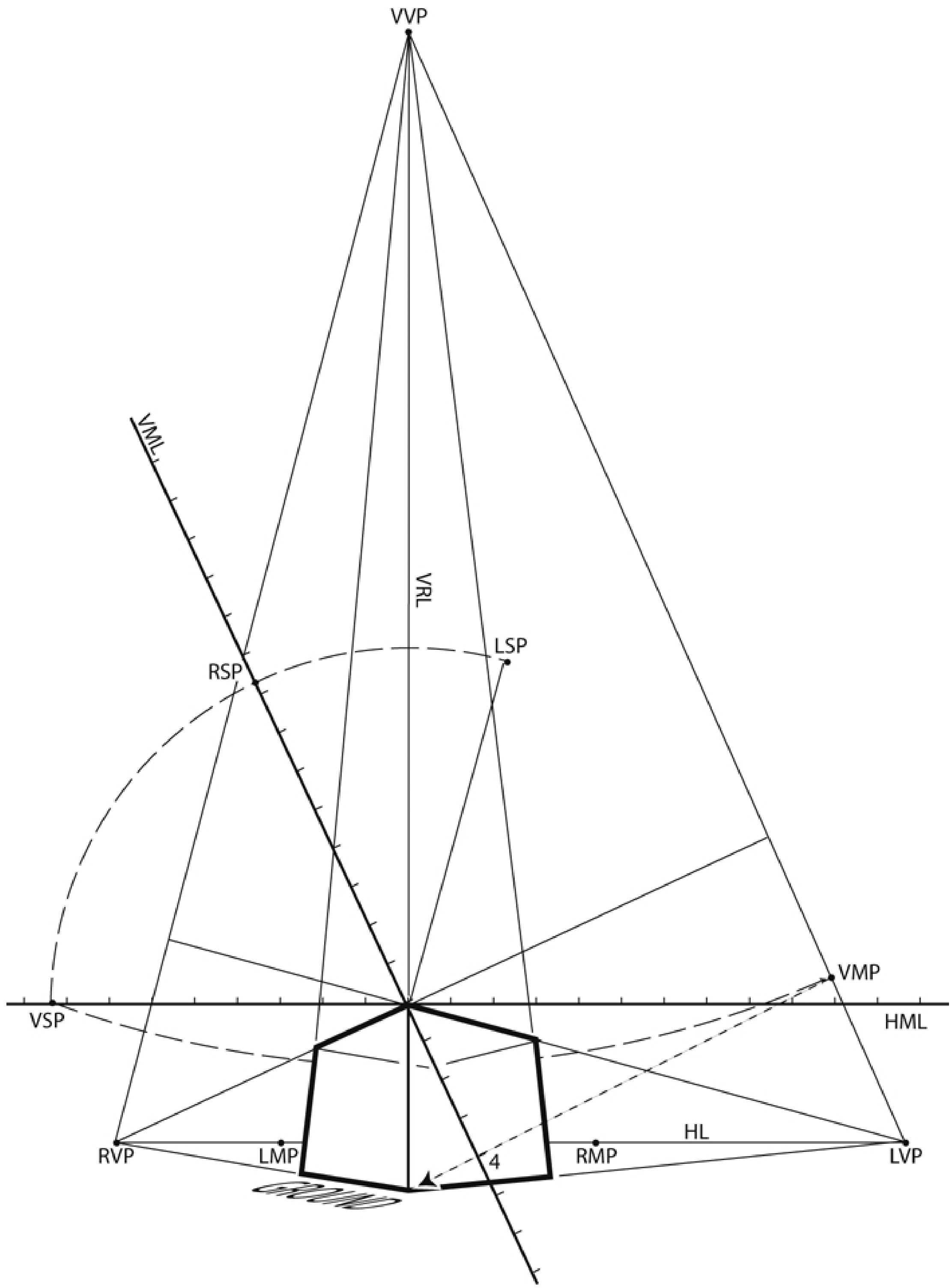
Worm's-Eye View

A bird's-eye view diagram turned upside-down becomes a worm's-eye view diagram. The diagram is the same, only rotated 180°. All the guidelines for constructing the diagram and for measuring objects still apply. When drawing a worm's-eye view there are, however, a couple of issues to keep in mind when measuring and establishing the ground plane.

Establishing the Ground Plane

Measuring Width

The horizontal measuring line is tangent to the center of vision. The center of vision, being above the horizon line, requires that all horizontal measurements take place above the ground plane (Figure 17.22). For example, if the horizontal measuring line is 4 units above the ground plane, make all the horizontal measurements at that level.



[Figure 17.22](#) **This 5 unit square is drawn on the same plane as the measuring line.**

Measuring Height

For an object to be sitting on the ground (as opposed to floating above the ground), its base must be at or below the horizon line. Once the width and depth of the object has been established, measure down to the ground plane. The ground plane can be located anywhere. The farther that is measured down, the higher the horizontal measuring line will be. How tall the object is decides where the ground is (Figure 17.23).

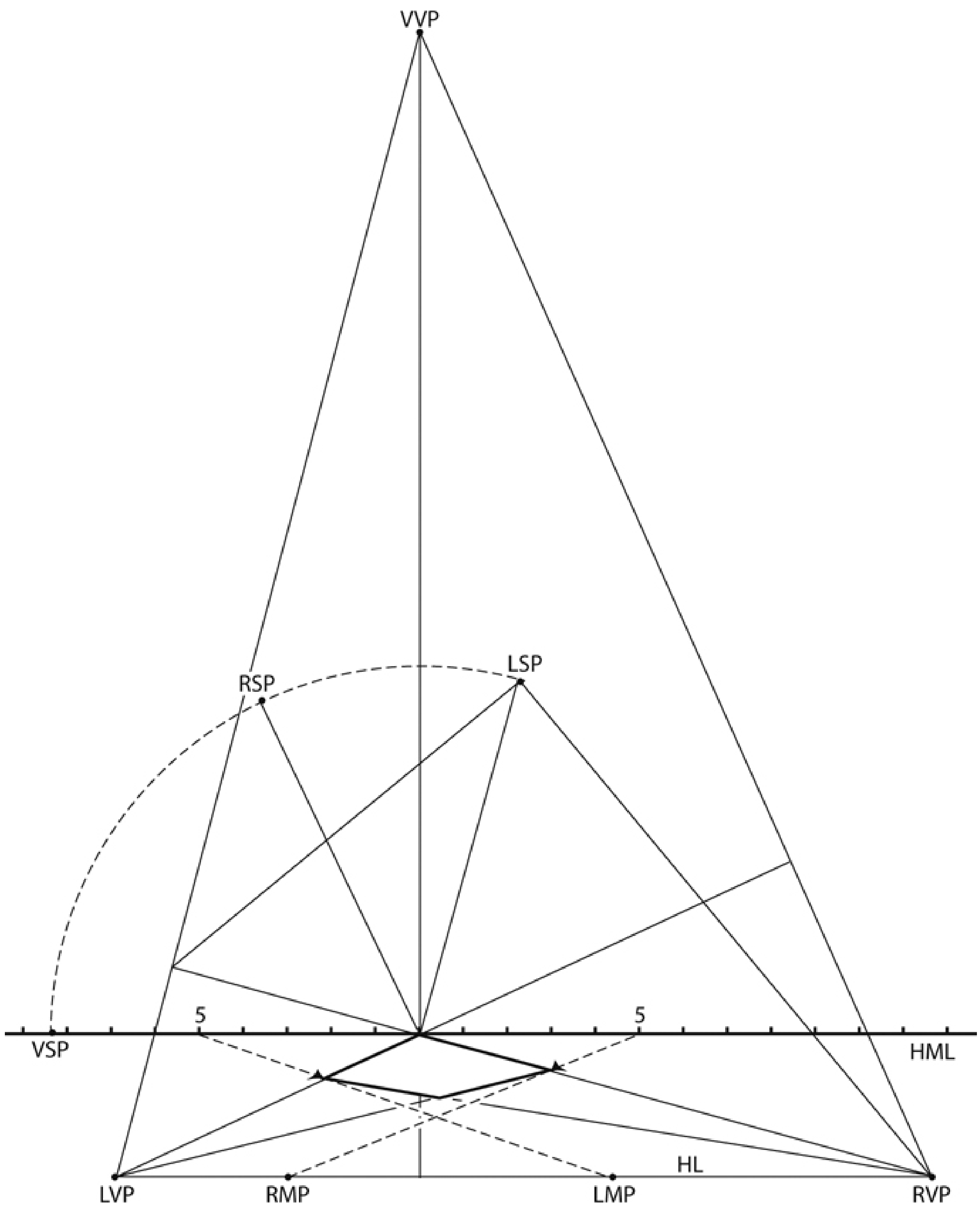
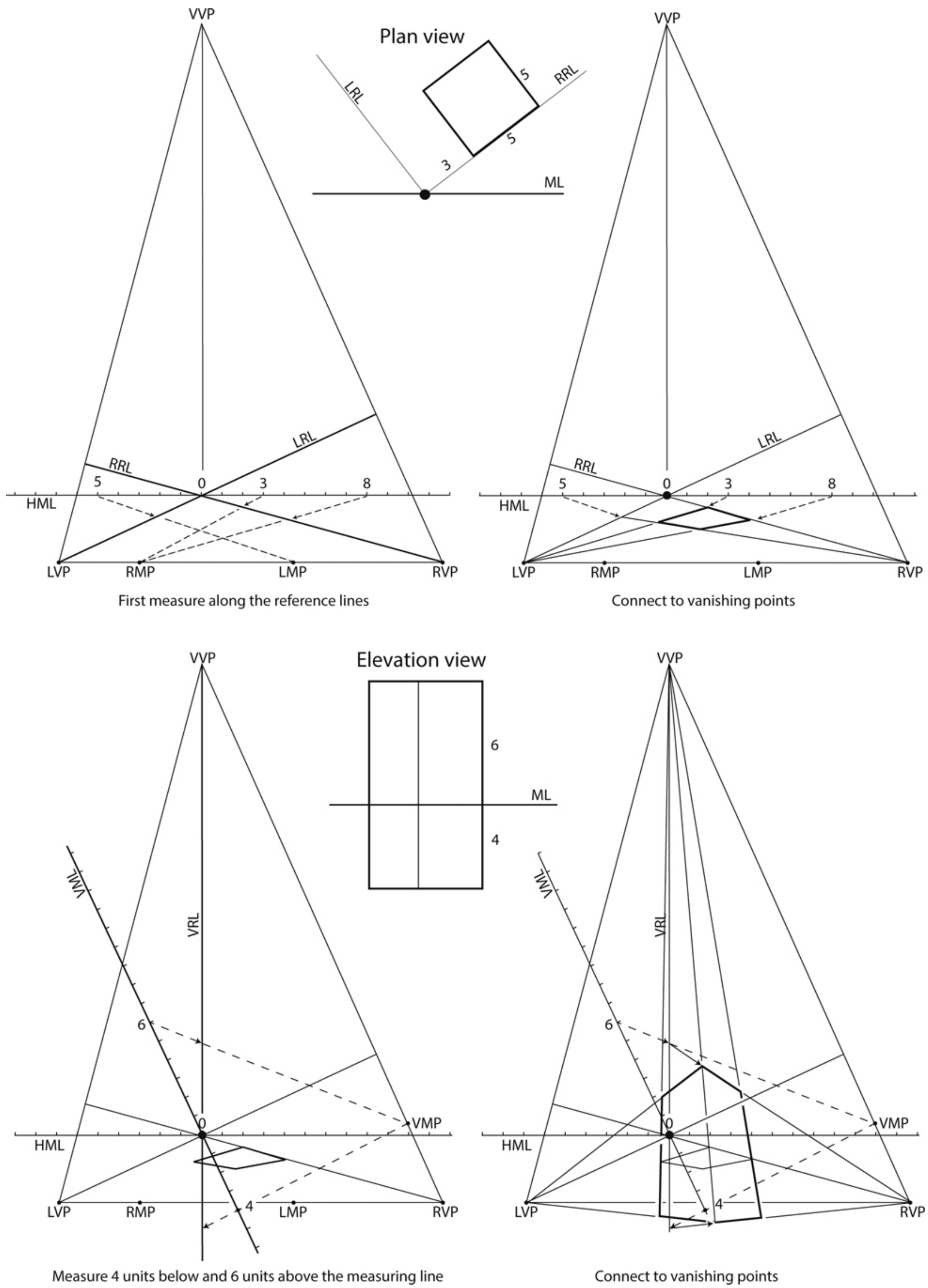


Figure 17.23 The top of this box and the horizontal measuring line are 4 units above the ground plane.

All of the previous examples have had the corner of the box touching the center of vision. Before moving on, here is an example where the box is in a different location. The plan and elevation views describe its position (Figure 17.24).



[Figure 17.24](#) Horizontal dimensions are measured at the level of the horizontal measuring line. Vertical dimensions are measured along the vertical reference line and then projected to their desired location.

18
Three-Point Angles

Now that the basics of three-point perspective have been covered (as unlikely as it may seem— yes, that was only the basics), vanishing points can be found and objects can be measured. But, using this knowledge, all objects would be parallel with each other. To draw objects at different angles requires different vanishing points. The following explains how to find them.

Left, Right, and Vertical Axes

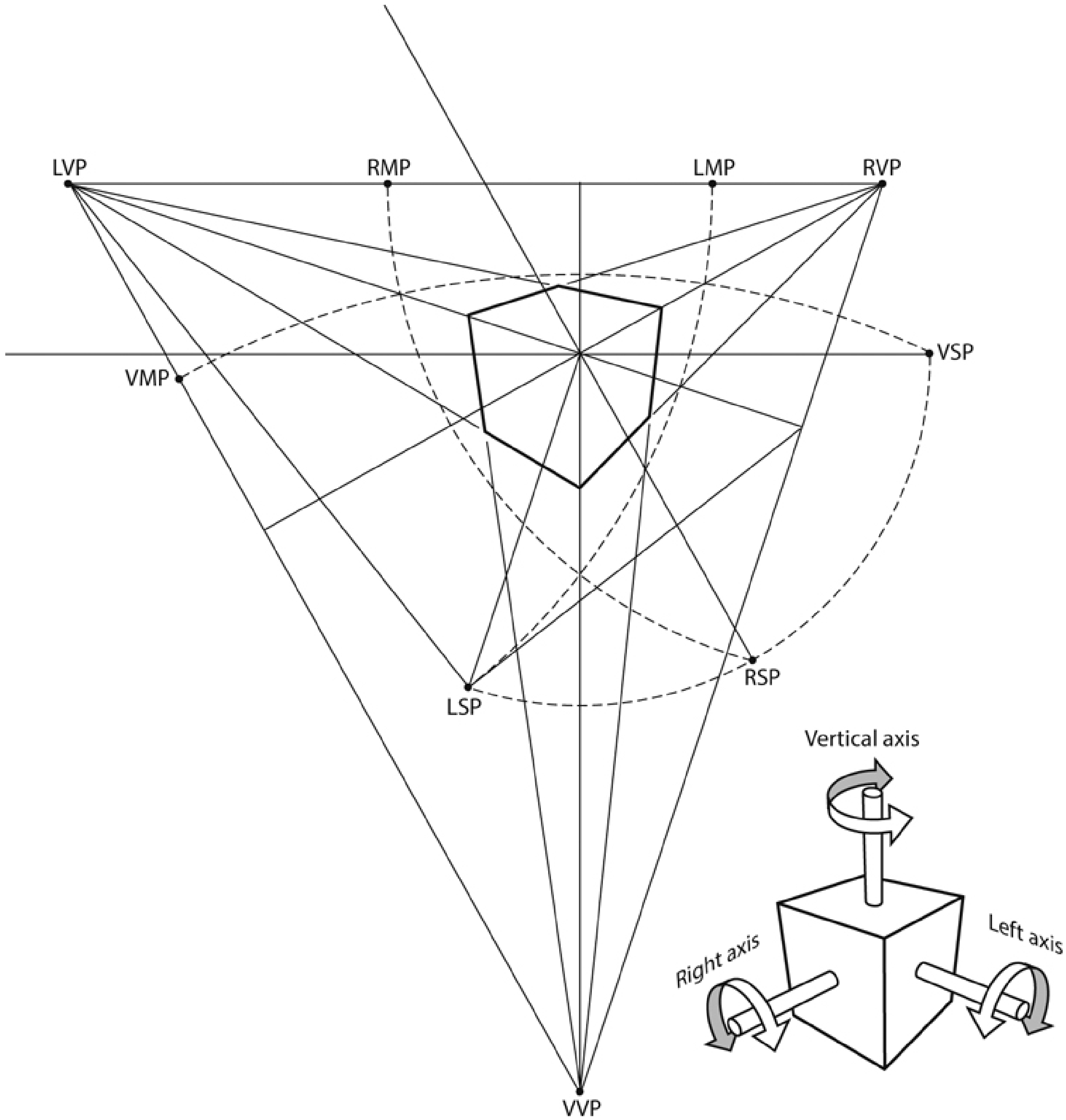
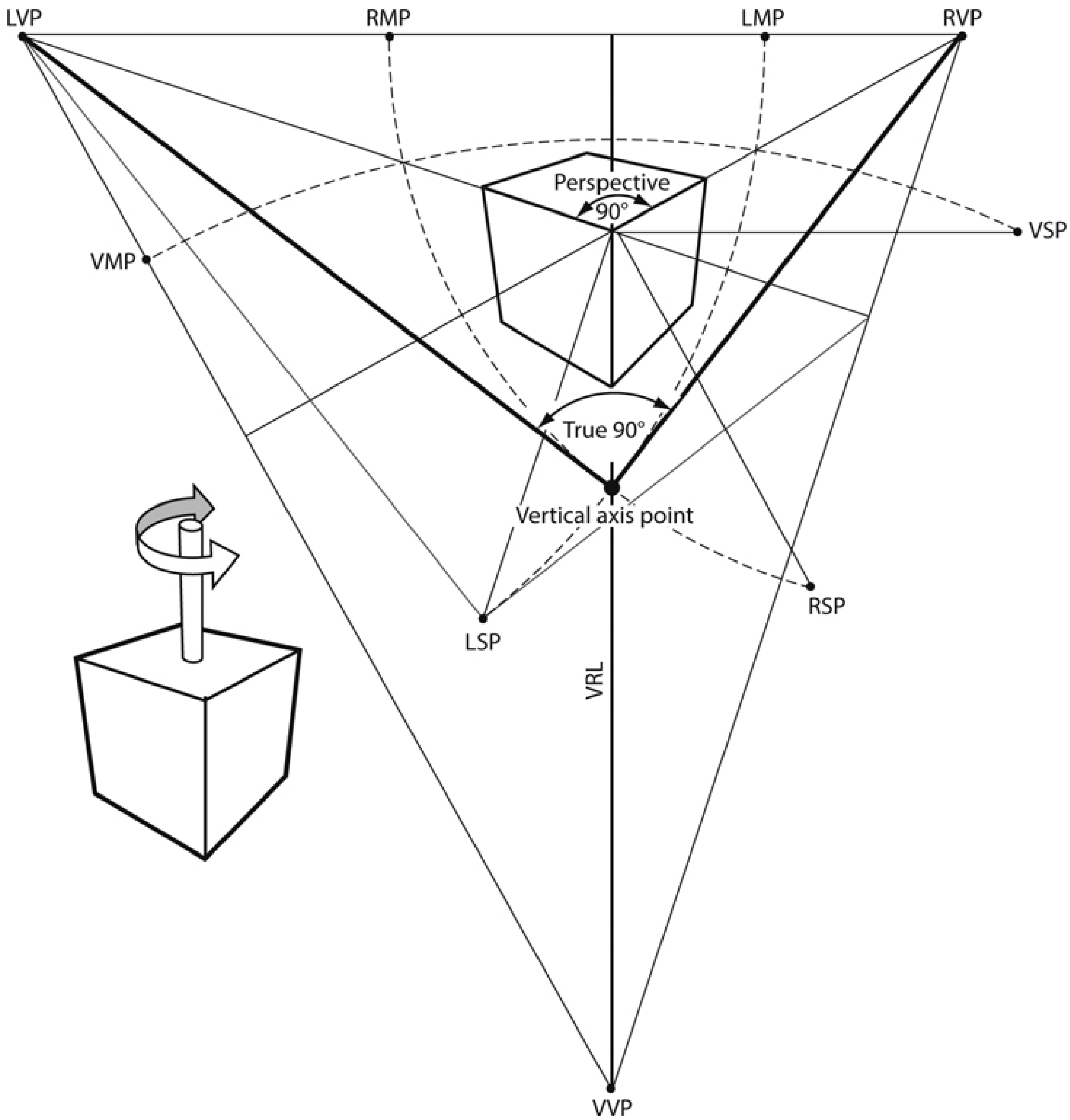


Figure 18.1 An object can rotate along three axis points.

An object can turn in three directions: along a vertical axis, a right axis, or a left axis. This coordinates with the vertical, right, and left vanishing points. The Cartesian coordinate system of X-, Y-, and Z-axis can be confusing when working with three vanishing points, so the terms left, right, and vertical will be used instead (Figure 18.1). These angles will be tackled one at a time.

Vertical Axis

As an object turns on a vertical axis, the left and right vanishing points change position. In one- and two-point perspective, the station point is used to find new vanishing points. Three-point perspective is different: the station point cannot be used. A different point is needed, a point of true angles, 90° from the left and right vanishing points. From this point, new left and right vanishing points can be found. This point is referred to as a vertical axis point (VAP). It is located on the vertical reference line, 90° from the right and left vanishing points. Use a triangle to draw a 90° corner along the vertical reference line. Make sure the legs of the triangle connect to the left and right vanishing points. To establish new left and right vanishing points, use this point as a station point when drawing in two-point perspective (Figure 18.2).



[Figure 18.2] **True angles for all horizontal lines are found at the vertical axis point.**

Vanishing Points

Any angle drawn from the vertical axis point creates a vanishing point that draws that same angle in perspective. For example, to rotate a box 20° clockwise, draw a true 20° angle at the vertical axis point, and project that angle to the horizon line ([Figure 18.3]).

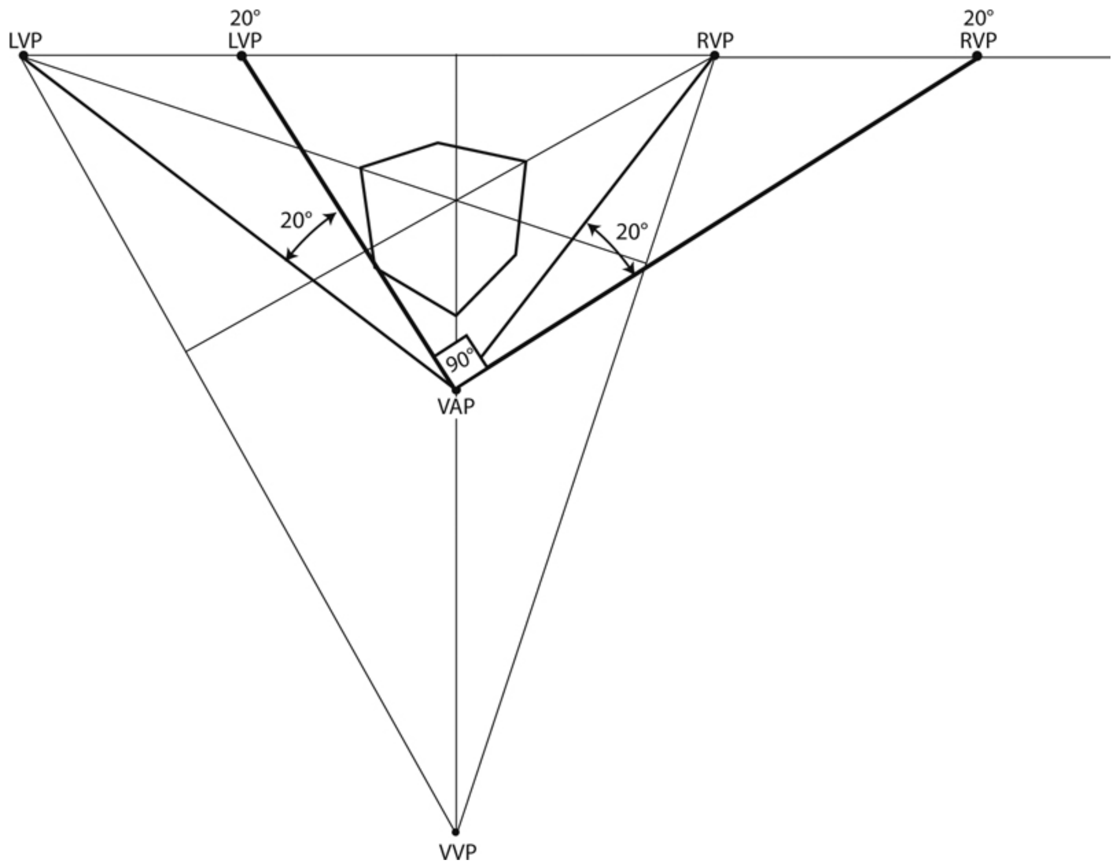


Figure 18.3 Angles projected from the vertical axis point create vanishing points that draw those same angles in perspective. These two new vanishing points are rotated 20° clockwise.

Measuring Points

New measuring points are needed for the new vanishing points. Measure the distance from the new vanishing point to the vertical axis point, and then project that distance to the horizon line (Figure 18.4). Measure width using the same procedures as in two-point perspective (Figure 18.5).

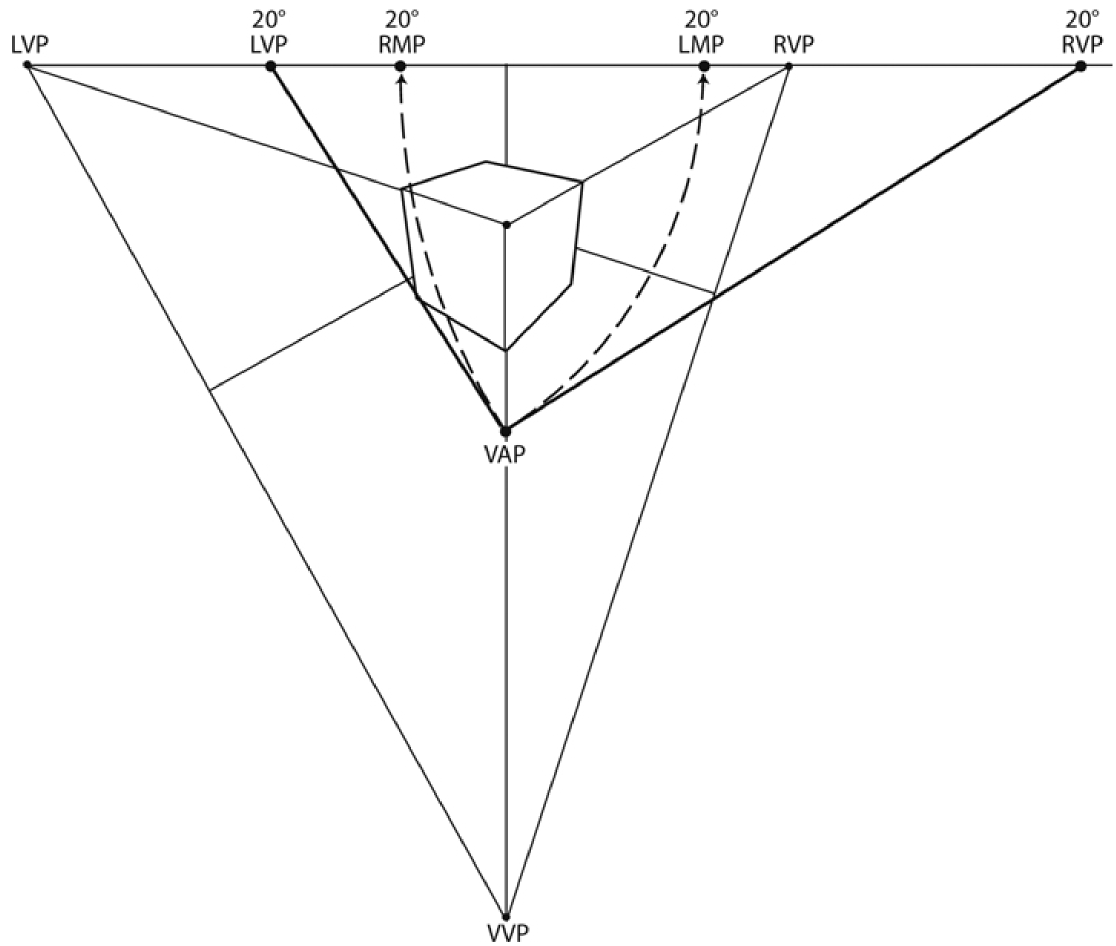


Figure 18.4 To locate measuring points, measure the distance from the vanishing points to the vertical axis point. Transfer that distance to the horizon line.

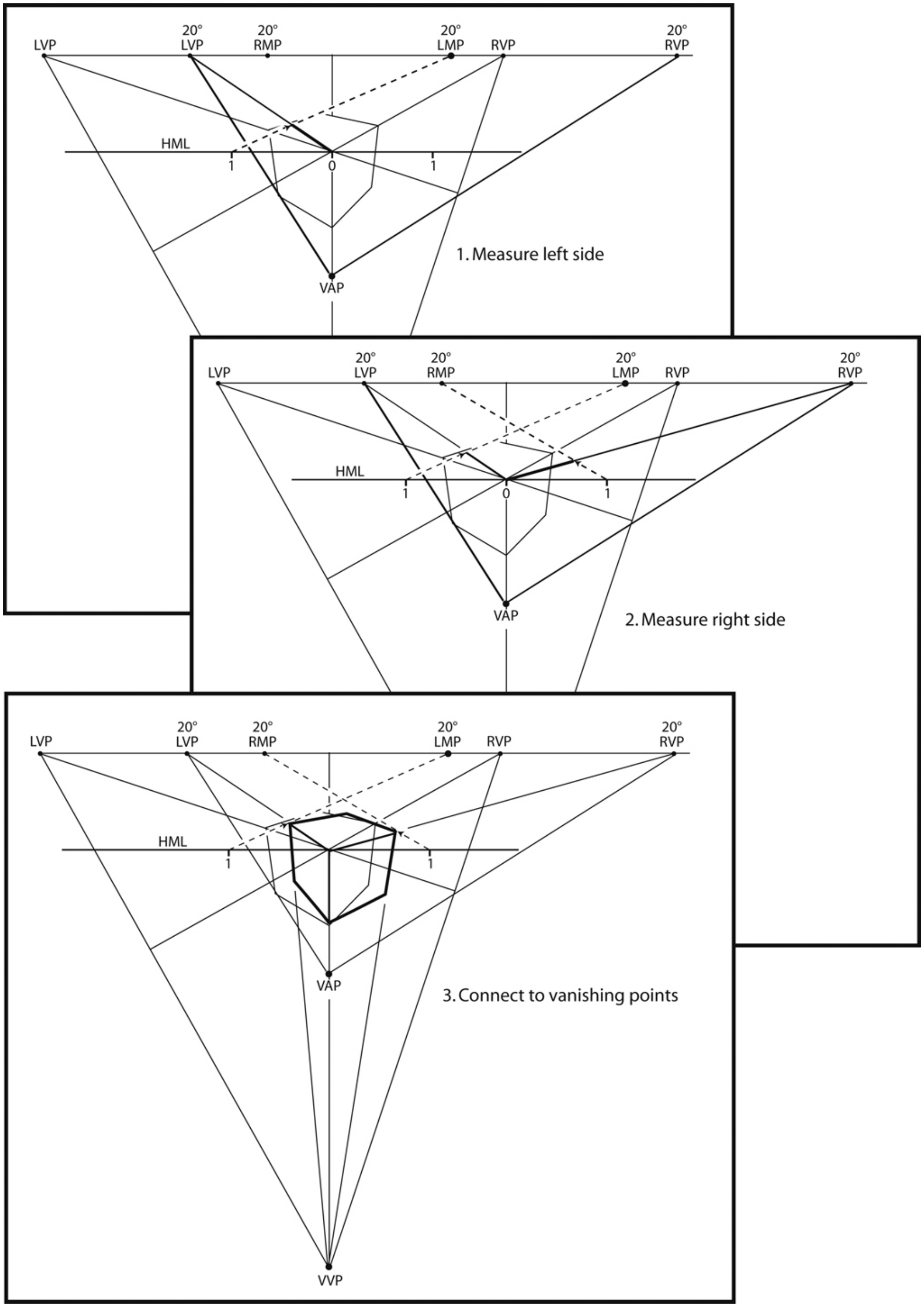


Figure 18.5 **Measuring horizontal lines is accomplished using the same technique as two-point perspective.**

One-Point Objects

If an object continues to turn along a vertical axis, eventually one side will be parallel with the picture plane (Figure 18.6).

This is a one-point object viewed in three-point perspective. Lines parallel with the picture plane have no vanishing point. Vertical lines still connect to the vertical vanishing point. Lines representing depth connect to the vertical reference point (Figure 18.7).

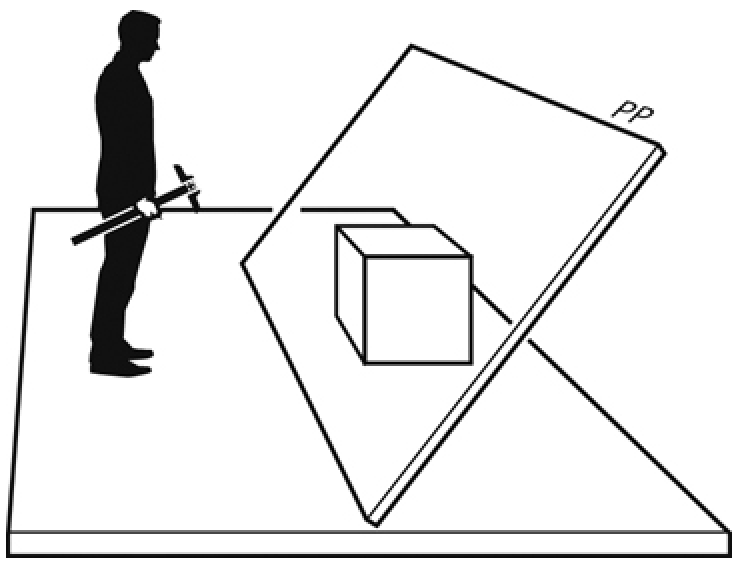


Figure 18.6 Looking down at a one-point cube.

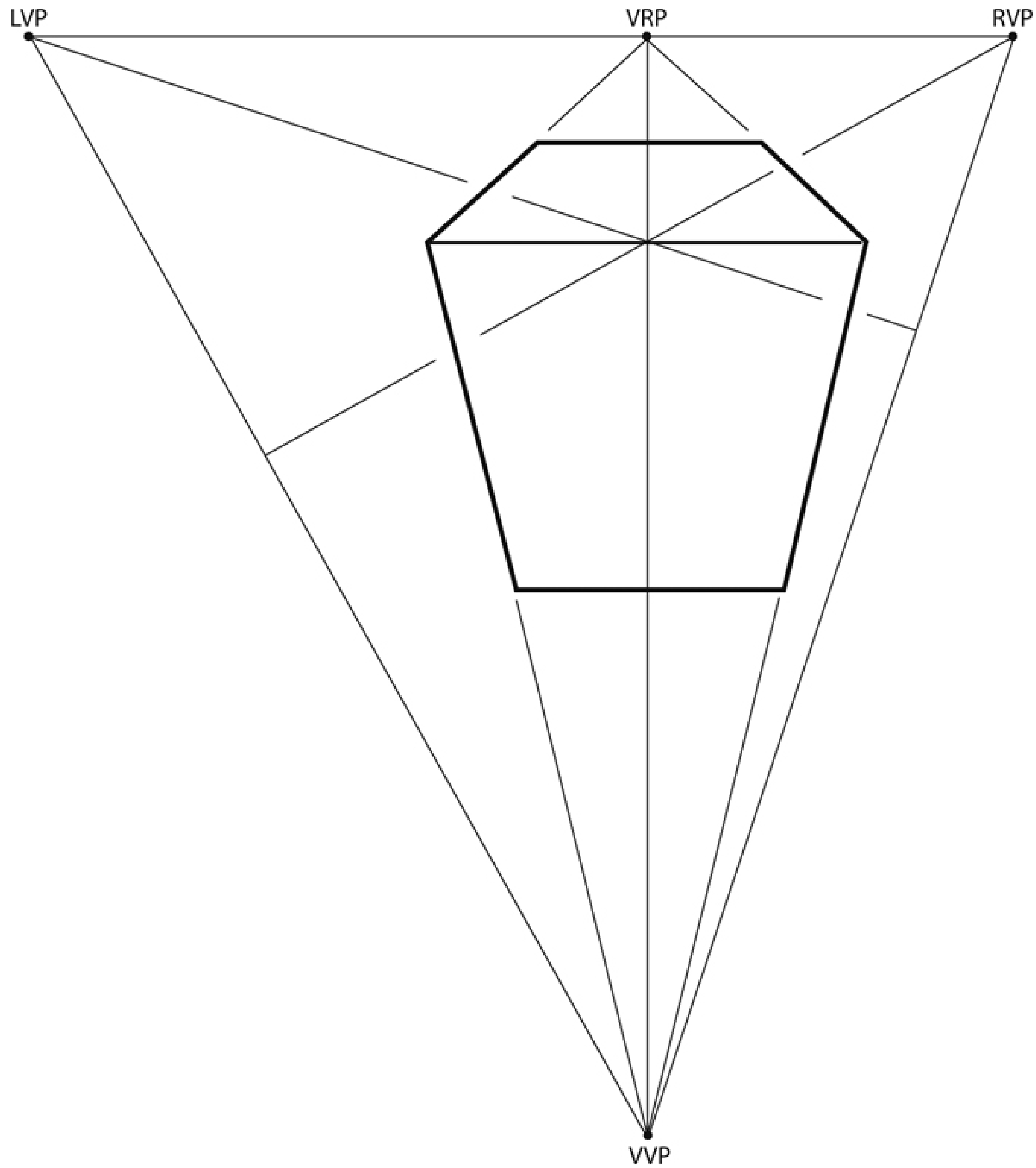


Figure 18.7 When drawing a one-point perspective box in three-point perspective, foreshortened lines connect to the vertical vanishing point and the vertical reference point. Horizontal lines are parallel with the picture plane and have no vanishing point.

Measuring Width

Width is not foreshortened. Measure these dimensions along the horizontal measuring line as if drawing in one-point perspective (Figure 18.8).

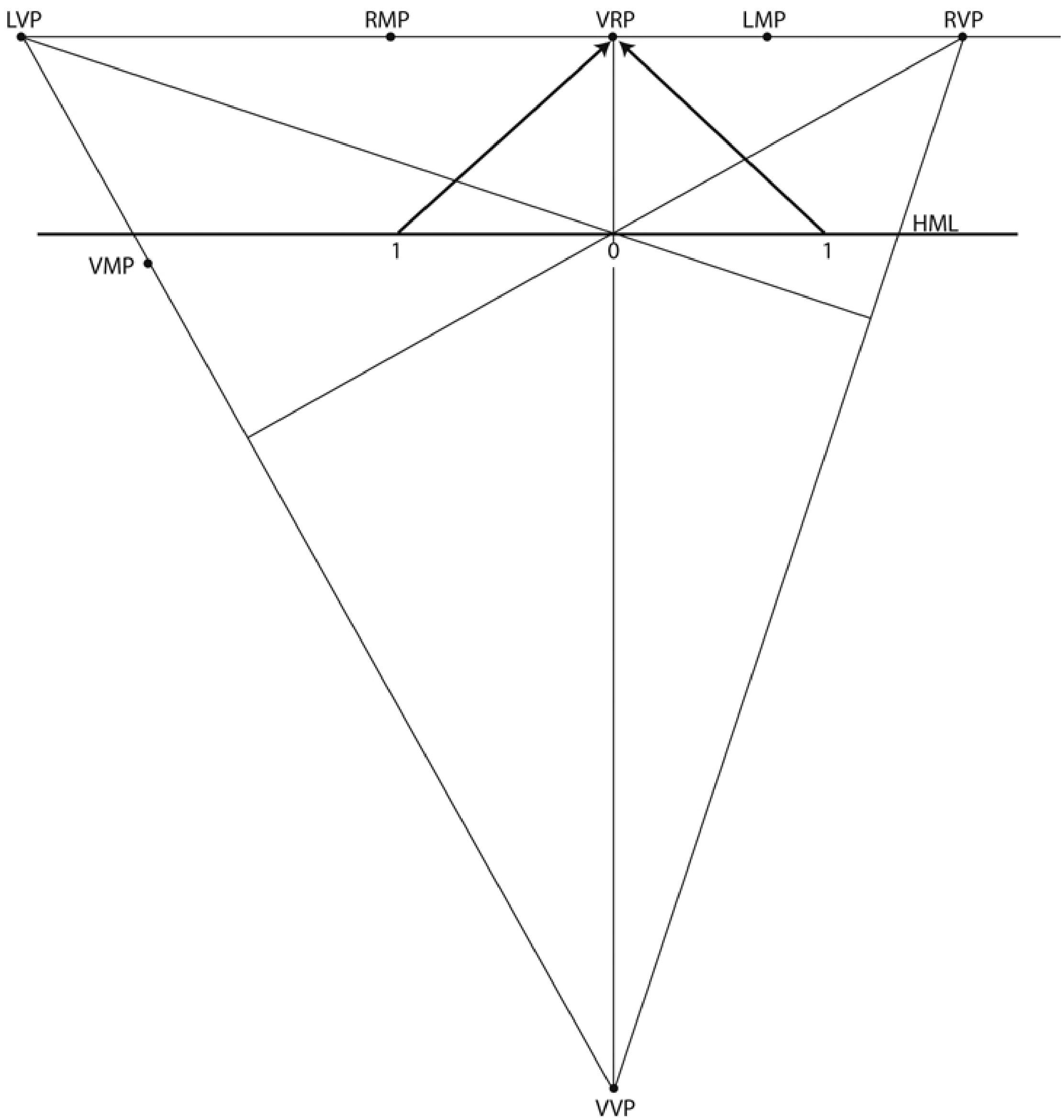


Figure 18.8 Measure width along the horizontal measuring line and connect to the vertical reference line. This box is 2 units wide—1 unit to the left, and 1 unit to the right—with its center at the center of vision.

Measuring Height

Measure vertical dimensions along the vertical reference line. The vertical reference line is on the same plane as the measuring line (Figure 18.9).

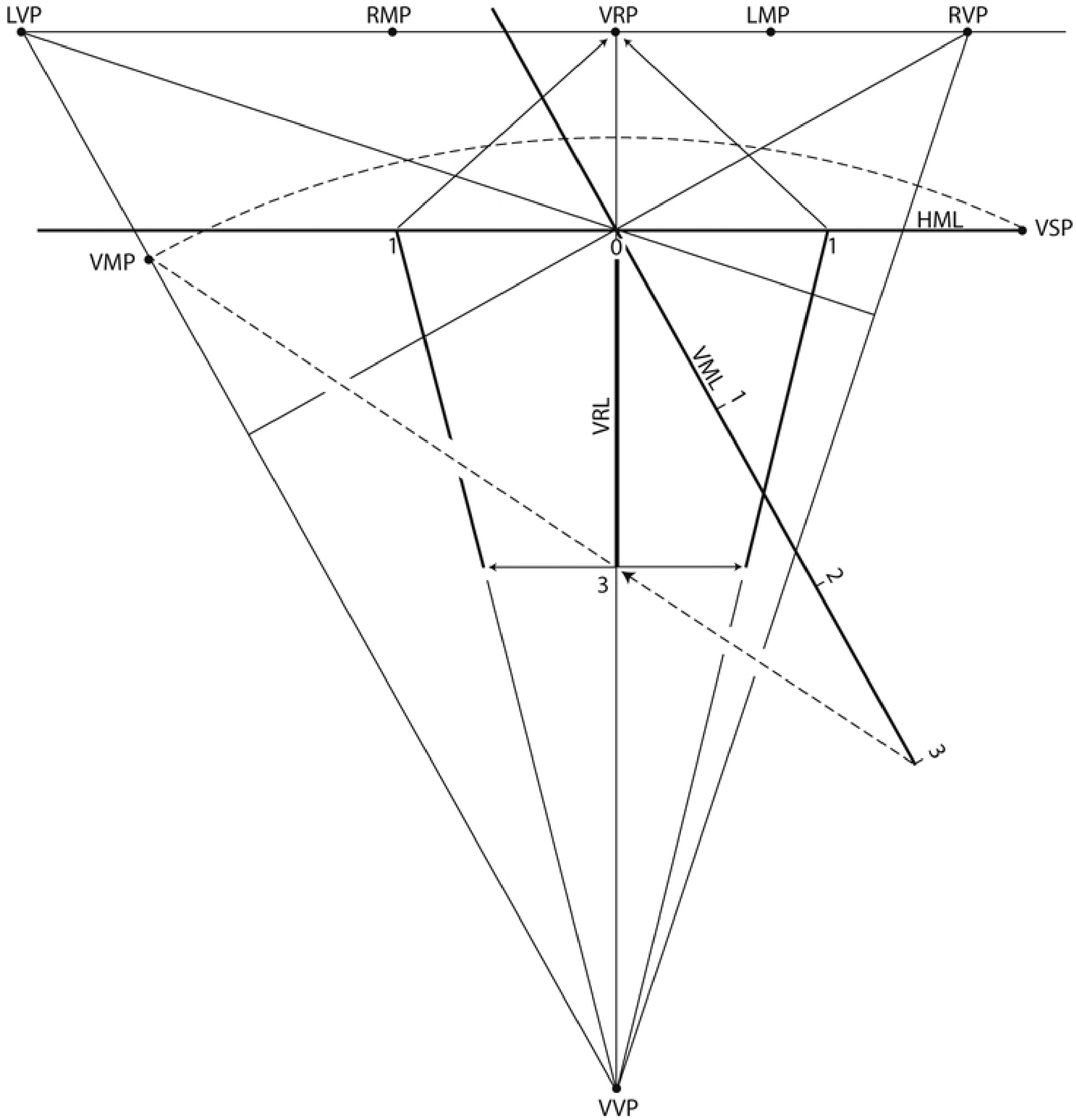


Figure 18.9 **Measure the height along the vertical reference line. The bottom of the box is 3 units below the horizontal measuring line.**

Measuring Depth

The vertical reference point serves as the vanishing point. To find the measuring point, use a compass. Measure the distance from the vertical reference point to the vertical axis point and transfer that distance to the horizon line. The measuring point can be placed on the right or left side.

With the measuring point in place, measure using the technique in one-point perspective (Figure 18.10).

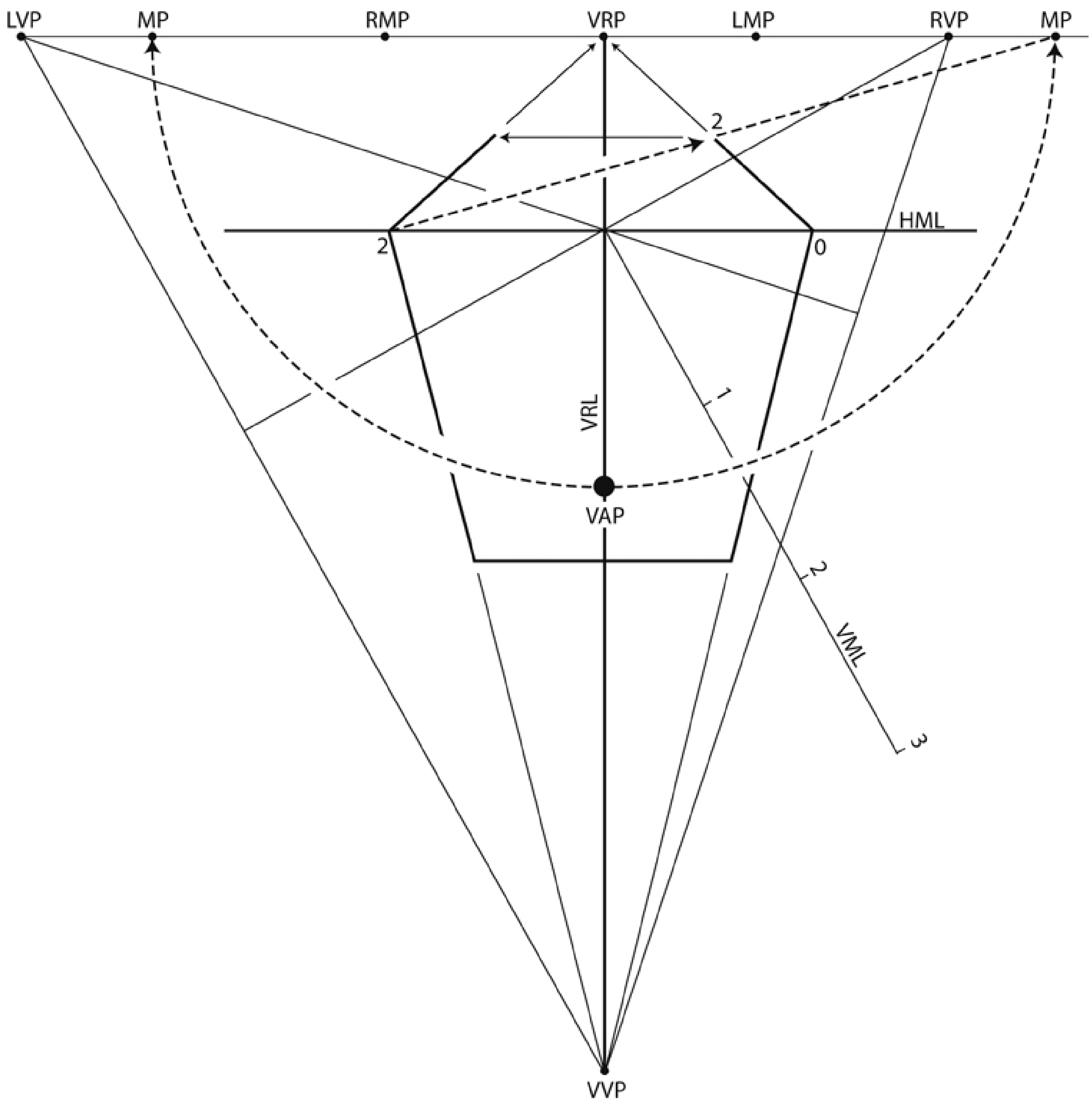


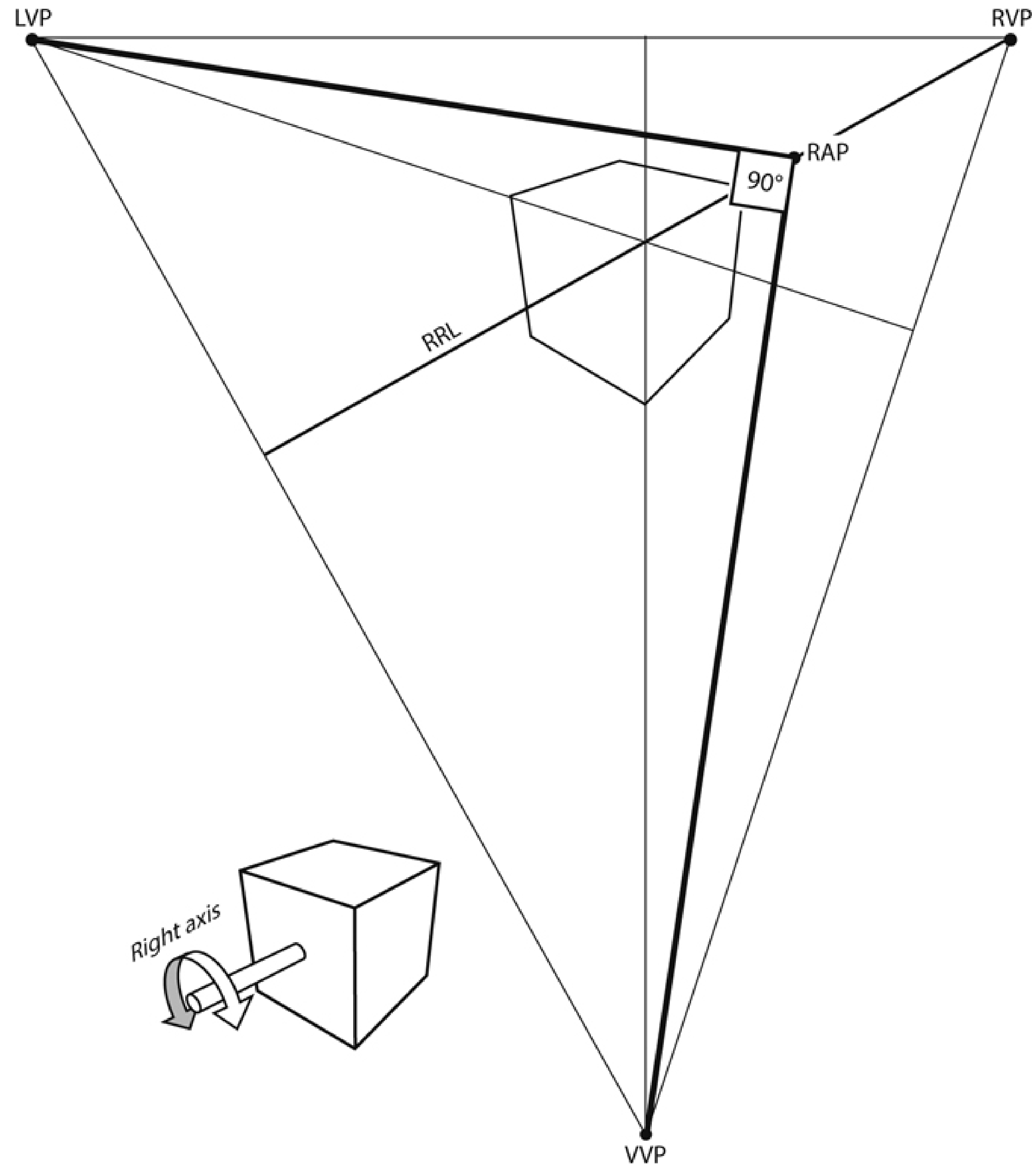
Figure 18.10 **Find measuring points using the vertical axis point and measure as per one-point perspective.**

Right Axis

In this next example, the box is rotating along an axis that aligns with the right vanishing point. This box tilts backward to the left or forward to the right. All lines parallel with the axis of rotation still connect to the right vanishing point. As the object rotates, the left and vertical vanishing points change position, and the right vanishing point remains in its place. To rotate an object along a right axis, new left and vertical vanishing points are needed.

Right Axis Point

To move the location of the left and vertical vanishing point, a right axis point (RAP) is needed. The right axis point is the point of true angles between the left and vertical vanishing points. Use a triangle and align its legs to the left and vertical vanishing points. Place the 90° corner of the triangle on the right reference line. This marks the location of the right axis point (Figure 18.11).



[Figure 18.11](#) The right axis point must be placed on the right reference line, 90° from the left and vertical vanishing points.

Vanishing Points

A 90° angle placed at the right axis point creates two vanishing points that draw 90° angles in perspective. The new vanishing points are placed along the line that connects the left and the vertical vanishing points. To accommodate the new vanishing points, this line may be extended to whatever length necessary (Figure 18.12).

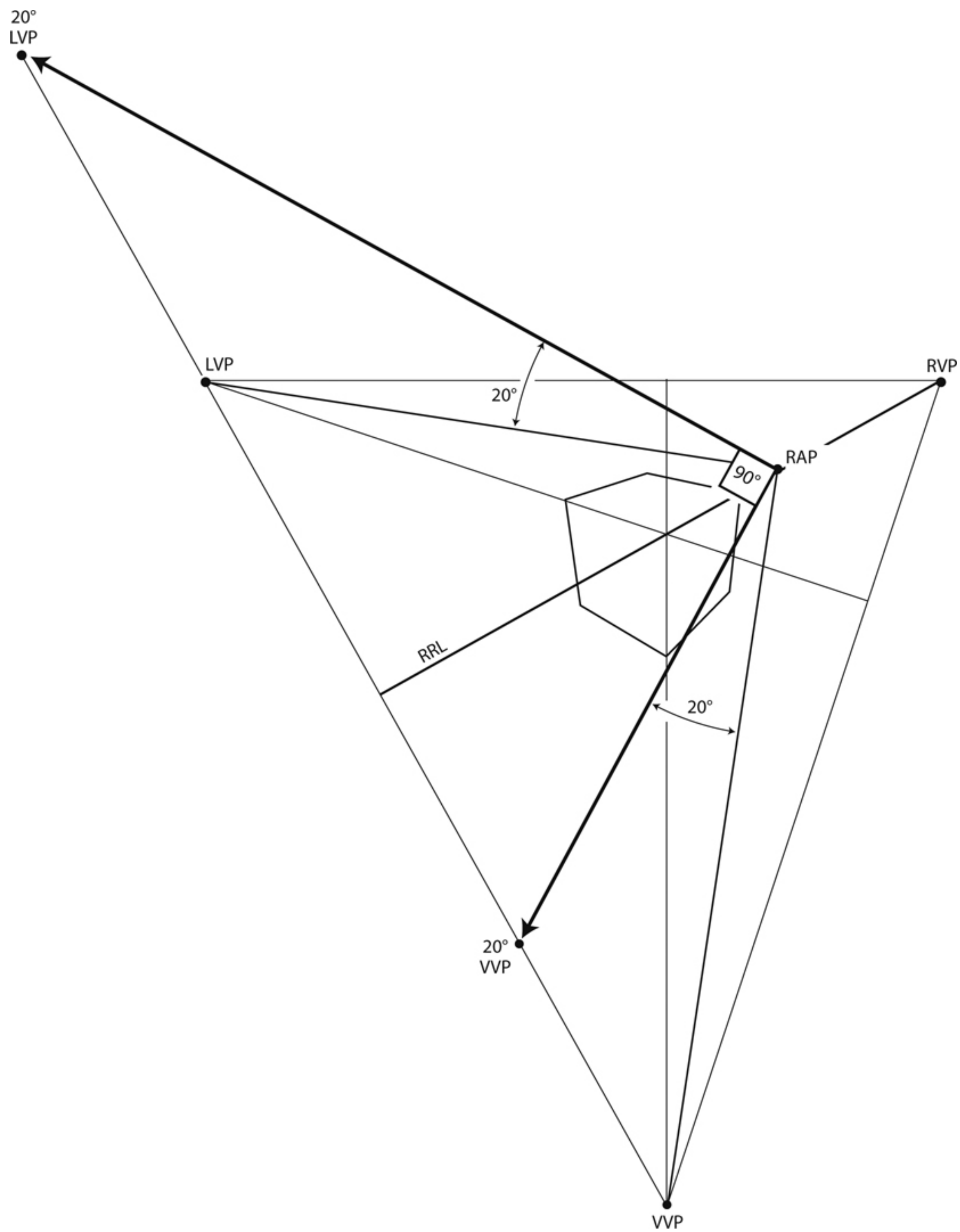


Figure 18.12 Using the right axis point, create two new vanishing points by rotating the 90° angle. These new vanishing points are rotated 20° clockwise from the original.

Measuring Points

The measuring points are placed on the same line as the vanishing points. Measure the distance from the vanishing point to the right axis point. The distance from the vanishing point to the measuring point will be the same length (Figure 18.13). Use a compass or ruler to transfer the distance.

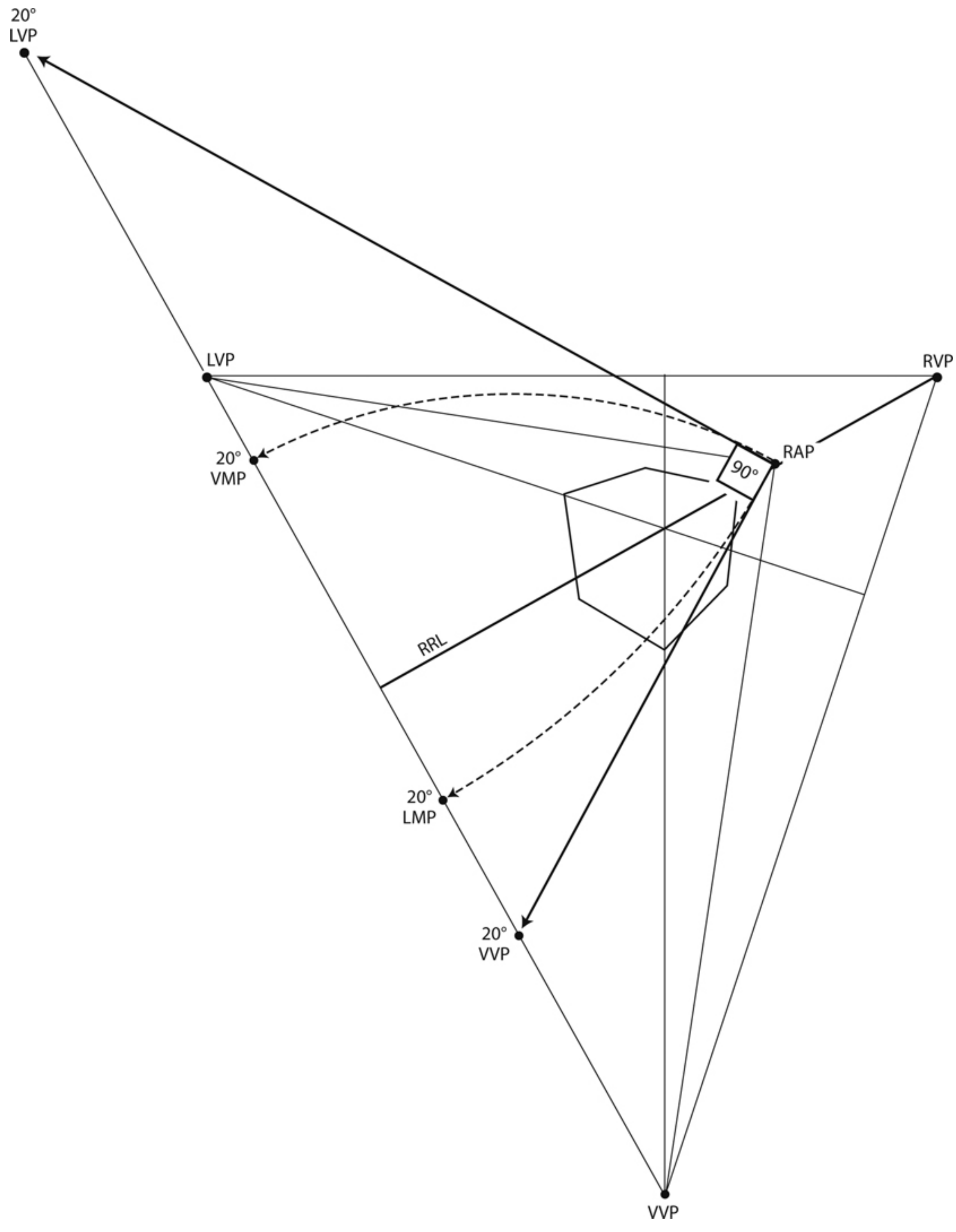


Figure 18.13 To establish the measuring points, measure the distance from the vanishing point to the right axis point. Project that distance to the line connecting the vanishing points.

Measuring Line

The measuring line must be parallel with the line the measuring points are on. Since the measuring points are on an angled line, the measuring line must be at the same angle (Figure 18.14).

The measuring procedure is the same—except it is sideways. Confirm that the object being measured is on the same plane as the measuring line. Determining this can sometimes be challenging; it takes some practice.

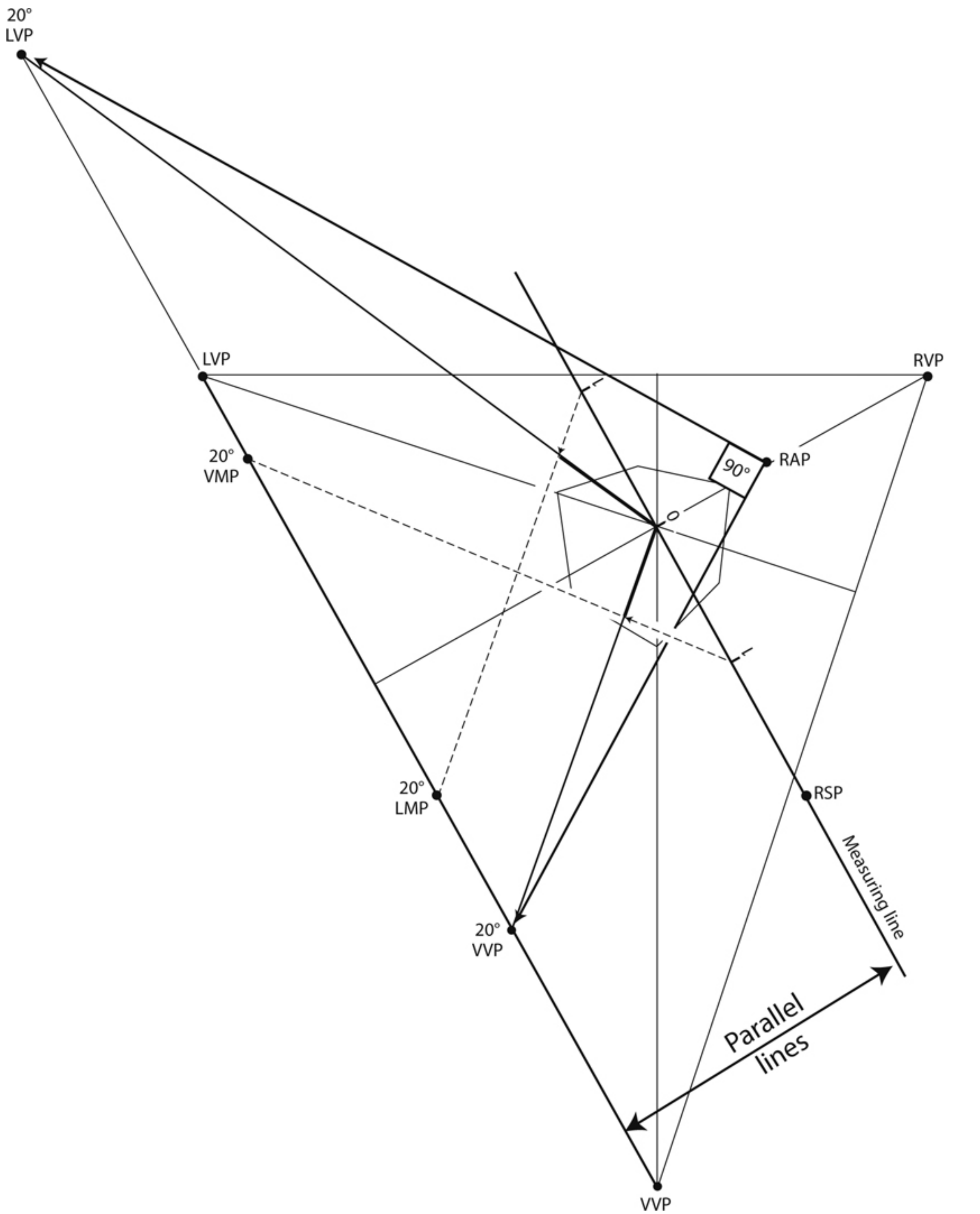


Figure 18.14 **The measuring line must be parallel with the line the measuring points are on (the line parallel with the right station point becomes the measuring line).**

Complete the Box

Connect lines to the new vanishing points. Lines parallel with the ground plane (parallel with the axis of rotation) connect to the right vanishing point (Figure 18.15).

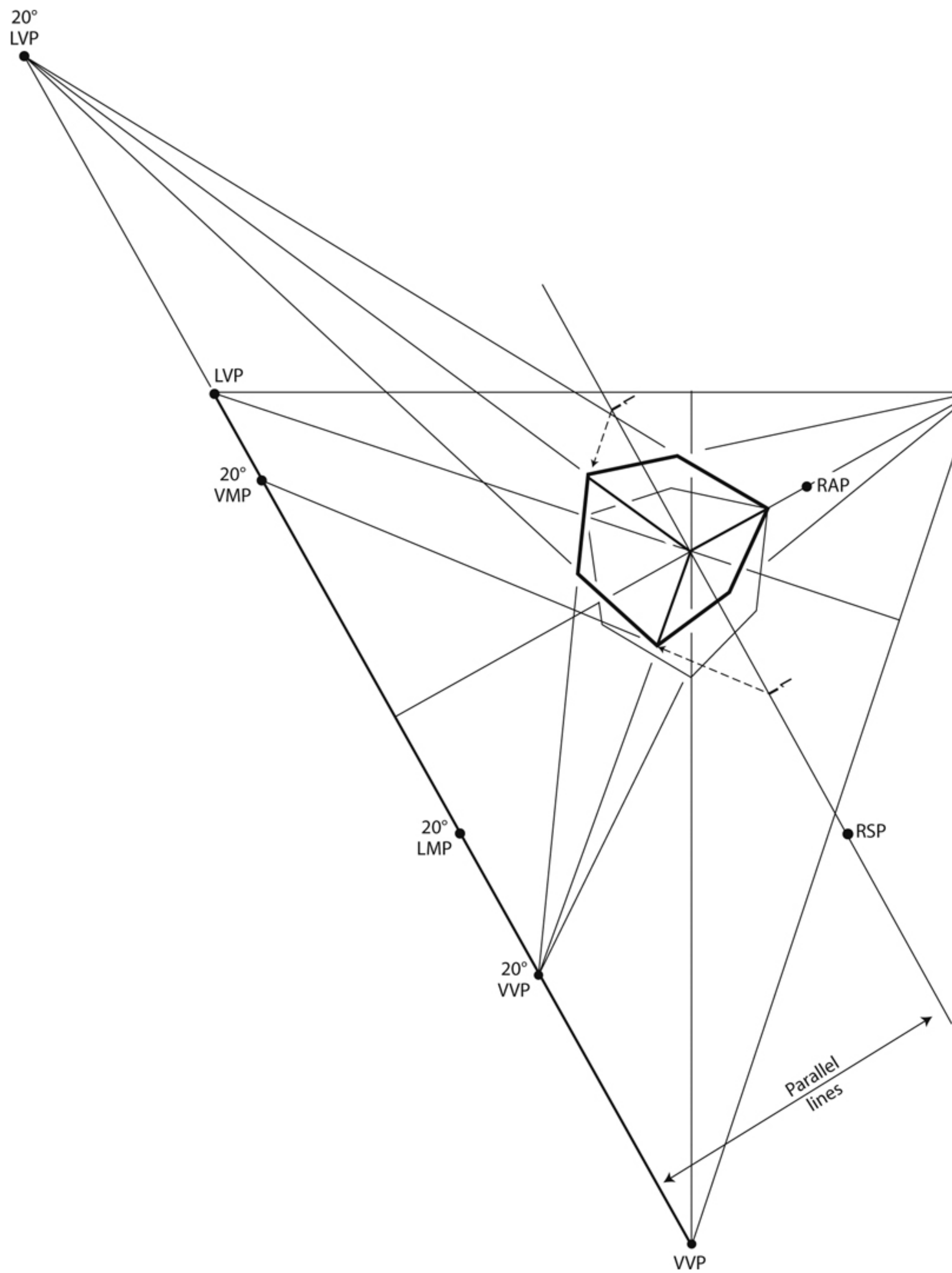
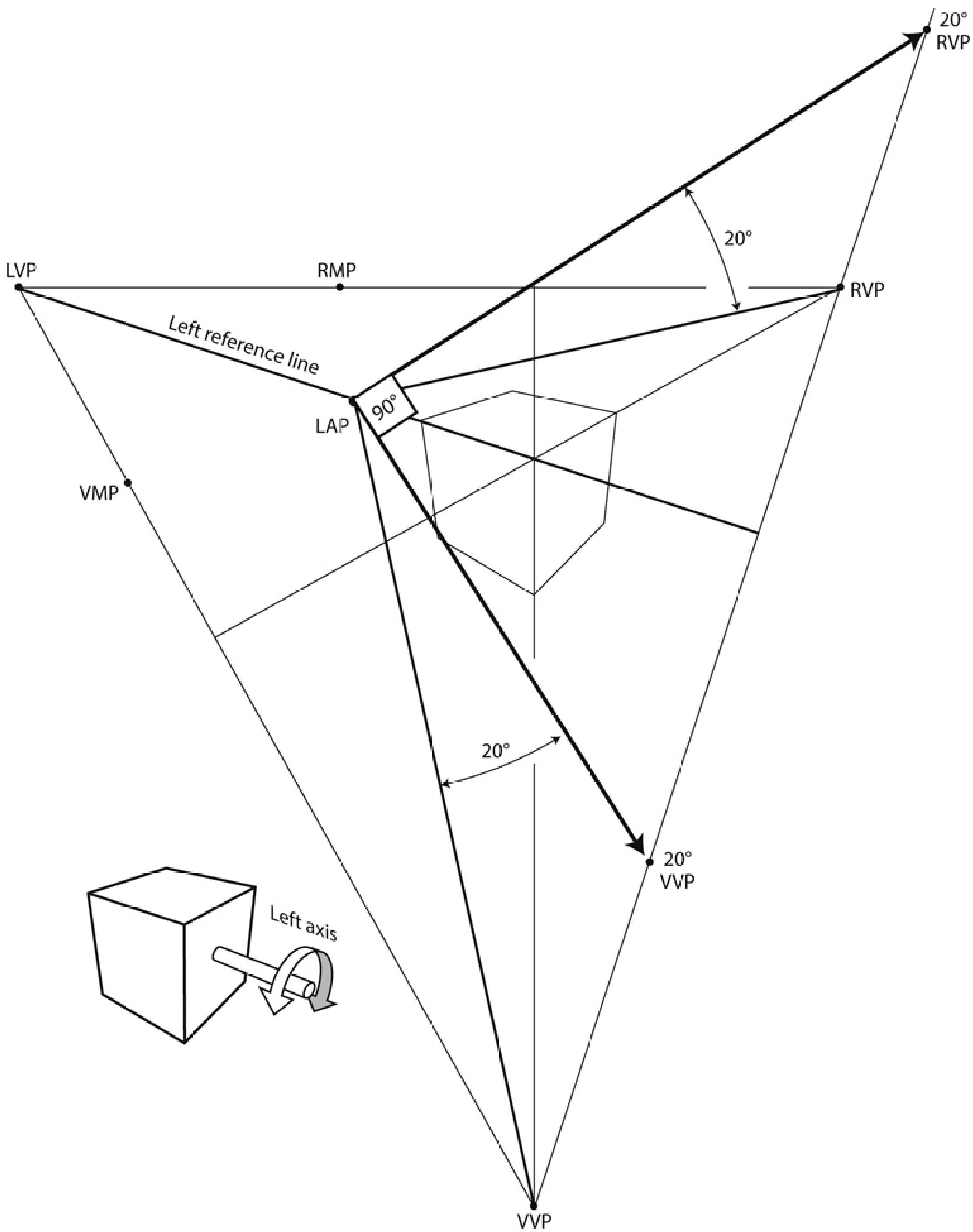


Figure 18.15 **Connect lines to vanishing points to complete the box.**

Left Axis

A left axis rotation follows the same procedure as a right axis rotation. There is a new axis point: the left axis point (LAP). It is located on the left reference line, 90° from the right and vertical vanishing points (Figure 18.16). There are also new measuring points (Figure 18.17) and a new measuring line, parallel with the line the measuring points are on (Figure 18.18). Connect lines to vanishing points to complete the box (Figure 18.19).



[Figure 18.16](#) **Use the left axis point to find the new vanishing points.**

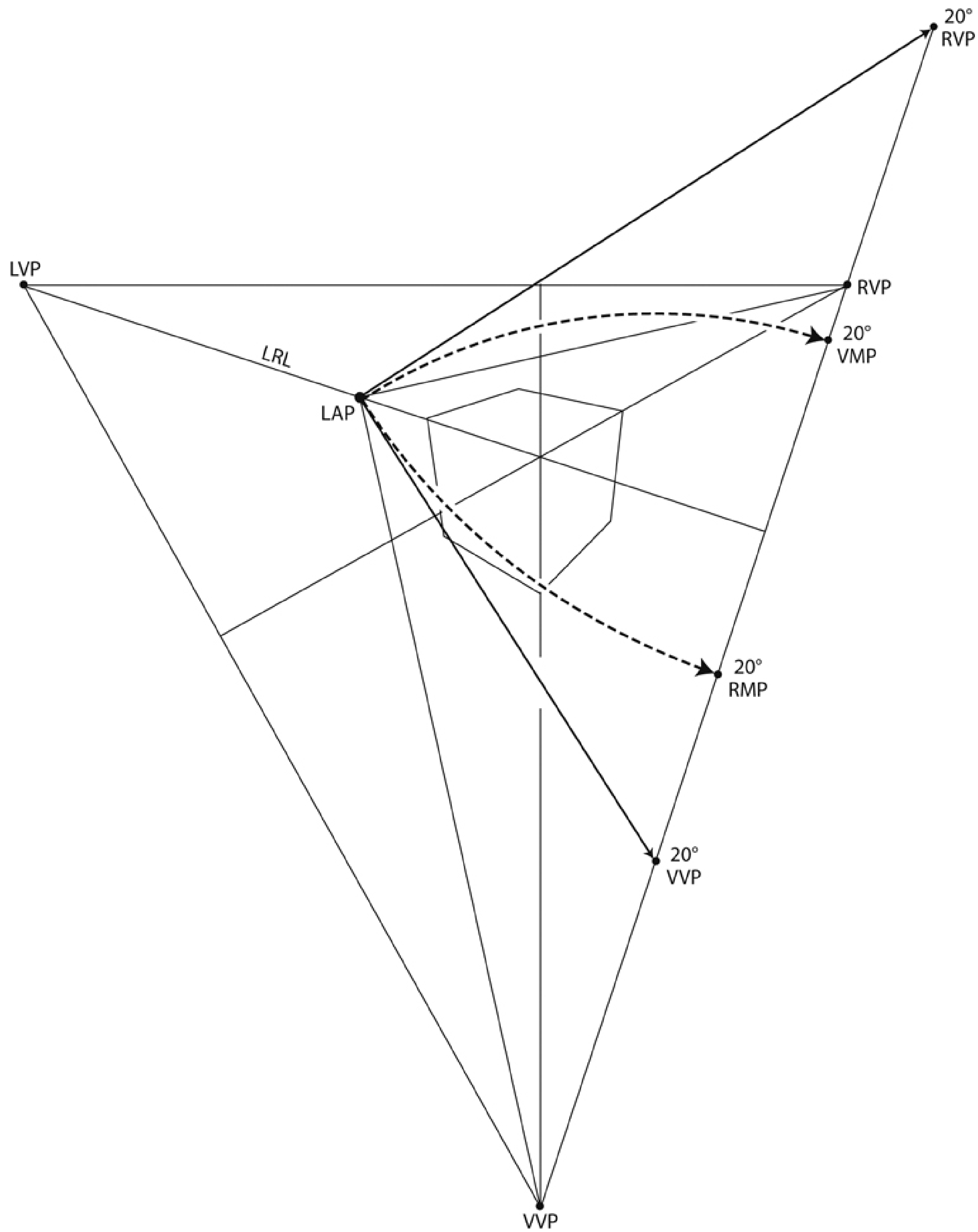


Figure 18.17 To establish the measuring points, measure the distance from the vanishing point to the left axis point. Project that distance to the line the vanishing points are on.

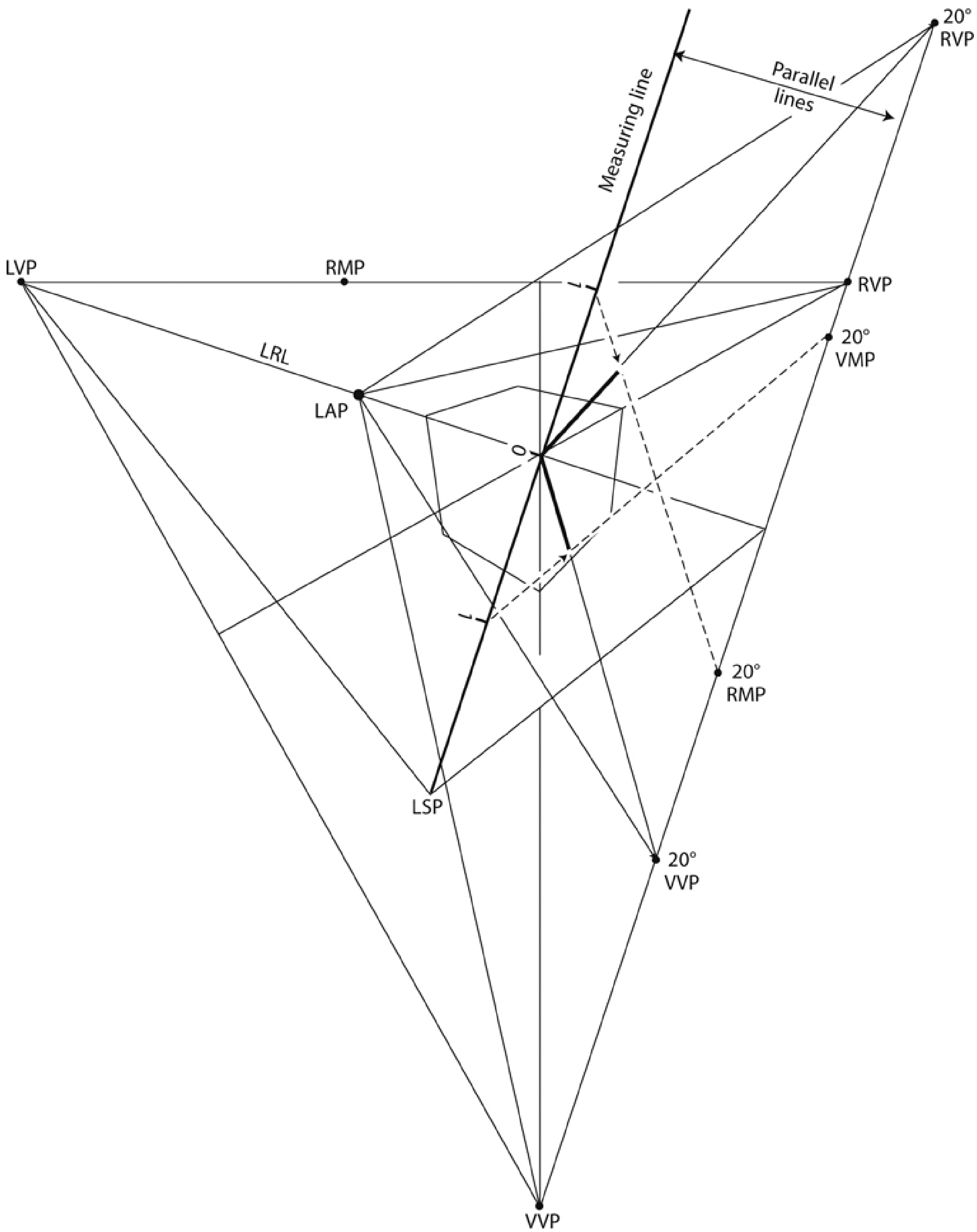


Figure 18.18 The measuring line must be parallel with the line the measuring points are on, and the line being measured must be on the same plane as the measuring line.

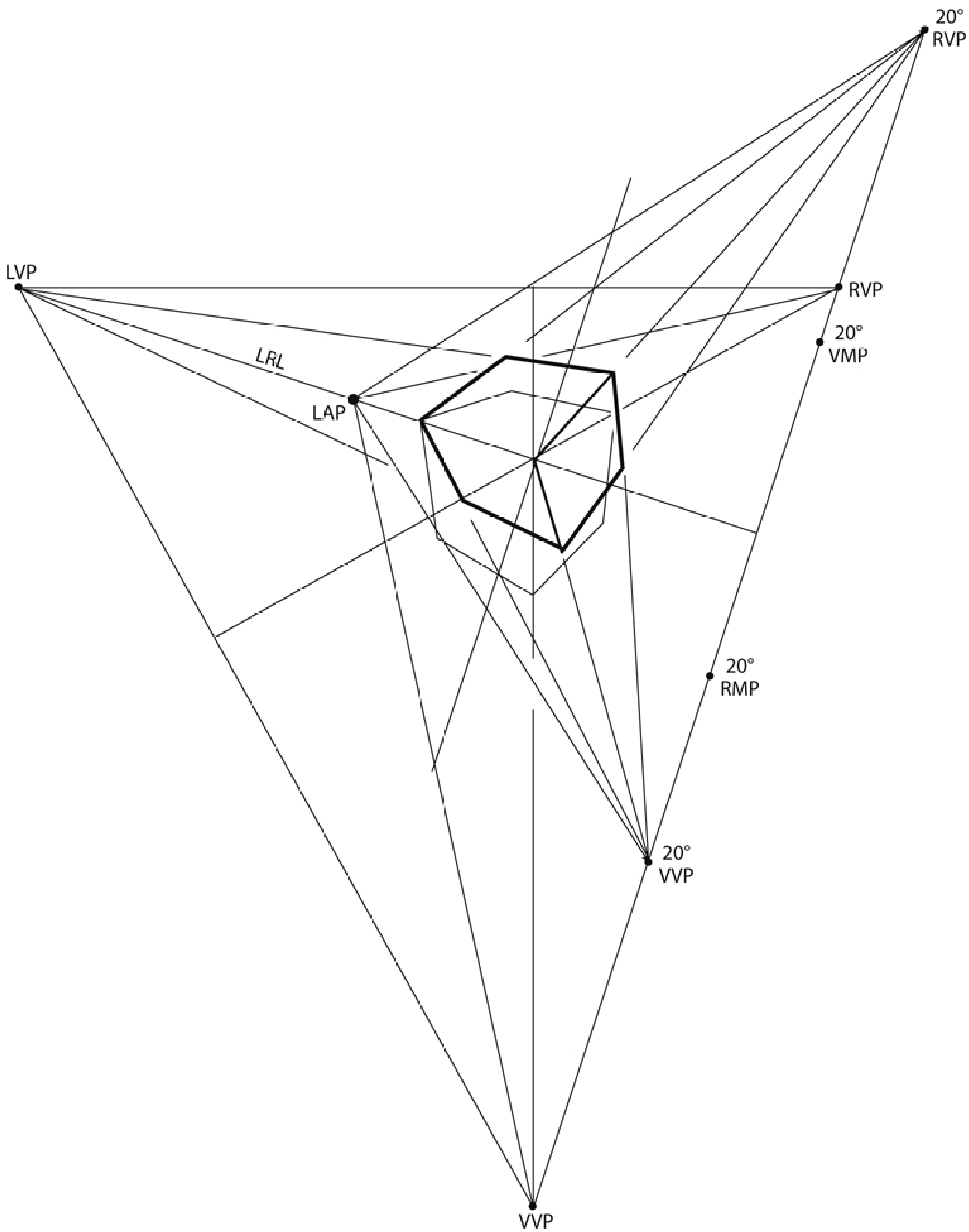


Figure 18.19 **Connect lines to vanishing points to complete the box.**

Cartesian Coordinate System

The Cartesian coordinate system is a graph that allows a point in space to be accurately plotted. In three-dimensional Cartesian space this is accomplished using three axes labeled *x*, *y*, and *z*. There are variations to these designations, but traditionally *x* and *y* are placed on a horizontal plane, with *y* representing width and *x* representing depth. Height is represented by *z*. This is the right-handed Cartesian coordinate system, and it is the system used in this book.

One-Point Angles

A one-point perspective box, seen in a three-point view, can tilt forward or backward (rotate along the *y*-axis), or tilt to the left or right (rotate along the *x*-axis) (Figure 18.20). These angles are approached differently than the others encountered thus far so it is worthwhile to address each of them.

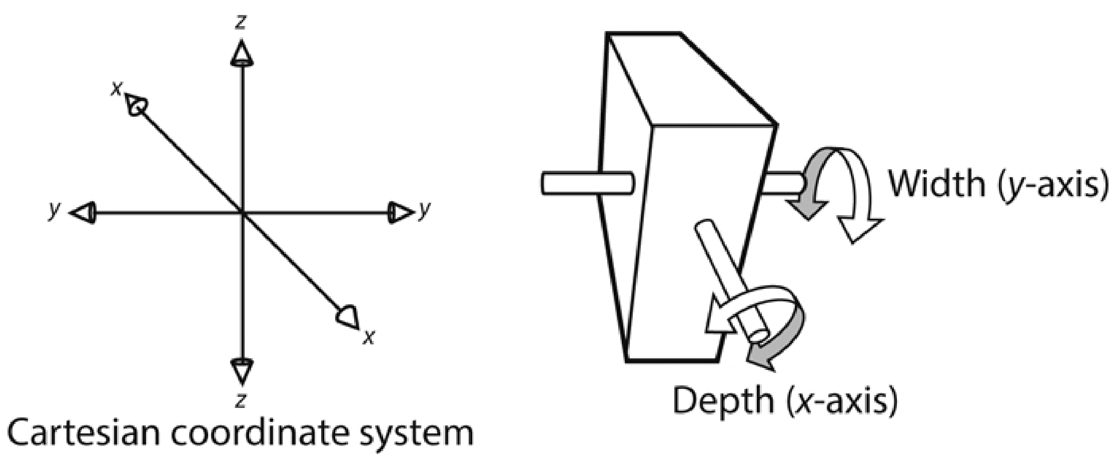
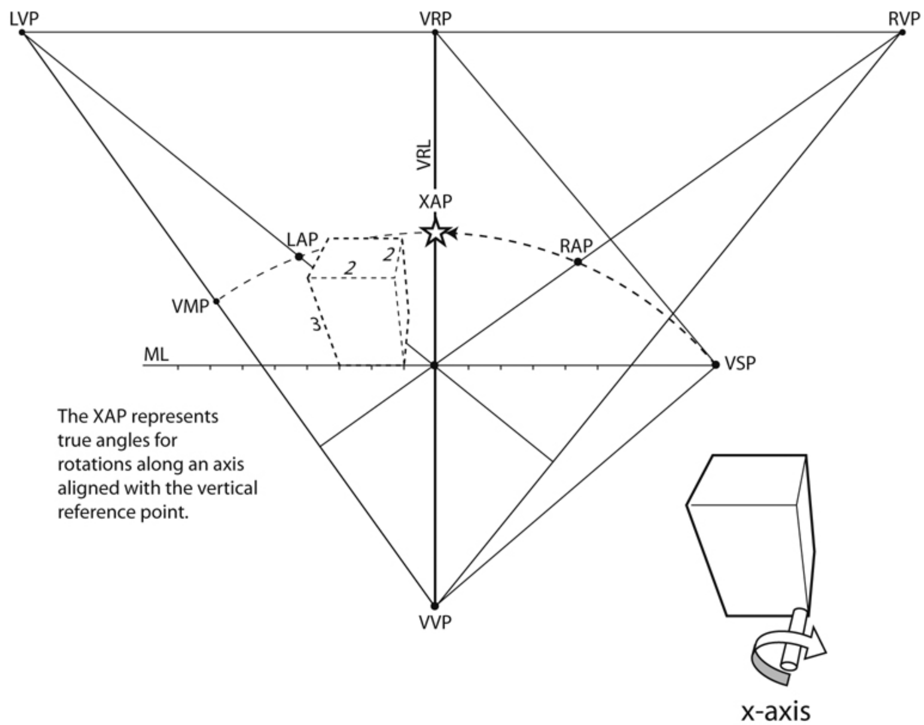


Figure 18.20 Using the Cartesian coordinate system to define one-point perspective angles.

x -Axis

The axis designated *x* rotates around a line connected to the vertical reference point. The object tilts to the left or right along this axis—this creates new vanishing points. To locate these new vanishing points, a point of true angles for *x* is needed.

The point of true angles for *x* is located along the vertical reference line. Measure the distance from the vertical vanishing point to the vertical station point. Transfer that distance to the vertical reference line (Figure 18.21). This is the point of true angles: the *x*-axis point (XAP).



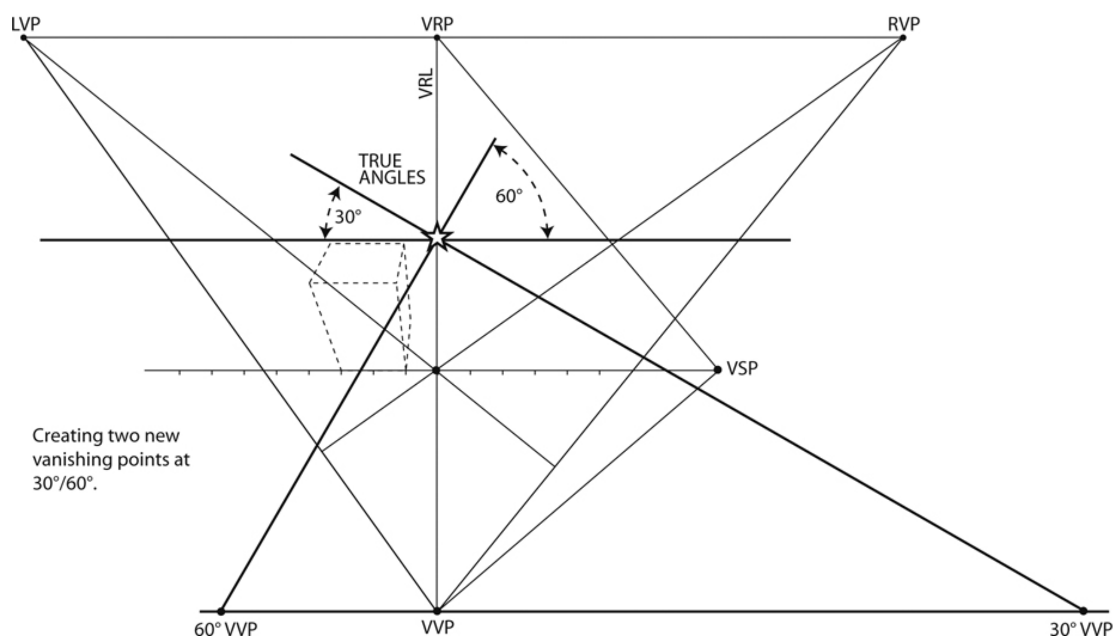


Figure 18.21 The 60° and 30° vanishing points are aligned horizontally with the vertical vanishing point.

Any 90° angle drawn from the *x*-axis point will create two vanishing points 90° apart. These vanishing points are aligned horizontally with the vertical vanishing point (Figure 18.22). Measuring points are also aligned horizontally with the vertical vanishing point and are found by measuring the distance from the vanishing point to the *x*-axis point (Figure 18.23). Connect lines to vanishing points to complete the box (Figure 18.24).

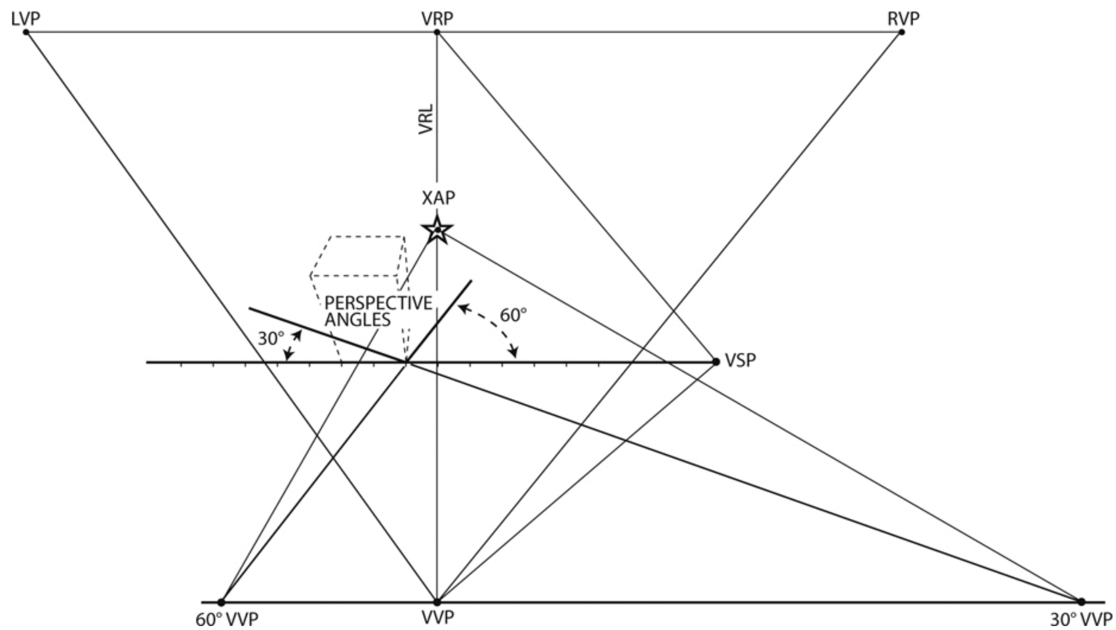


Figure 18.22 The perspective angles drawn from the two new vanishing points reflect the true angles plotted at the *x*-axis point.

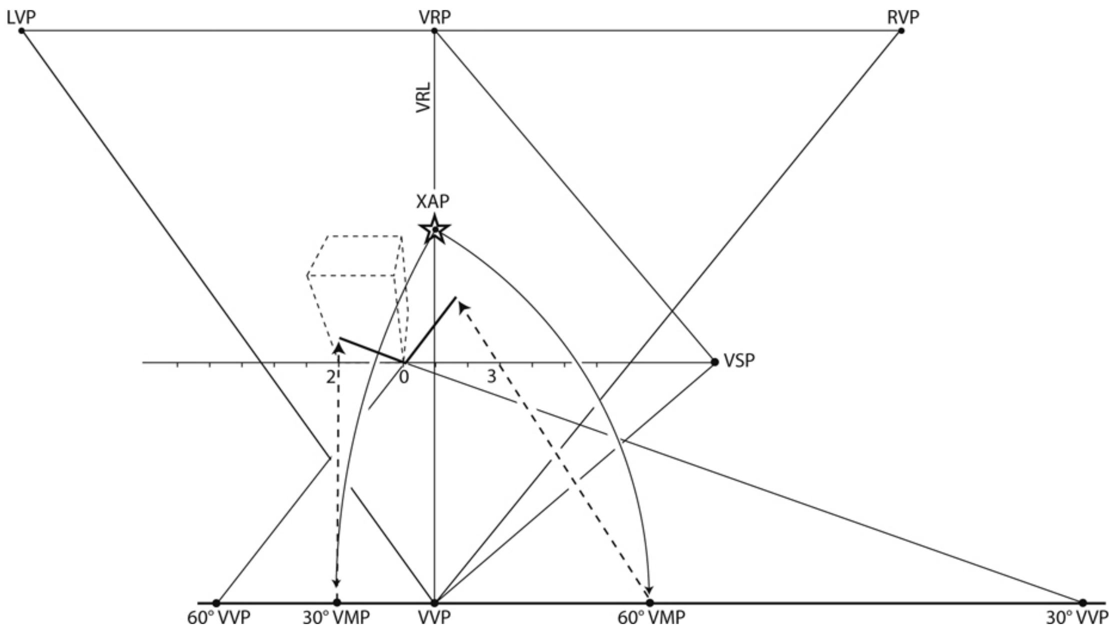


Figure 18.23 **Find measuring points by measuring the distance from the new vertical vanishing points to the *x*-axis point. Then transfer it to the horizontal line.**

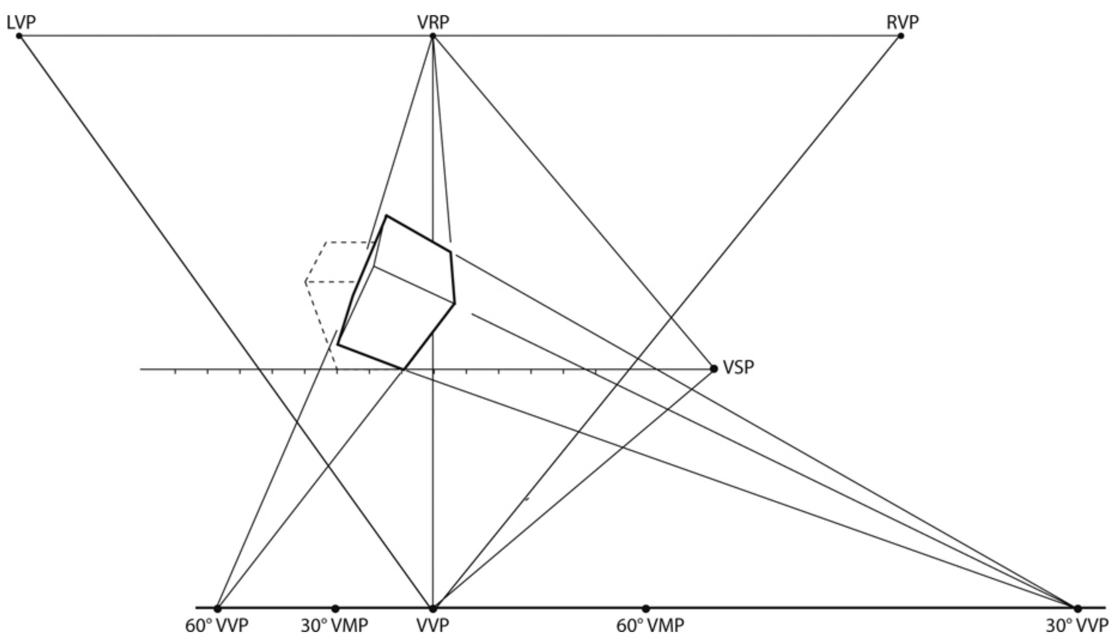
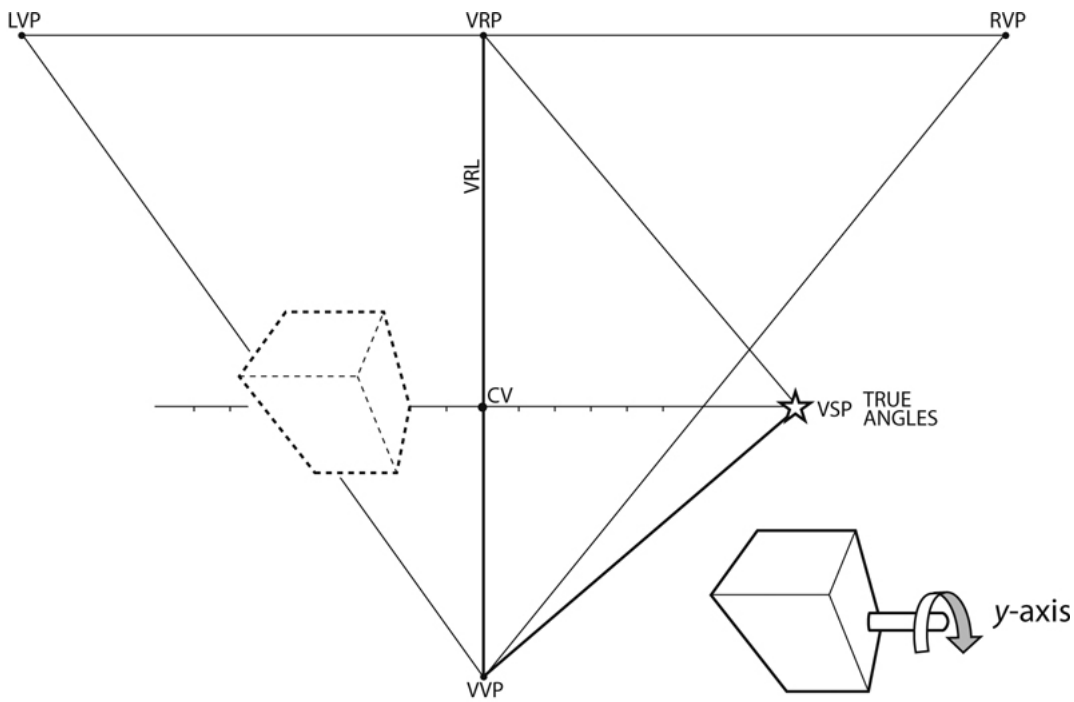


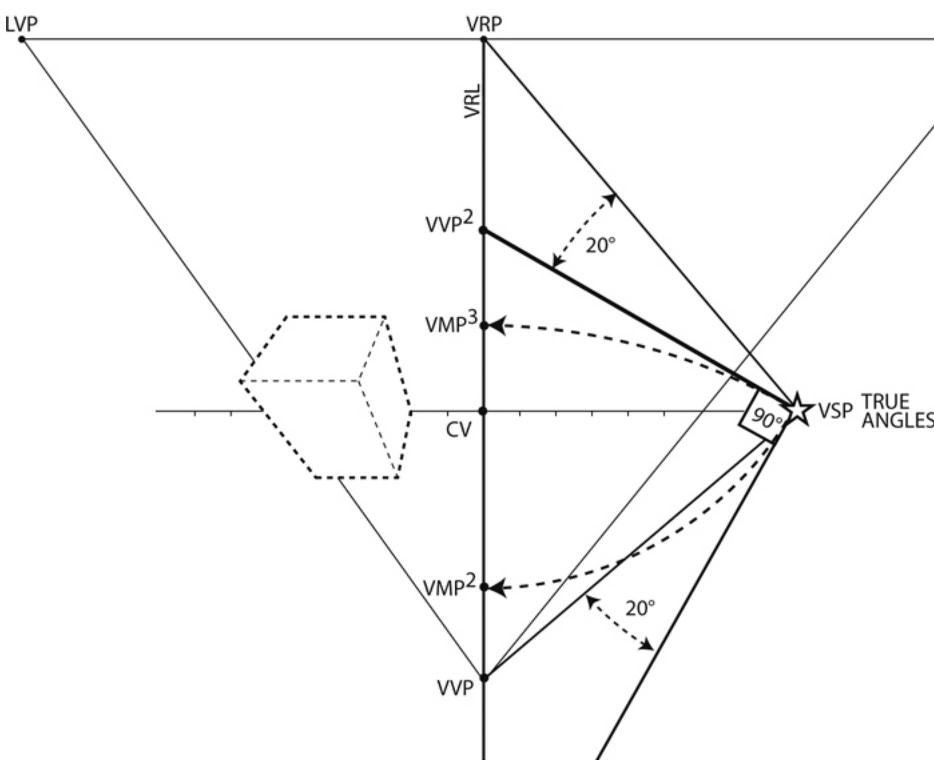
Figure 18.24 **Connect lines to vanishing points to complete the box.**

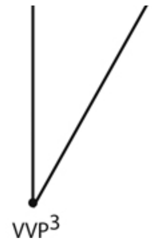
y-Axis

The final rotation is along a horizontal axis, the *y* coordinate. The box tips forward or backward.

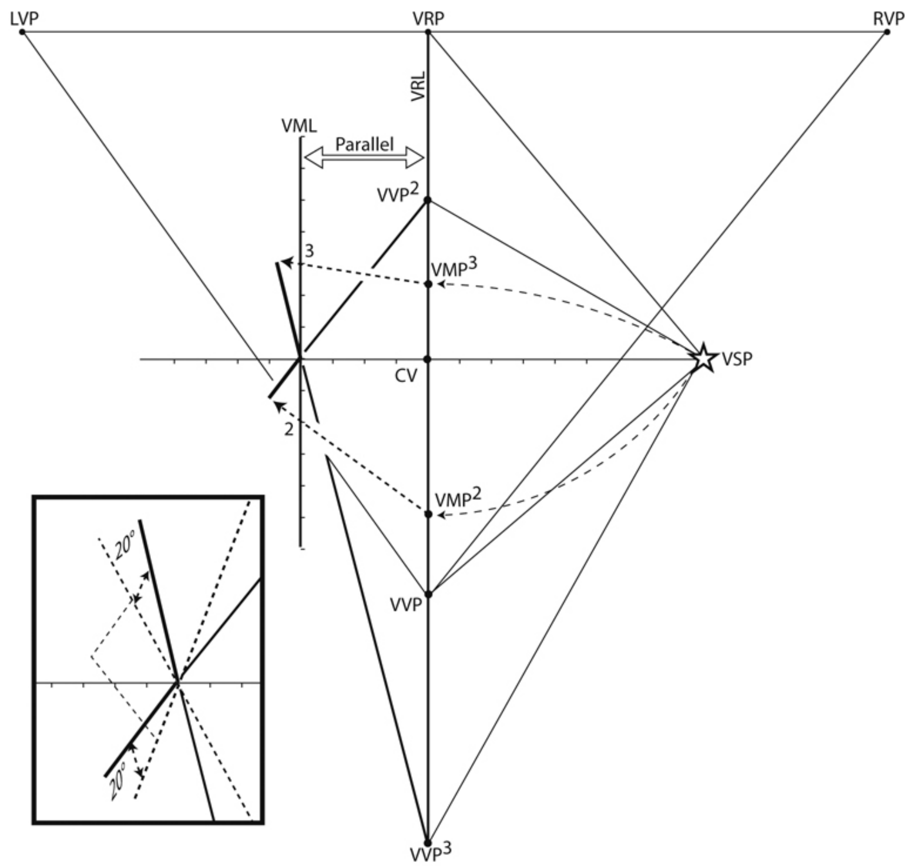
The point of true angles for the *y*-axis is the vertical station point (Figure 18.25, top).

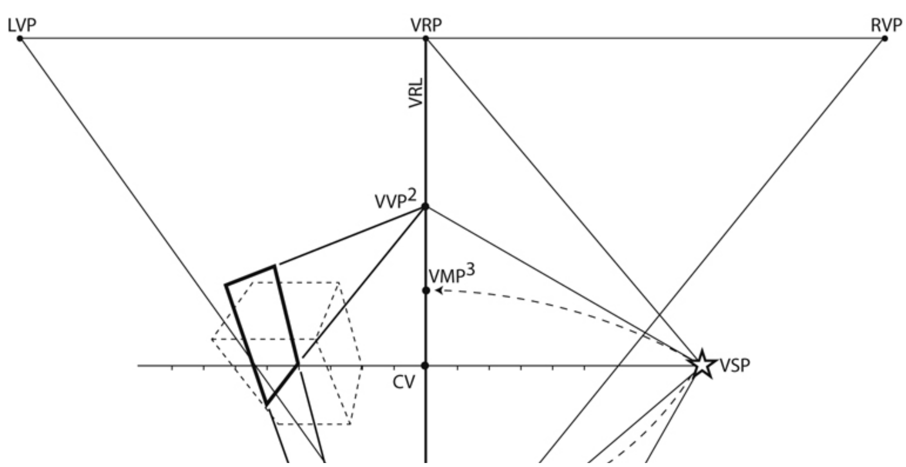






[Figure 18.25](#) **The vanishing and measuring points are placed on the vertical reference line. True angles are found at the vertical station point.**





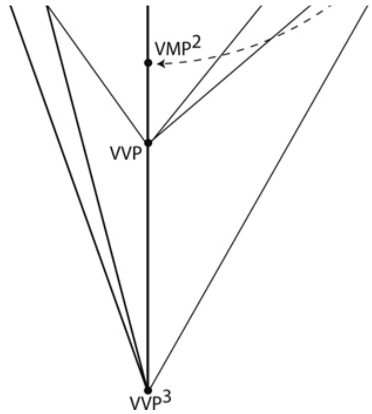


Figure 18.26 **Since the measuring points are on a vertical line the measuring line must also be vertical. This box is tilted 20° clockwise.**

The vanishing points and measuring points are placed on the vertical reference line (Figure 18.25, bottom).

Measure the height and depth using a measuring line that is parallel with the vertical reference line (Figure 18.26, top).

Connect to vanishing points to complete the left side of the box (Figure 18.26, bottom). Complete the box using horizontal lines for the thickness (Figure 18.27).

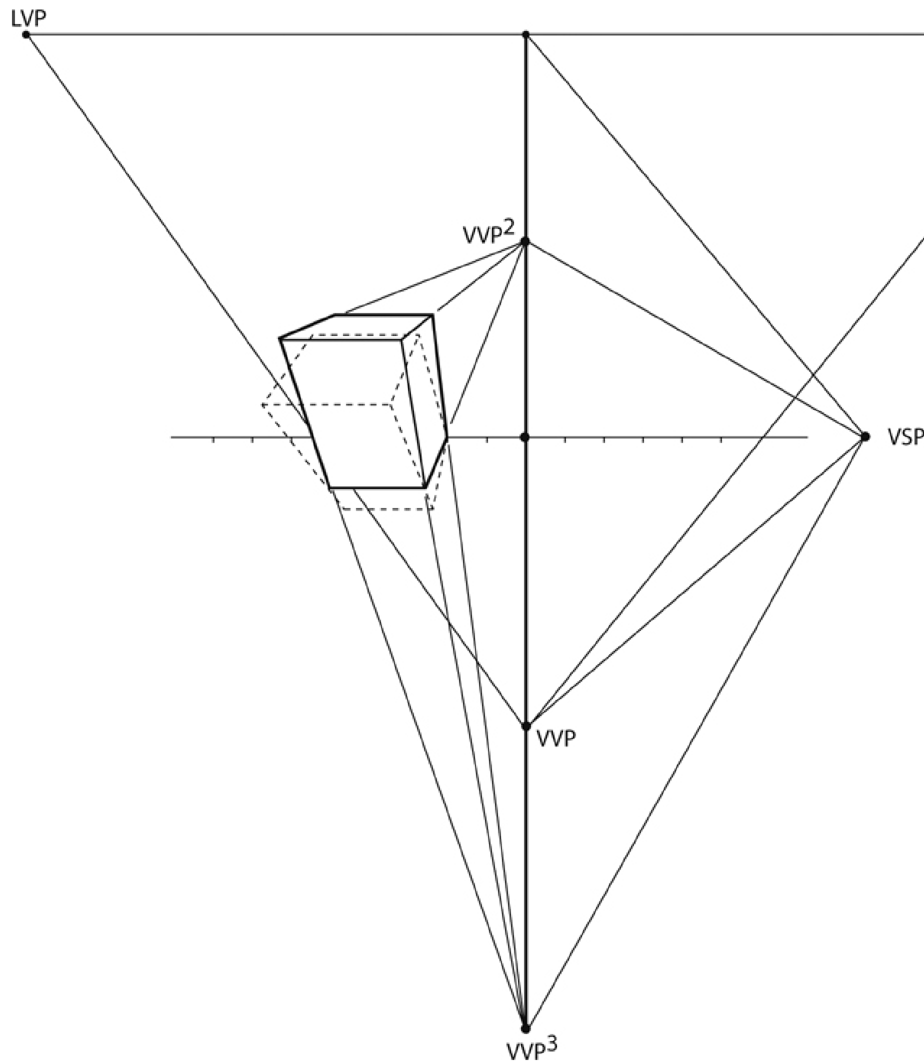


Figure 18.27 Connect to the vanishing points to complete the box.

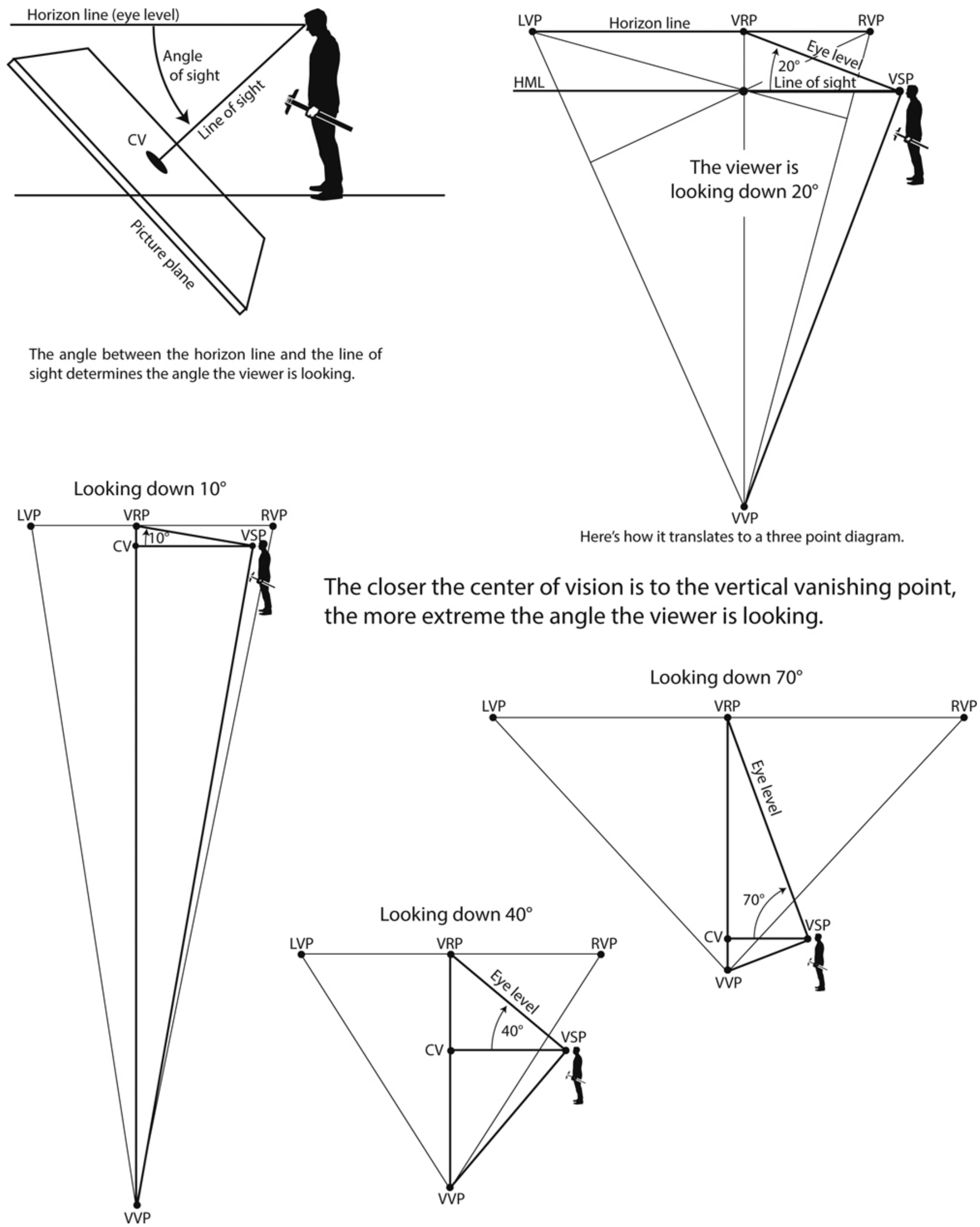


Figure 18.28 The angle of sight is determined at the vertical station point.

Three-Point Diagrams

At this point, the focus will change to a previous topic: creating a three-point diagram. Not generically, as before, but creating a diagram from a specific viewpoint. This could not have been done earlier. To create a diagram from a specific viewpoint, it is necessary to understand how axes points work.

Angle of Sight

First, consider the center of vision. In three-point perspective the viewer is looking up or down, but at what angle? The horizon line is at the eye level, and the vertical vanishing point is 90° from the horizon line. The center of vision is somewhere between them. To place the center of vision at a specific location, so that the viewer is looking at a precise angle, use the vertical station point. There is 90° between the vertical vanishing point and the vertical reference point, 90° between looking straight down and looking straight ahead. If the viewer is looking at a specific angle, place that angle at the vertical station point. For example, if the viewer is looking down 20°, draw a 20° angle down from the vertical station point (Figure 18.28).

Angle of Object

The next step is to orient the angle of the object to the picture plane. The vertical axis point works the same as the station point in two-point perspective. The vertical axis point determines the object's angle to the picture plane. For example, if the object should be turned 60°/30° to the picture plane, draw those angles at the vertical axis point (Figure 18.29, right).

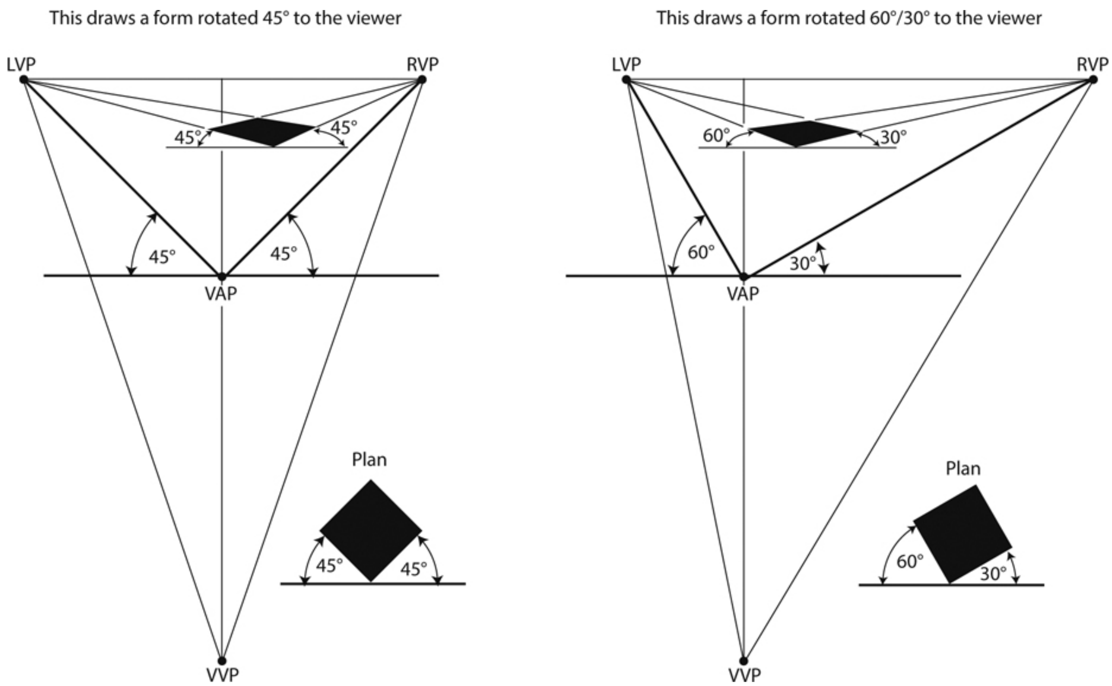


Figure 18.29 **The angle of the object to the picture plane is defined at the vertical axis point**.

First Steps

If a specific angle is desired for a three-point diagram, it is best to start with the vertical station point (Figure 18.30). Work out the diagram from there, establishing the center of vision, then using the vertical axis point to define the angle of the object to the picture plane.

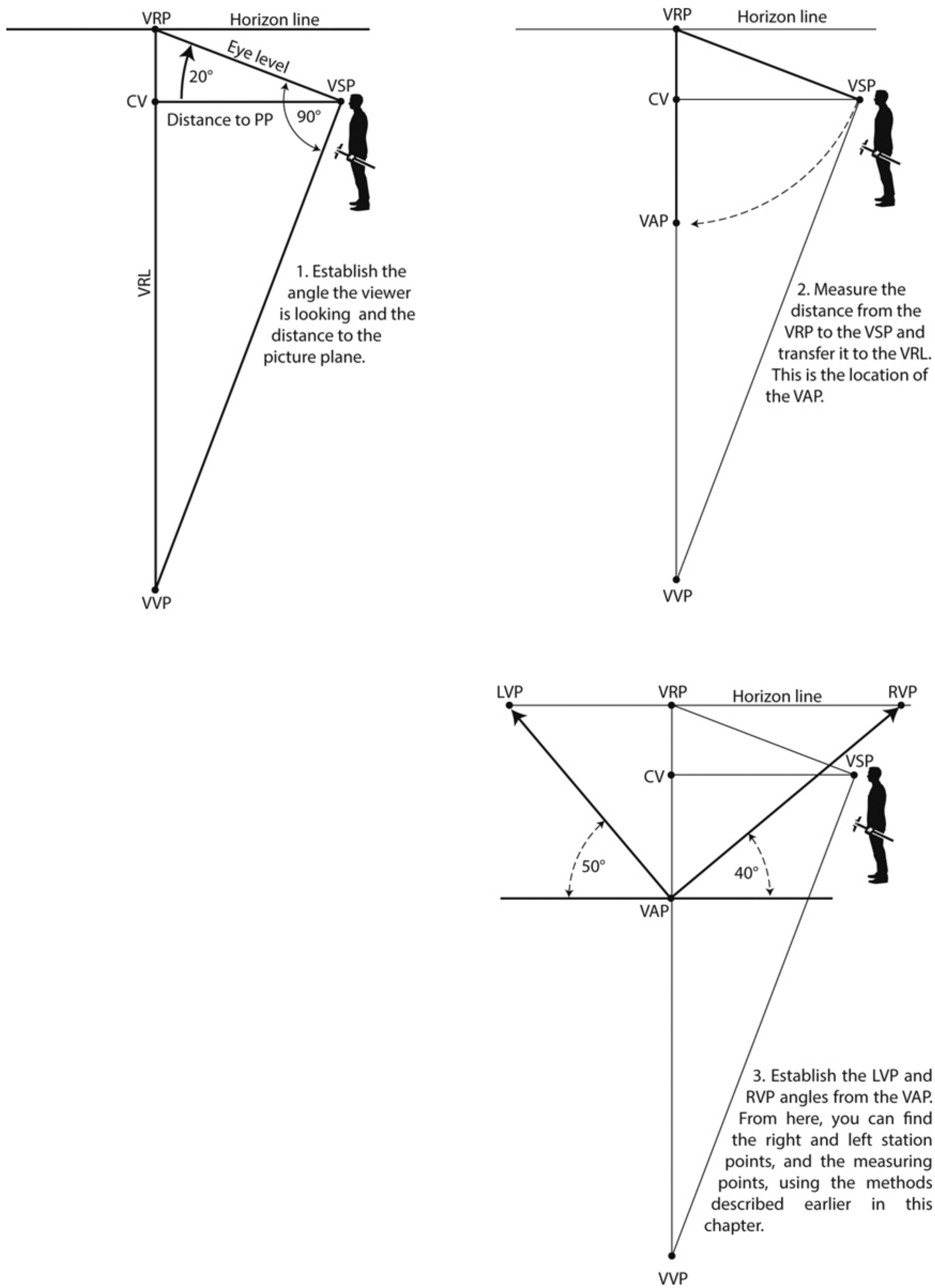
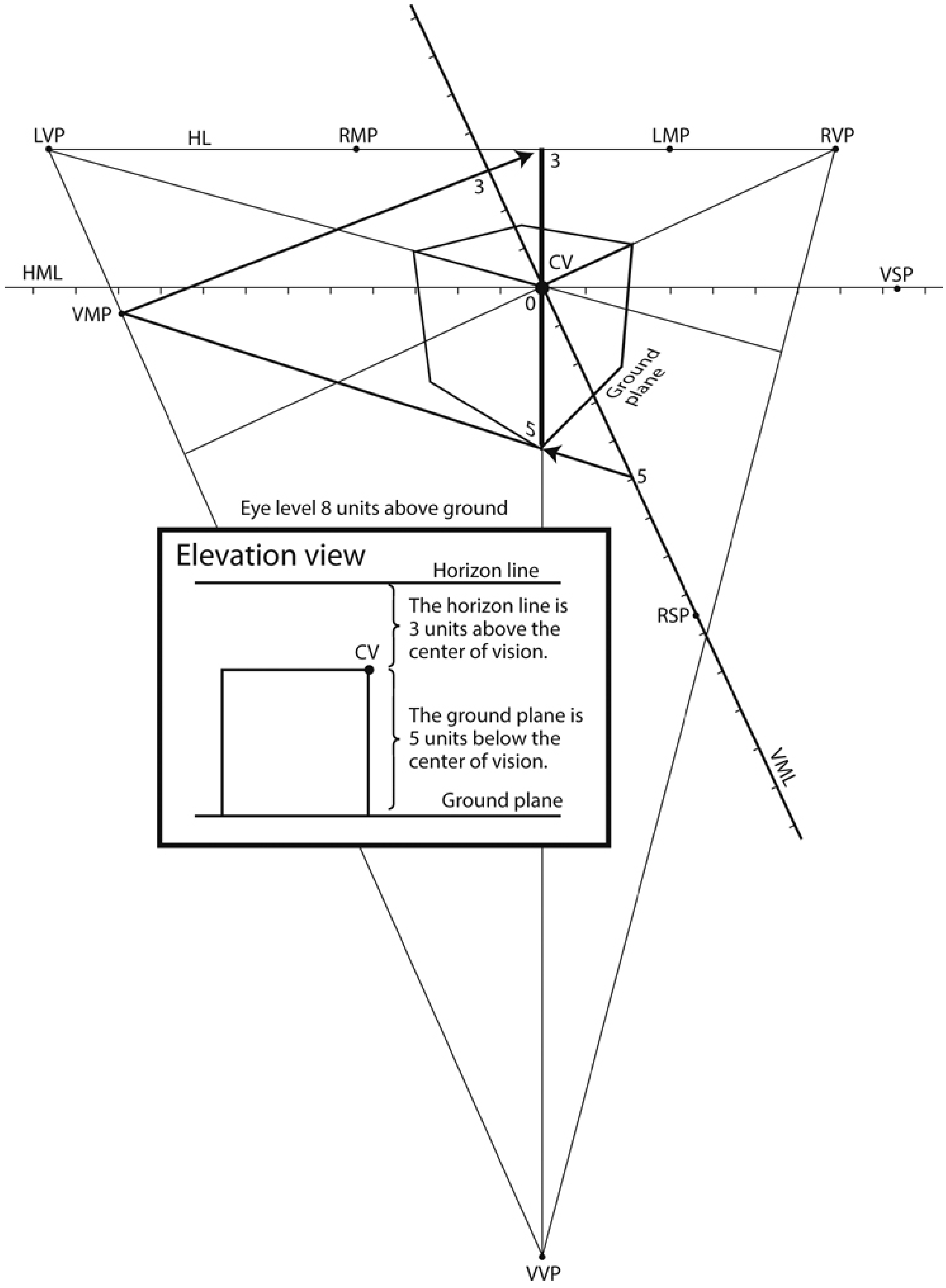


Figure 18.30 This three-point diagram begins at the vertical station point. The viewer is looking down at a 20° angle. The object being drawn is at a 50°/40° angle to the picture plane.

Eye Level

The viewer's eye level depends on the placement of the ground plane.

The horizontal measuring line is placed at the center of vision, which can be—but is not necessarily—on the ground. The ground plane is determined by the drawing. Decide where the ground is, then measure the distance from the ground to the horizon line to determine the eye level (Figure 18.31).



[Figure 18.31](#) **In this example, there are 5 units between the ground and the center of vision. There are 3 units between the center of vision and the horizon line. The eye level is 8 units above ground.**

19
Combining One- and Three-Point Perspective

Imagine the following scenario: a person is on a balcony looking down at a photographer. The photographer is taking their picture. The scene below is in three-point perspective—except for the camera. The camera is facing the person on the balcony, so the camera is in one-point perspective. There are endless scenarios where a one-point object can be envisioned in a three-point scene. This chapter combines a three-point view with a one-point object (Figure 19.1).

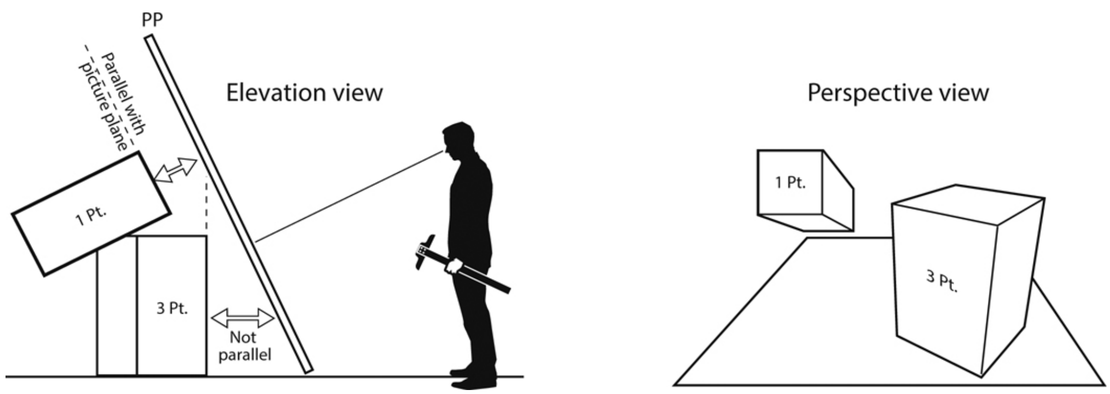


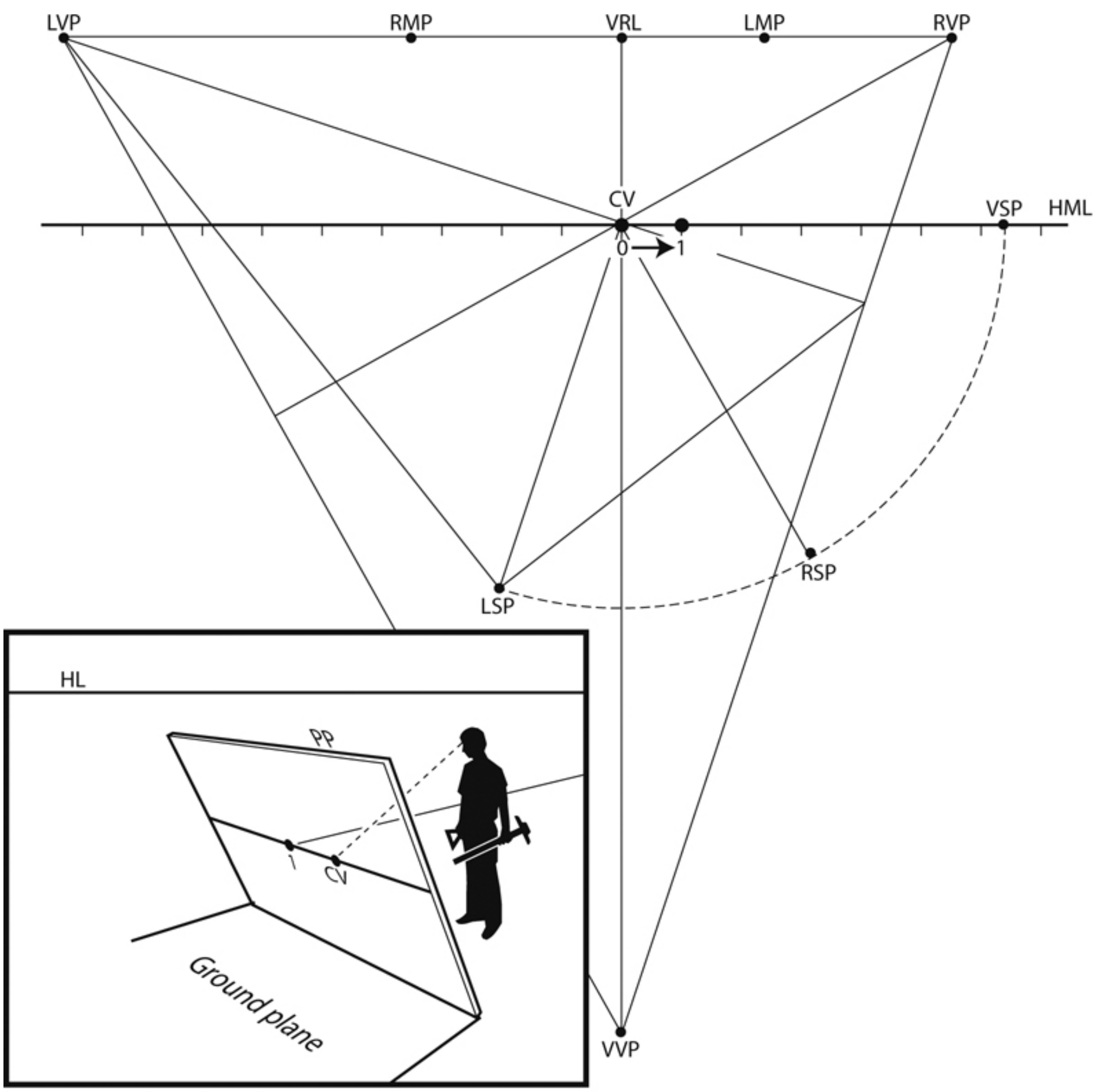
Figure 19.1 When combining a one-point object in a three-point view, the front of the one-point box is parallel with the picture plane. In one-point perspective, horizontal and vertical dimensions are parallel with the picture plane. In three-point perspective, all dimensions are foreshortened.

One-Point Vanishing Point

Objects in one-point perspective have vertical and horizontal dimensions parallel with the picture plane, and foreshortened lines connect to the center of vision.

The Placement

To illustrate this, draw a one-point box, 2 units wide, 2 units high, and 4 units deep. It is 1 unit to the right of the center of vision and 4 units behind the picture plane. First, measure 1 unit to the right of the center of vision (Figure 19.2). Then, measure 4 units behind the picture plane (Figure 19.3). Make this point the bottom left corner of the box.



A view from behind the picture plane showing 1 unit to the right of the center of vision.

[Figure 19.2](#) Measure 1 unit to the right of the center of vision.

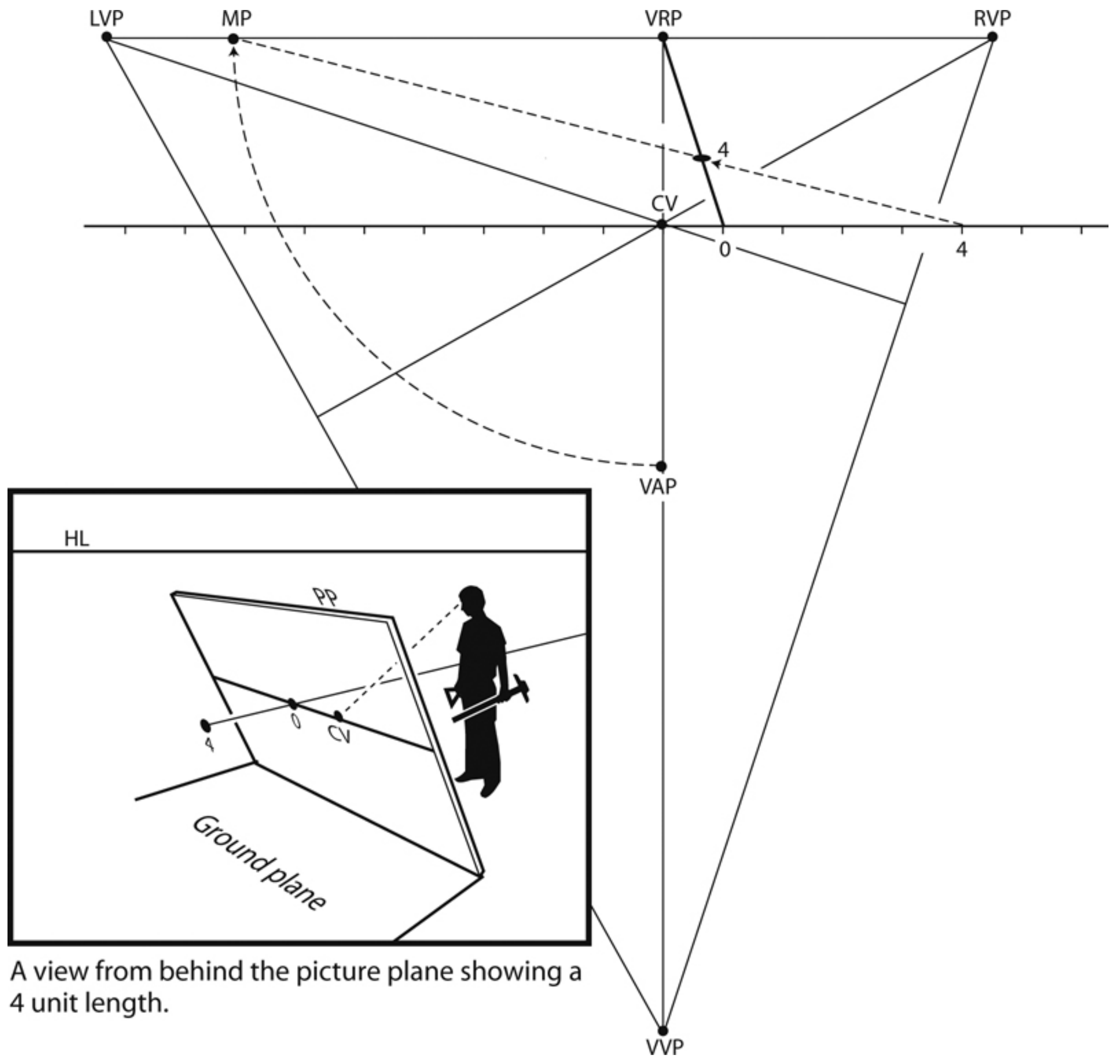


Figure 19.3 Measure 4 units behind the measuring line. An angle 90° to the measuring line connects to the vertical reference point. A measuring point for the vertical reference point is needed.

Width

The box is 2 units wide. Measure the width along the measuring line and project the distance backward, using the vertical reference point (Figure 19.4).

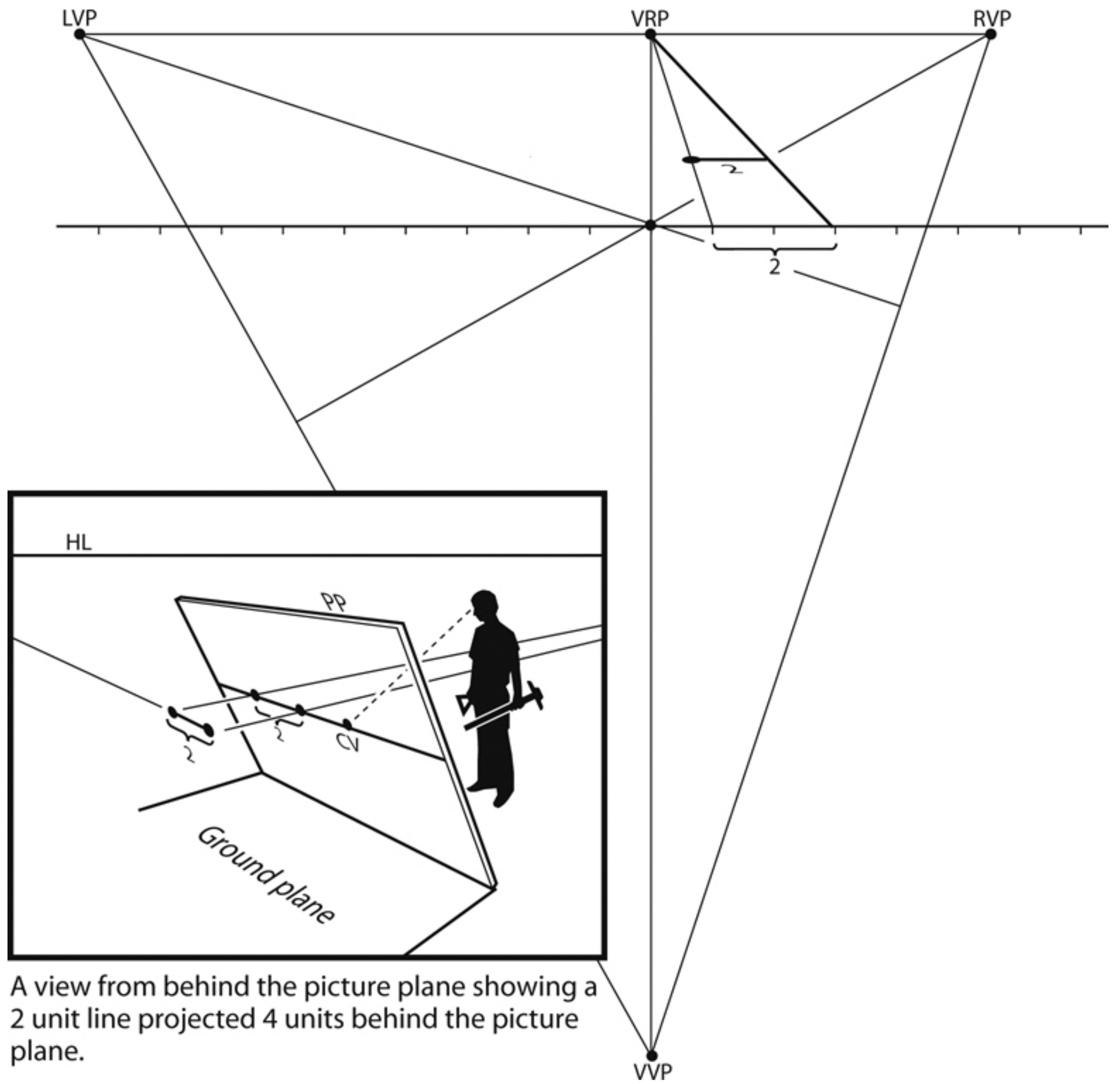


Figure 19.4 Horizontal dimensions are parallel with the picture plane. Measure 2 units along the horizontal measuring line. Project the 2 units to the desired location.

Superimposing Diagrams

Now the width and location of the one-point box has been established. All of this was done using the three-point diagram. But to draw the one-point box, and to measure its depth, requires a one-point diagram. A one-point diagram needs to be superimposed on the three-point diagram. Combining a one-point diagram and three-point diagram is not difficult, but there are, however, two important rules to remember.

First, a person can only look in one place at any given time, so there can be only one center of vision. The center of vision in the one-point diagram must be in the same place as the center of vision in the three-point diagram. Second, a person can be in only one place at a time, thus the distance from the viewer to the picture plane must remain constant. The distance from the center of vision to the station point must be the same in the one-point diagram as it is in the three-point diagram. Keep these two important rules in mind when superimposing multiple perspective diagrams (Figure 19.5).

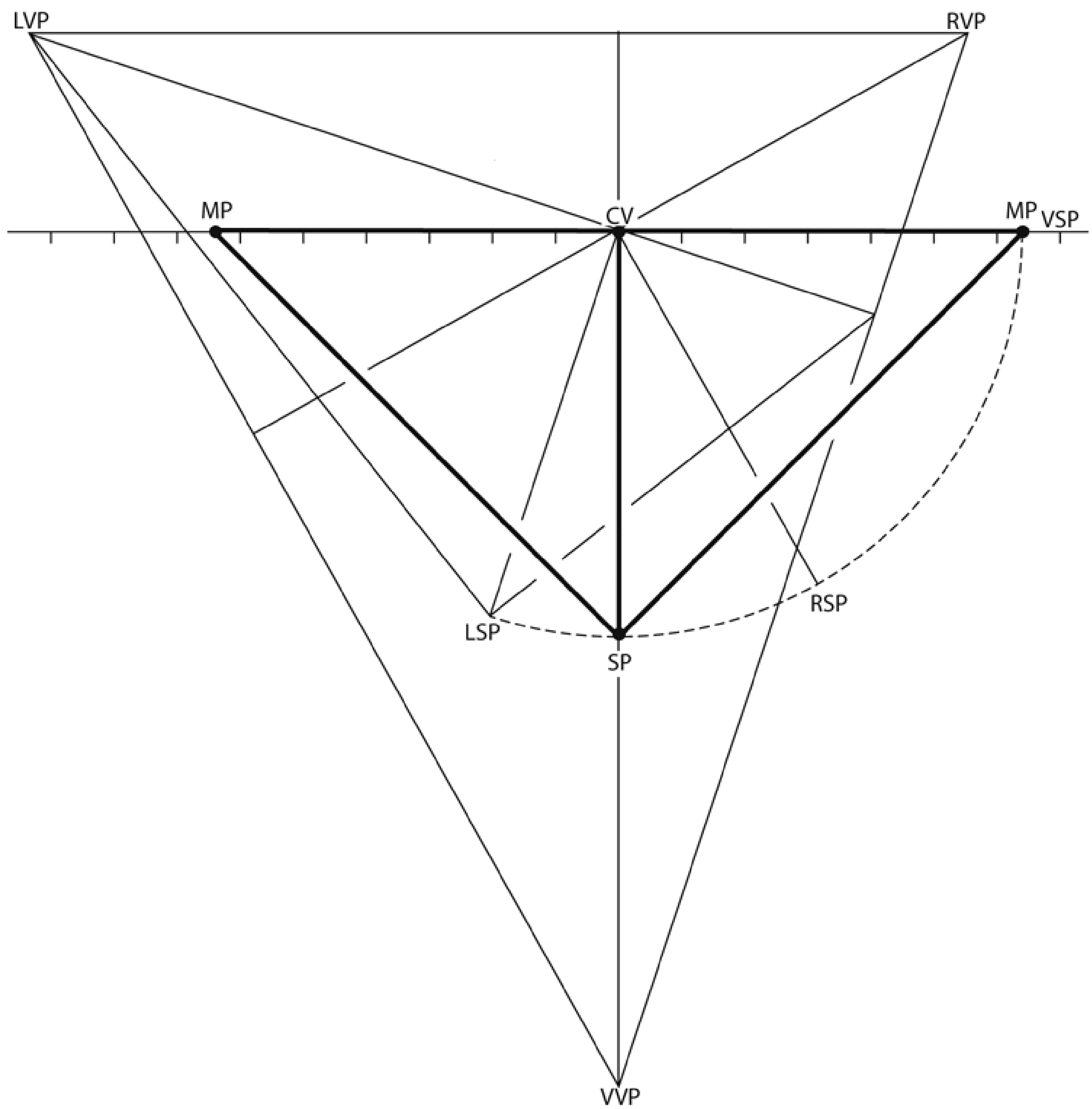


Figure 19.5 A one-point diagram can be superimposed on a three-point diagram if both diagrams use the same center of vision and the distance from the station point to the picture plane remains constant.

Depth

Move the measuring line so that it is on the same plane as, and touching, the line being measured. Measure the depth using one-point perspective guidelines (Figure 19.6).

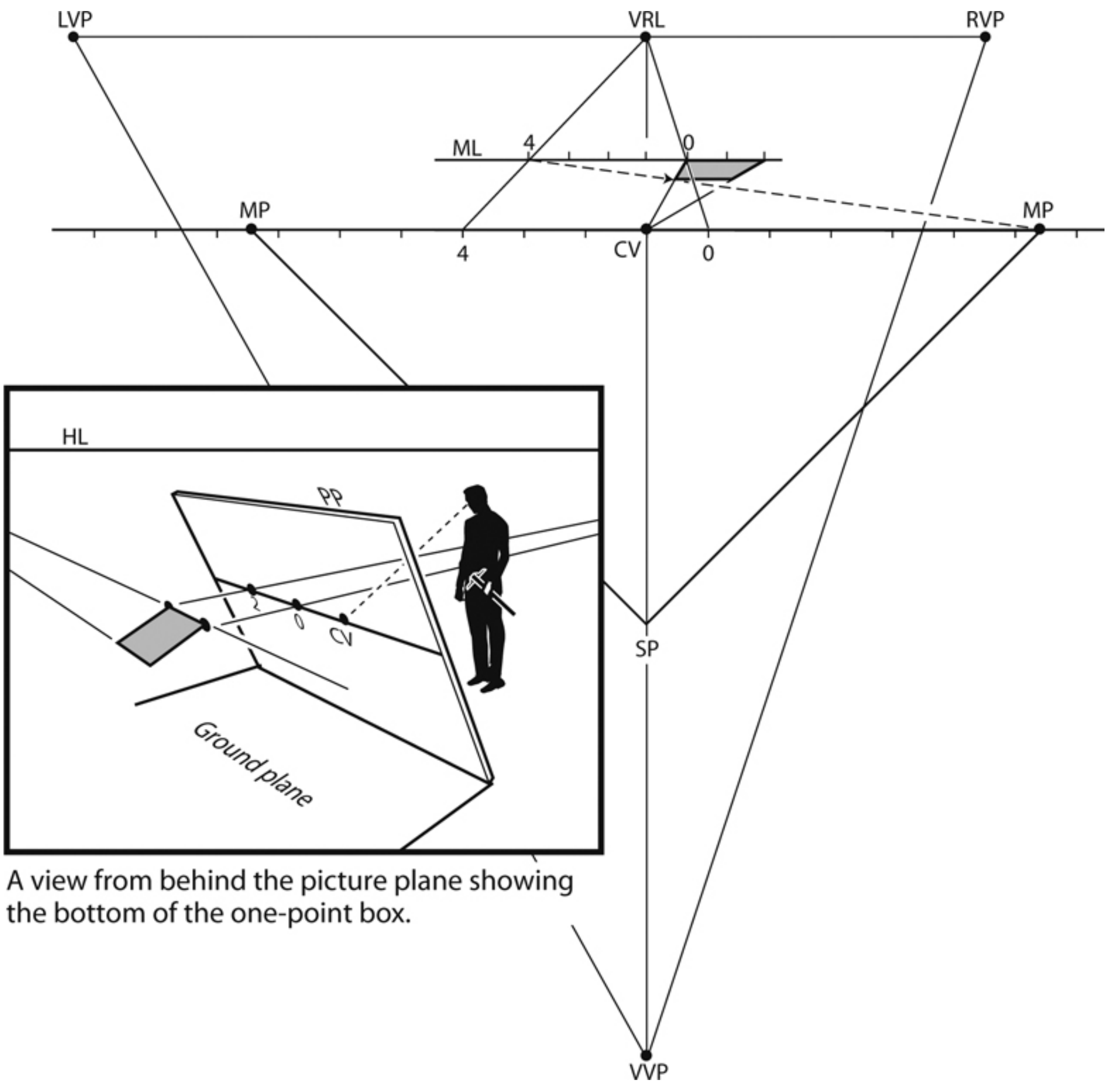


Figure 19.6 Lines drawn to the center of vision are 90° from the picture plane (but at an angle to the ground plane). Measure depth as measured in one-point perspective. Proper placement of the measuring line is vital.

Height

Turn the measuring line vertically to measure height (Figure 19.7). The height is parallel with the picture plane and is not foreshortened. Measuring points are not required to measure height and width in one-point perspective.

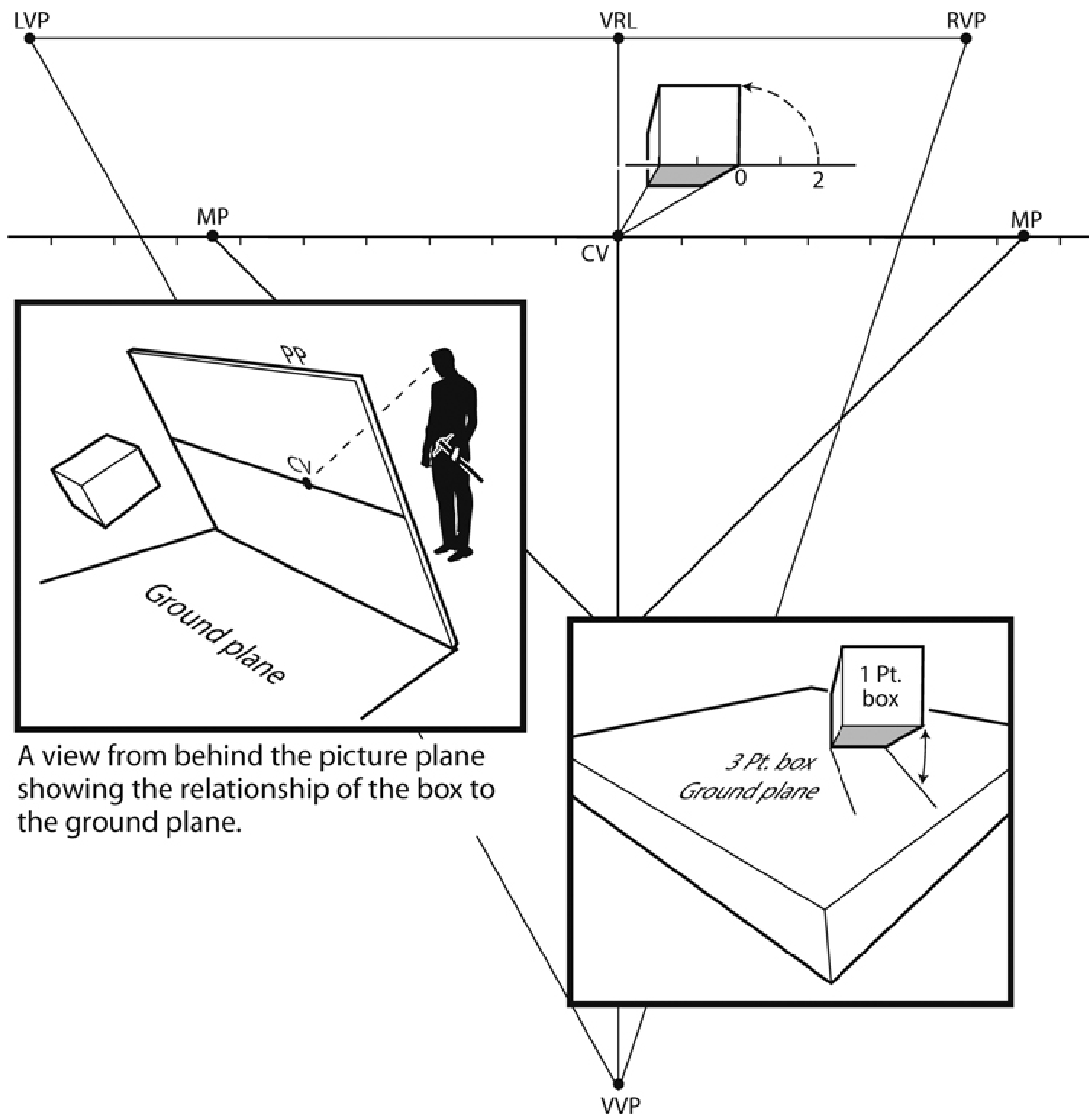
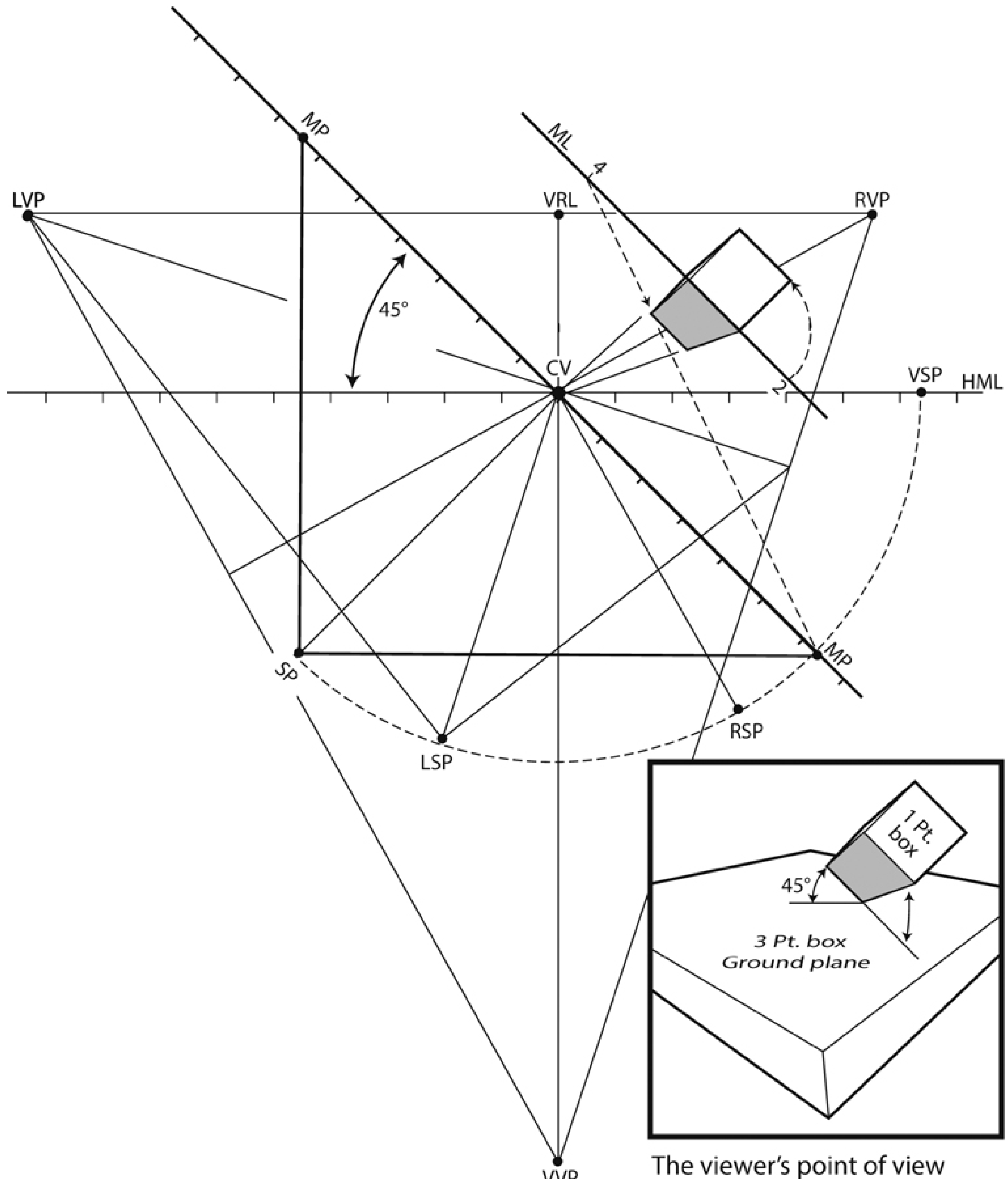


Figure 19.7 Finish the box by rotating the horizontal measuring line 180° to establish the height.

Rotating a One-Point Object

The one-point diagram can be rotated to any angle using the center of vision as an axis point. The angles are true angles. For example, if the one-point diagram is rotated 45° clockwise, the box being drawn will be rotated a true 45° clockwise (Figure 19.8). Remember, there can be only one center of vision, and the distance from the station point to the picture plane remains constant.



The viewer's point of view showing the relationship of the box to the ground plane.

Figure 19.8 The box can be rotated along the *y*-axis by rotating the diagram to the desired angle.

20
Combining Two- and Three-Point Perspective

In two-point perspective, the vertical lines are parallel with the picture plane (Figure 20.1). The procedures to combine two-point and three-point diagrams follow the same rules that apply to combining one-point and three-point diagrams. For this example, draw a 3 unit cube. This cube is located 2 units to the right of the center of vision and 4 units below the measuring line (Figure 20.2).

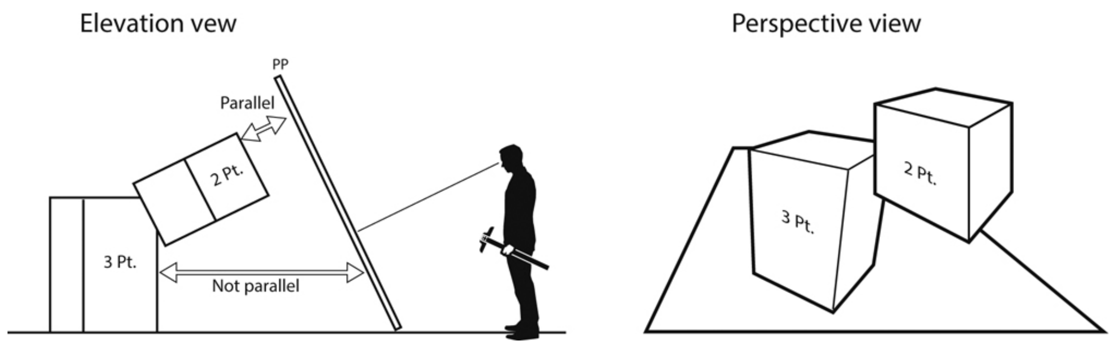


Figure 20.1 **A two-point object in a three-point view.**

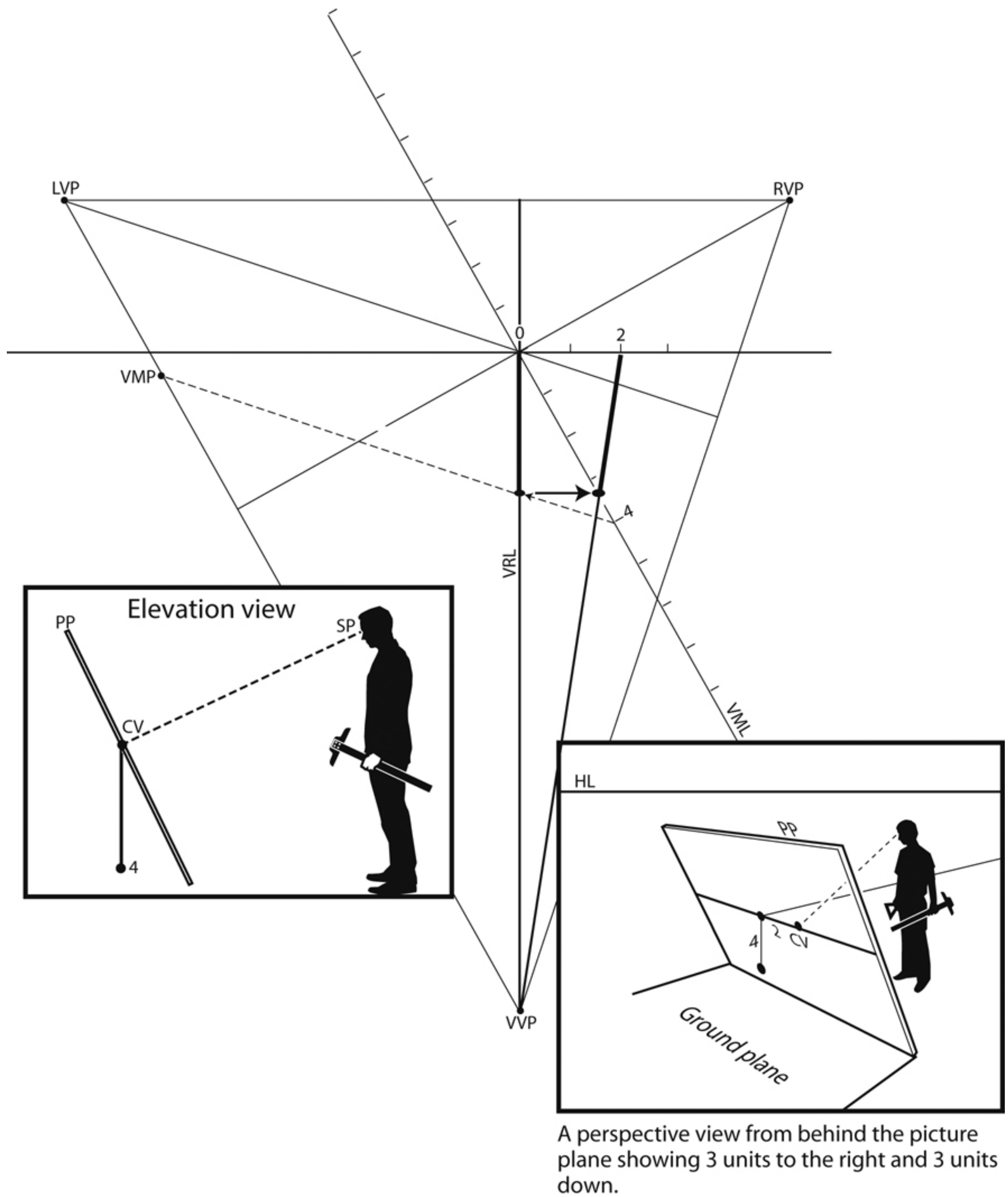


Figure 20.2 The box is 2 units to the right and 4 units below the center of vision.

First, using the horizontal measuring line, measure 2 units to the right of the center of vision. Then, using the vertical measuring line, measure 4 units below the center of vision. This point represents the top, front corner of the two-point box.

Superimposing Diagrams

The location of the two-point box was established using the three-point diagram (all these dimensions are in three-point perspective). To draw the two-point perspective box, a two-point perspective diagram is needed. When superimposing the two-point diagram, remember that there is only one center of vision (the two diagrams share the same center of vision), and the distance from the viewer to the picture plane remains constant.

Use the two-point diagram's station point to establish the left and right vanishing points. For example, if the cube is at a 45° angle to the picture plane, draw 45° angles from the station point (Figure 20.3).

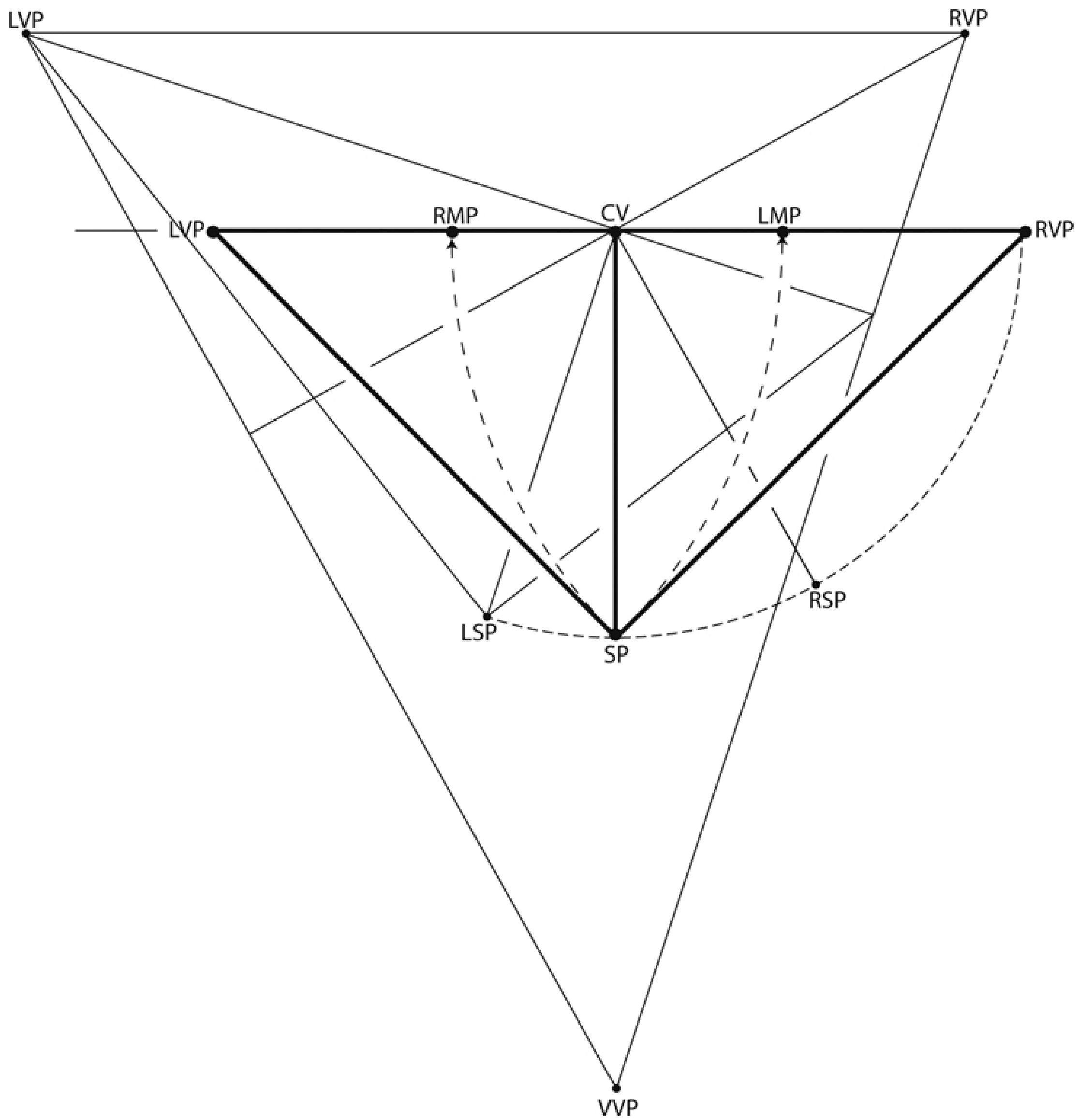


Figure 20.3 A two-point diagram superimposed on a three-point diagram. The left and right vanishing points are set up at 45°.

Measuring

Take care that the measuring line and the object being measured are on the same plane. This can be challenging. In this example, the measuring line must be moved down to the level of the box. Use the vertical vanishing point to project the measuring line down 3 units, touching the line being measured (Figure 20.4).

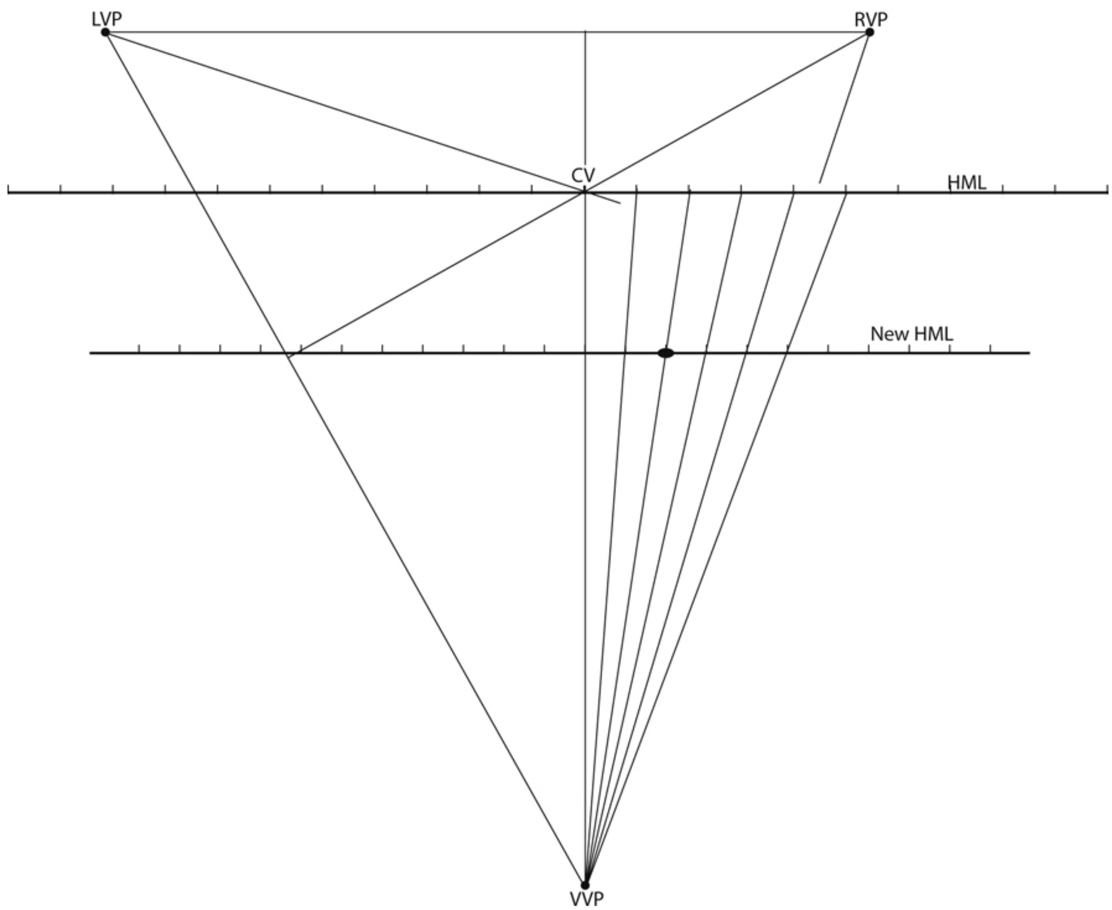


Figure 20.4 Move the measuring line down 4 units, so that it is on the same plane as the line being measured.

Once the measuring line is in place, follow two-point perspective procedures to measure the width and depth (Figure 20.5).

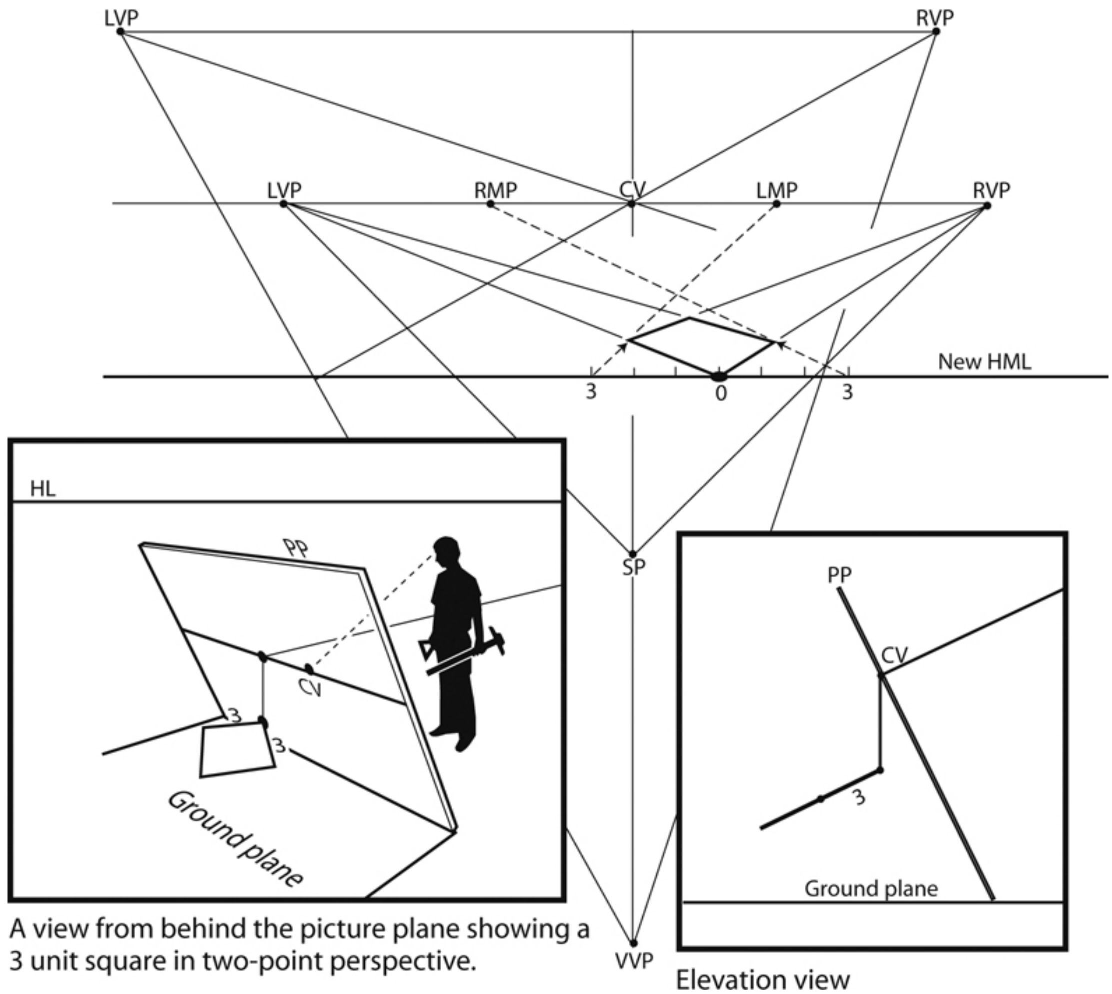


Figure 20.5 Measuring horizontal dimensions follows the same procedures as outlined in two-point perspective.

Vertical dimensions are not foreshortened, so turn the measuring line 90° to measure height (Figure 20.6).

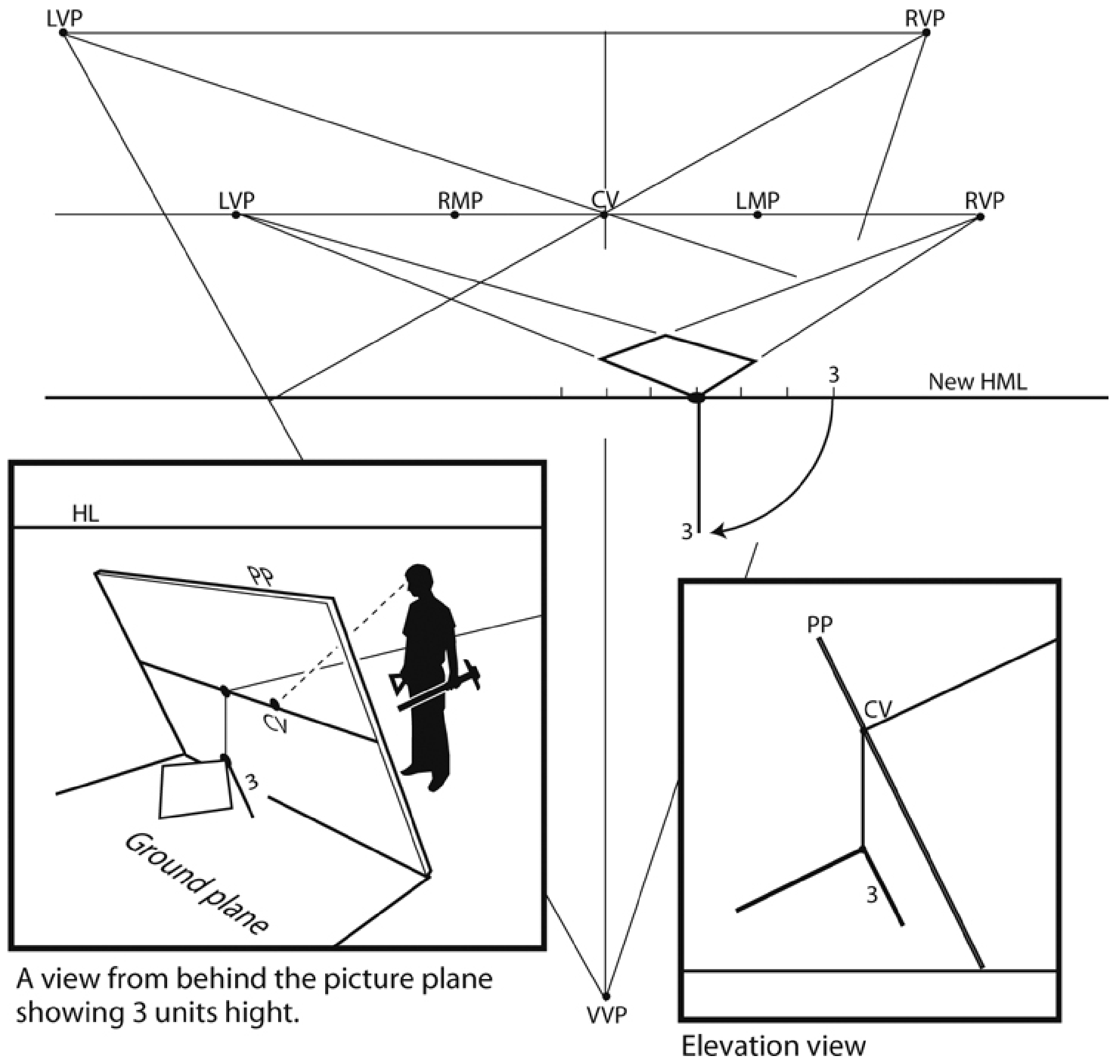
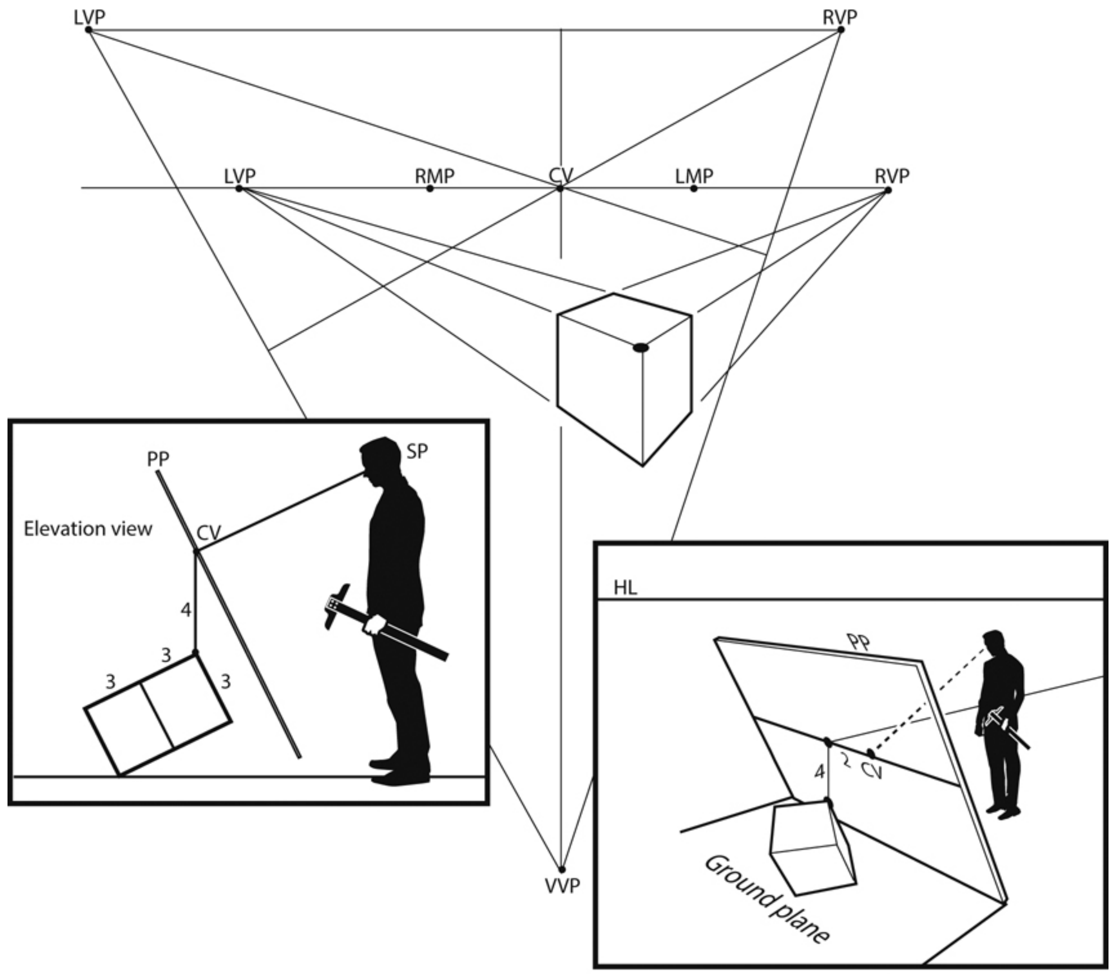


Figure 20.6 Height is parallel with the picture plane and not foreshortened. Turn the measuring line vertically.

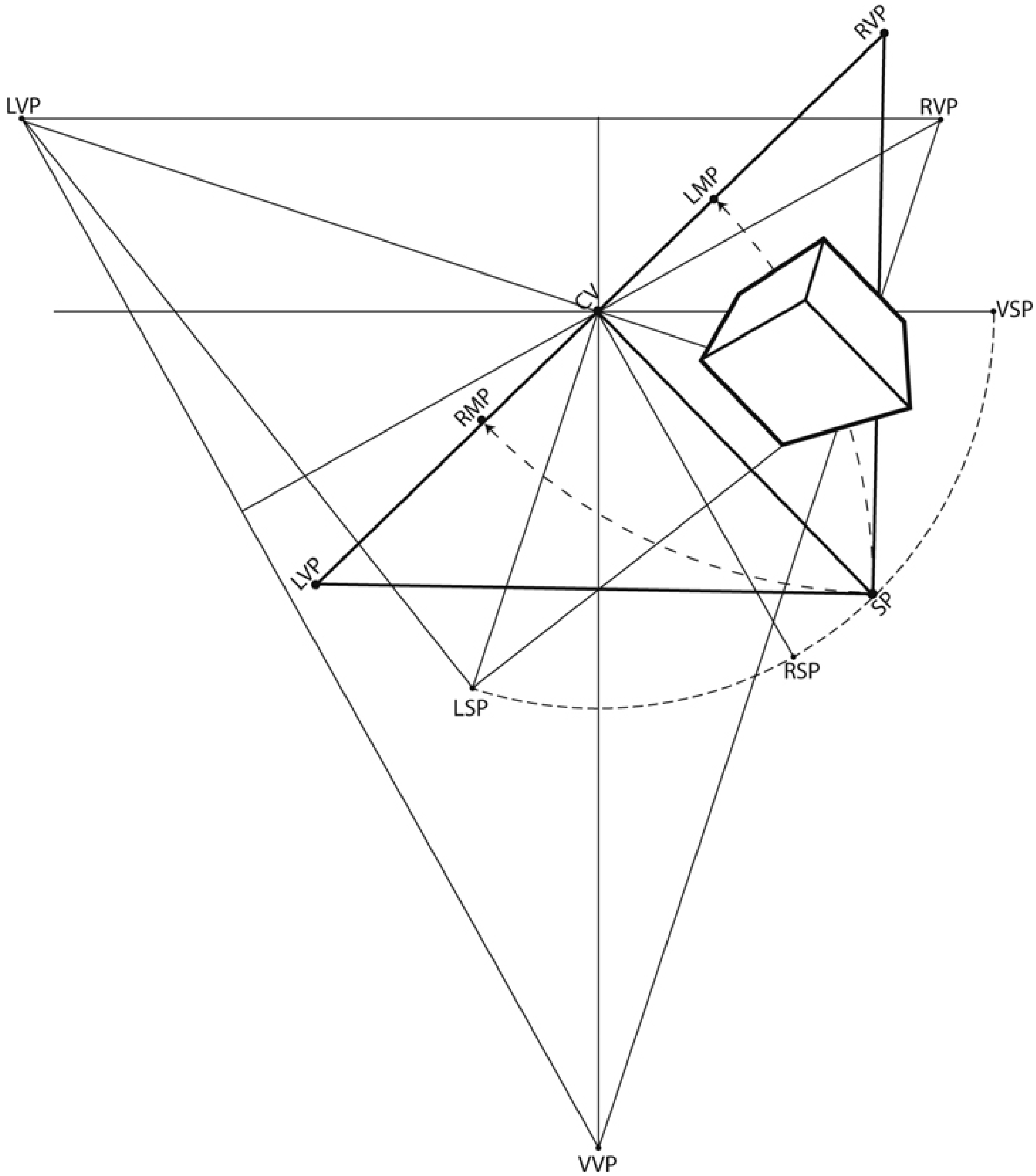
Complete the Box

Connect foreshortened lines to the right and left vanishing points to complete the box (Figure 20.7).



[Figure 20.7](#) **Connect lines to vanishing points to finish the box.**

The diagram can be rotated to any angle using the center of vision as an axis point ([Figure 20.8](#)).



[Figure 20.8](#) **To rotate the object being drawn, rotate the two-point diagram**.

Using these techniques, one-, two-, and three-point perspective can now be combined in the same illustration ([Figure 20.9](#)).

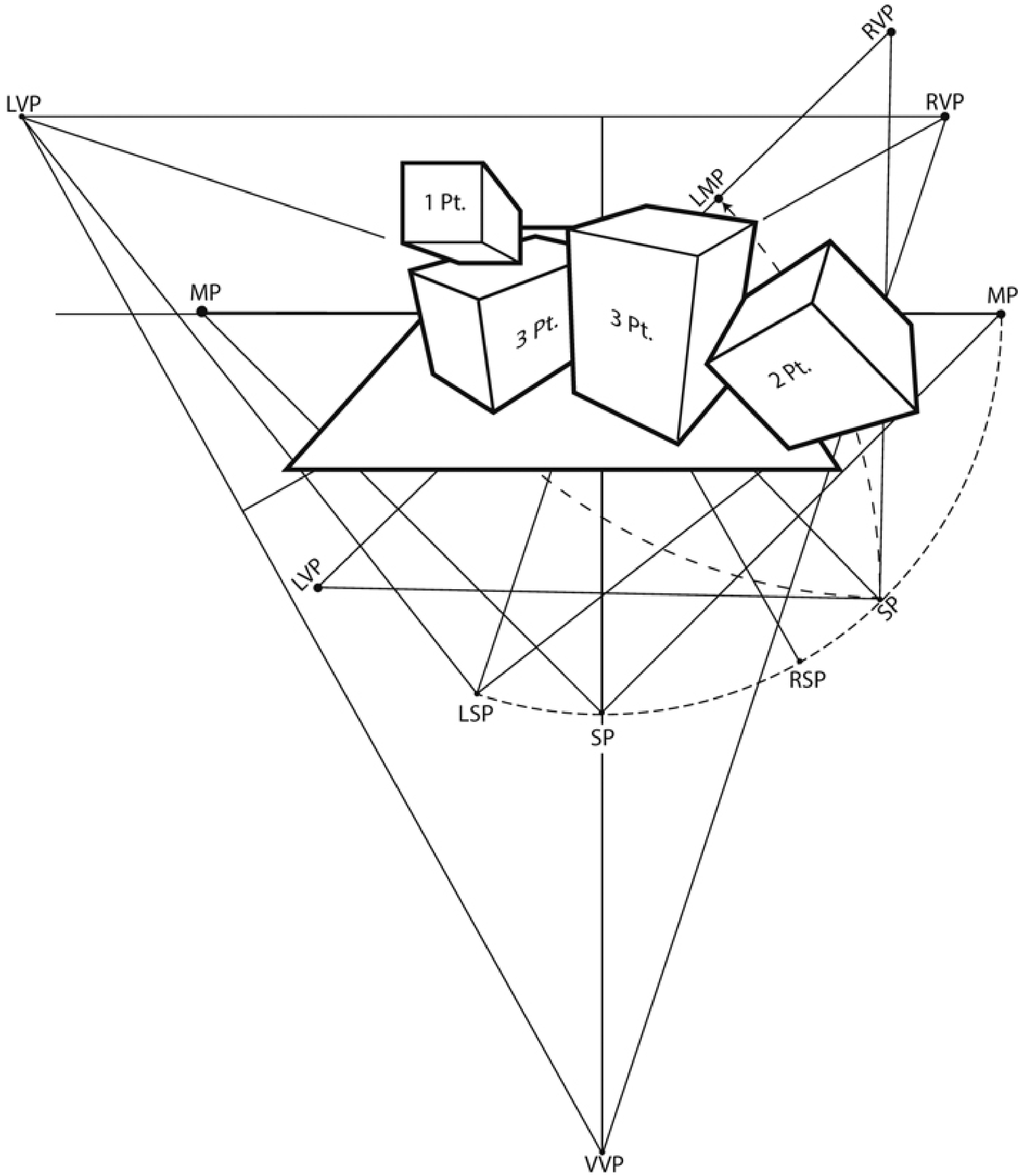


Figure 20.9 **Combining one, two, and three-point perspective.**

21
Combining Three-Point Perspective Diagrams

It may seem like every possible topic has been explored at this point. But, there is more: superimposing multiple three-point diagrams (none of the objects share vanishing points, and no angles are parallel with the picture plane). To do this successfully, the two cardinal rules must be followed: there is only one center of vision, and the distance between the viewer and the picture plane must remain constant (Figure 21.1). In addition to the two cardinal rules, all superimposed three-point diagrams must maintain the angle relationships discussed in Chapter 17 (Figures 17.3–17.13).

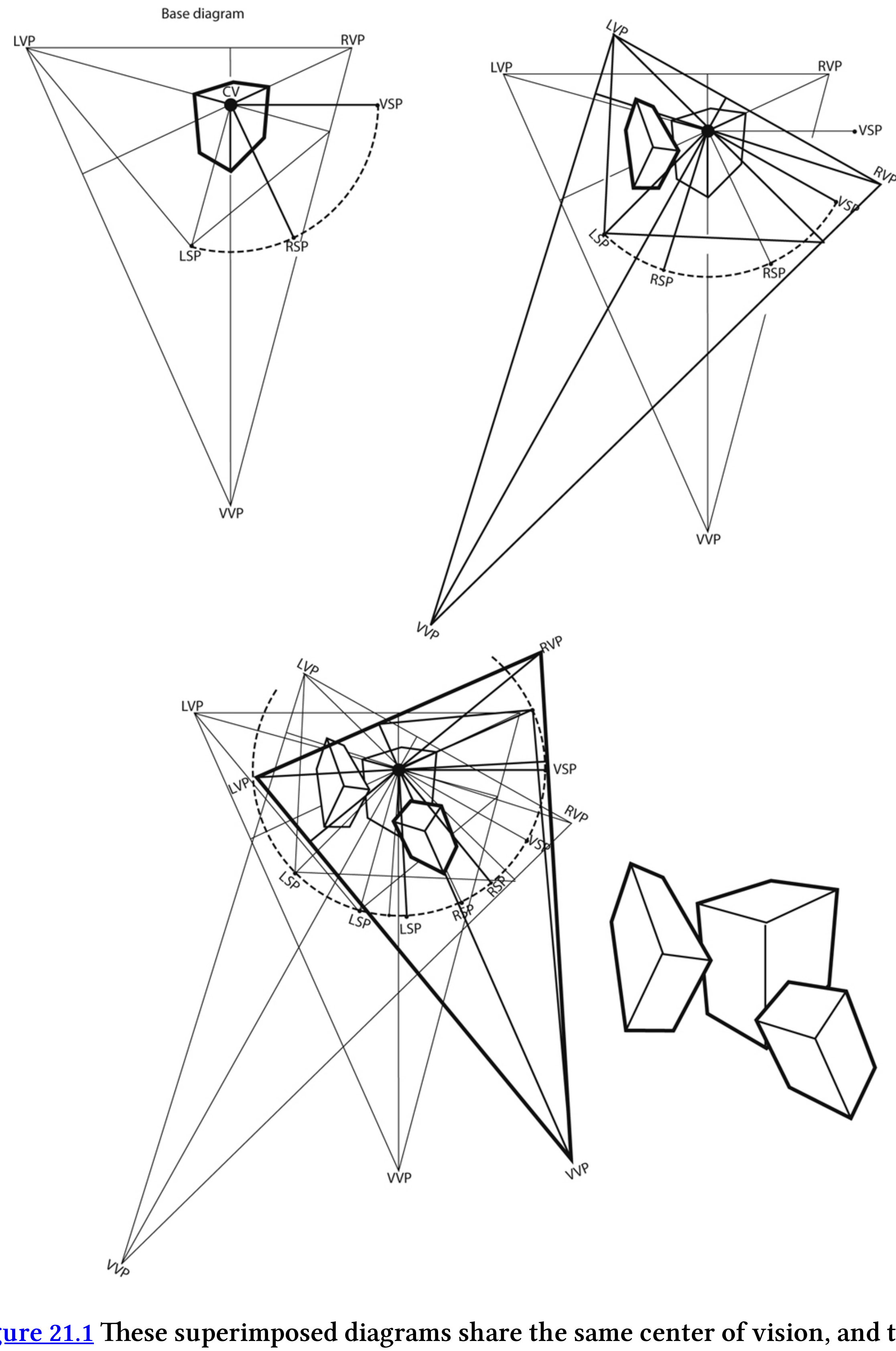


Figure 21.1 These superimposed diagrams share the same center of vision, and the distance from the picture plane to the station point is consistent throughout.

22
Compound Inclines in Two-Point Perspective

It is time to revisit the world of two-point perspective, where the viewer is looking parallel with the ground plane, and the center of vision is located on the horizon line. It is backtracking, but a good understanding of three-point perspective is needed to solve this next problem. In the next example, the box is tilted. Not tilted as drawn before, with one side parallel with the ground plane, and one side connecting to a vanishing point on the horizon line. This is a compound incline, a shape tilted so that no plane is parallel with the ground, and no lines connect to vanishing points on the horizon line. If this shape was touching the ground, it would do so at a corner. Imagine a pair of dice bouncing along a table. When they stop moving, the dice are in one- or two-point perspective. When moving, they are likely at an incline. It can be a simple incline where one surface is parallel with the ground plane, or a compound incline where no surface is parallel with the ground plane.

The Compound Incline Box

For this example, draw a box. The back of the box is 30° above the ground plane, and the box then rotates counterclockwise 30° (Figure 22.1). This is best approached as a two-step process.

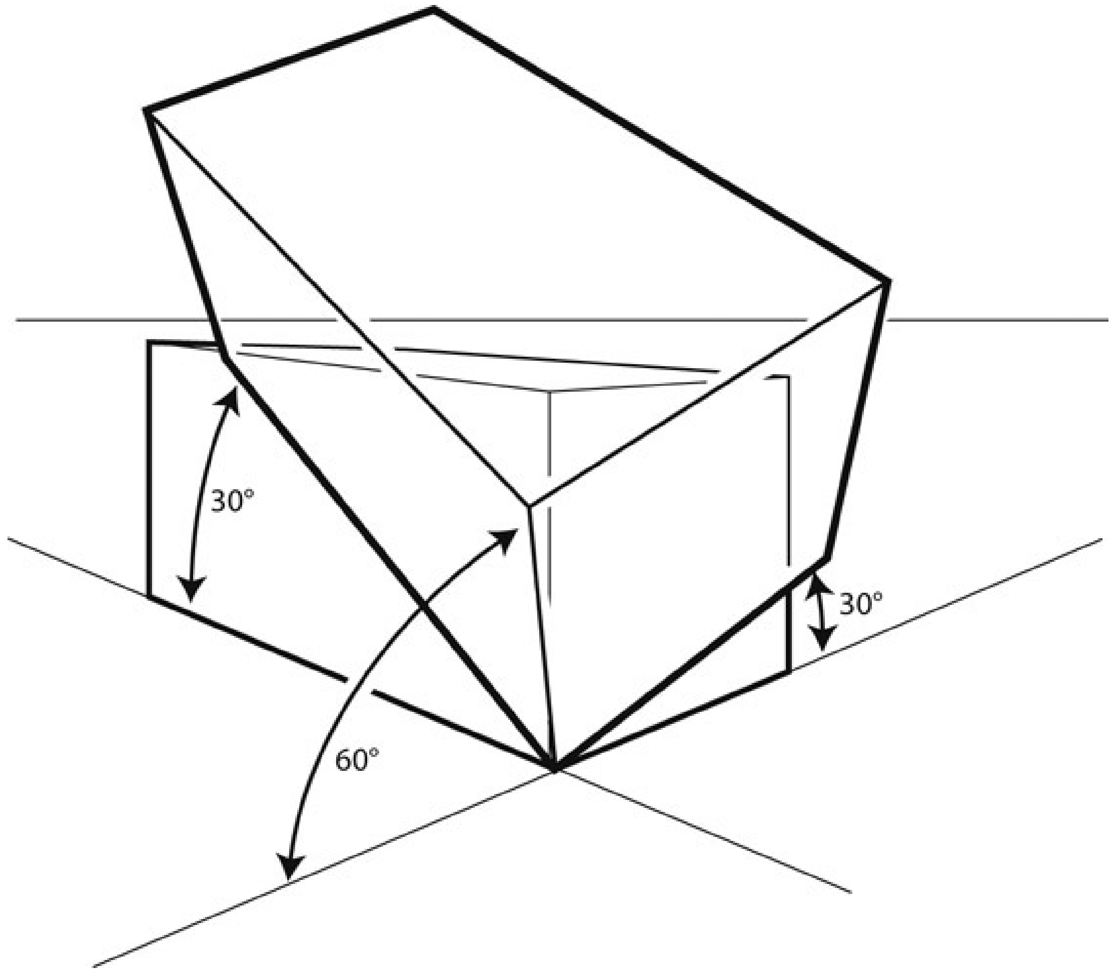
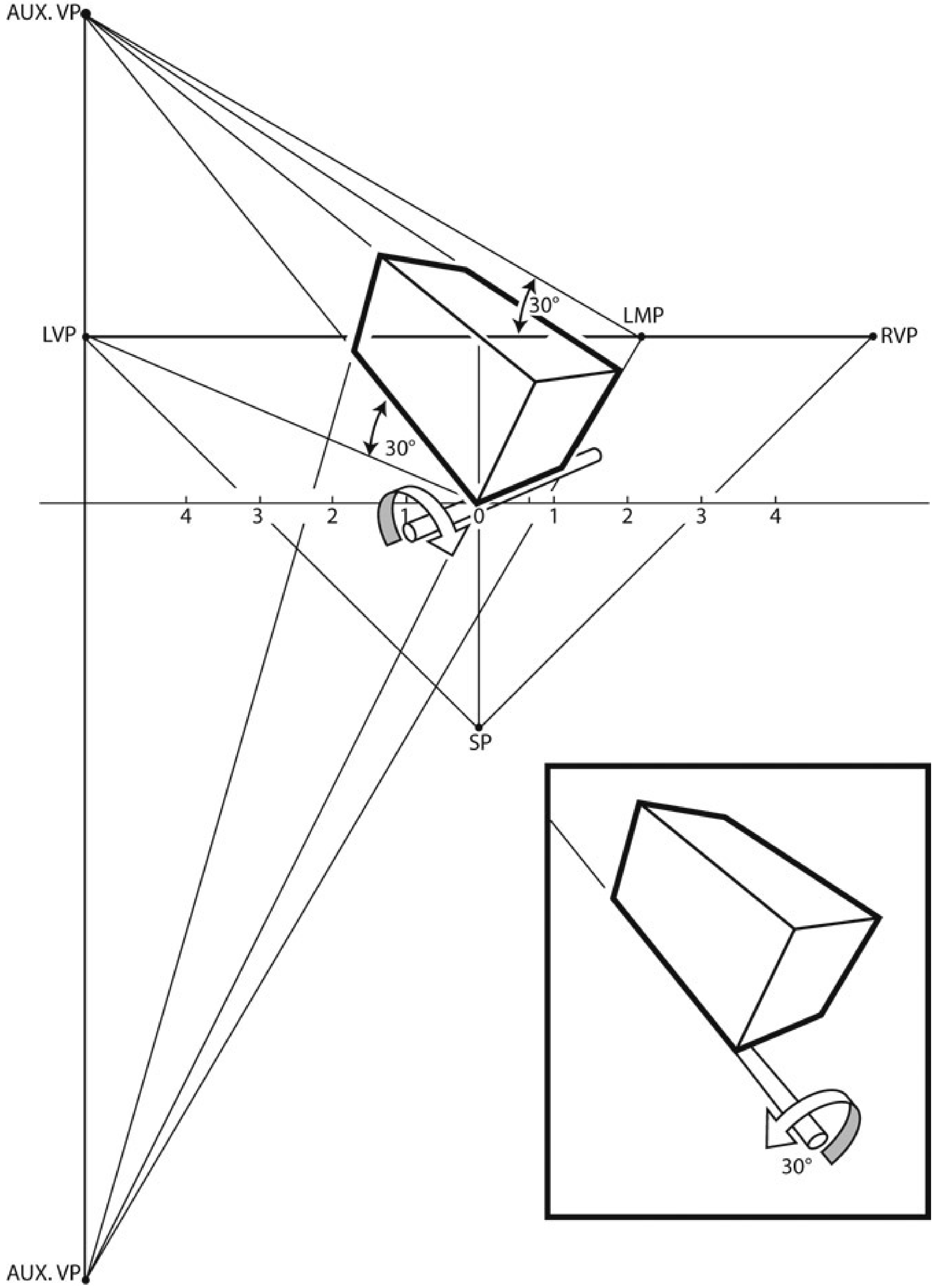


Figure 22.1 **No dimensions are parallel with the ground plane.**

The Incline

The box is angled 30° from the ground plane. Follow the procedures outlined in Chapter 10. After drawing the incline (Figure 22.2), the box rotates counterclockwise 30°. The axis of rotation is aligned with the upper auxiliary vanishing point (Figure 22.2, inset).



[Figure 22.2] **First, using a right axis point, lift the box 30° from the ground plane. Then, rotate the box 30° counterclockwise along an axis aligned to the upper auxiliary vanishing point.**

The Rotation

As the box rotates along this axis, the right and the lower auxiliary vanishing points change position. This is where the three-point procedures come into play. Create reference lines following the three-point perspective guidelines. (The diagram is the same as a three-point diagram. The only difference between the two diagrams is the labeling, see Figure 22.3.)

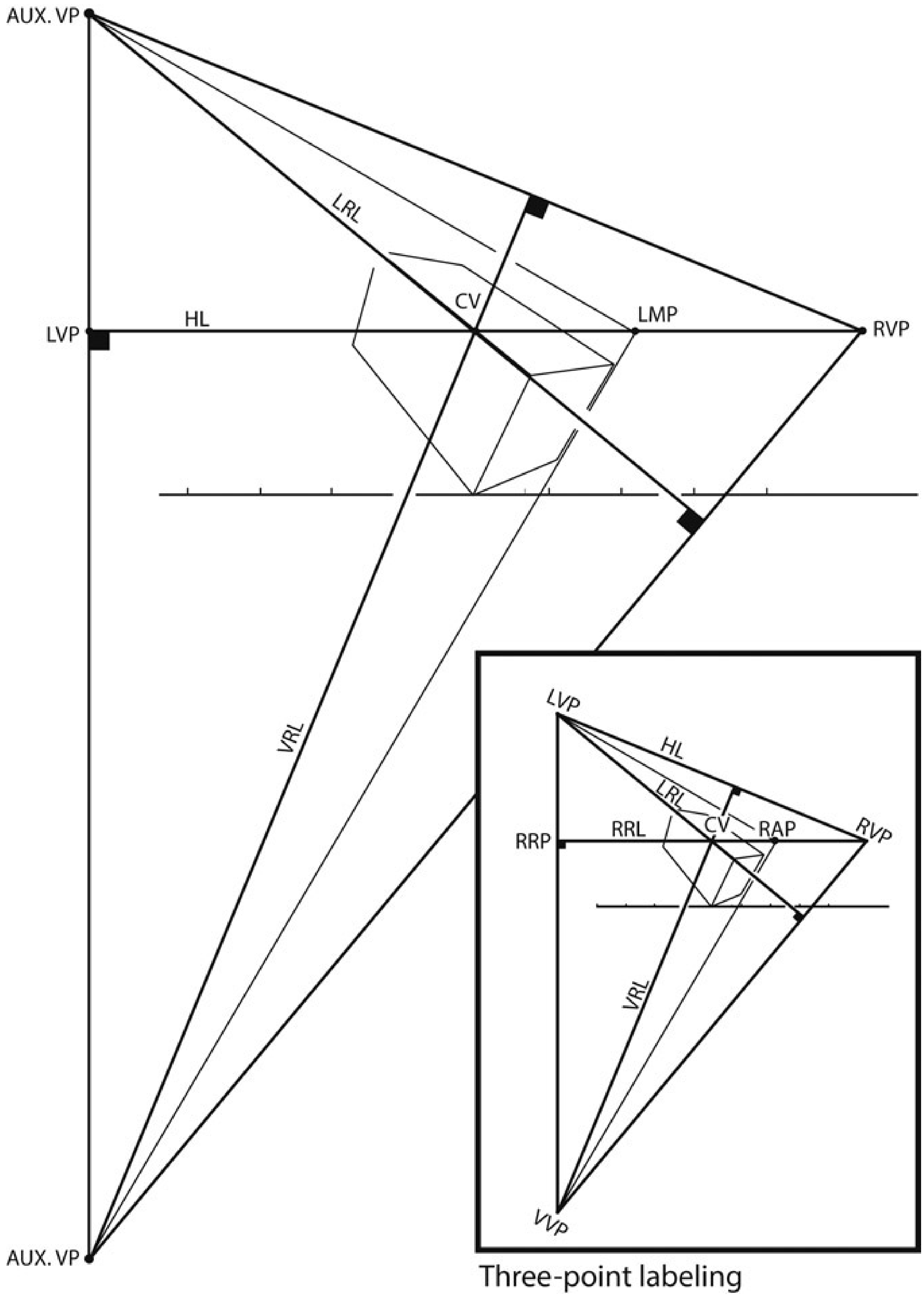
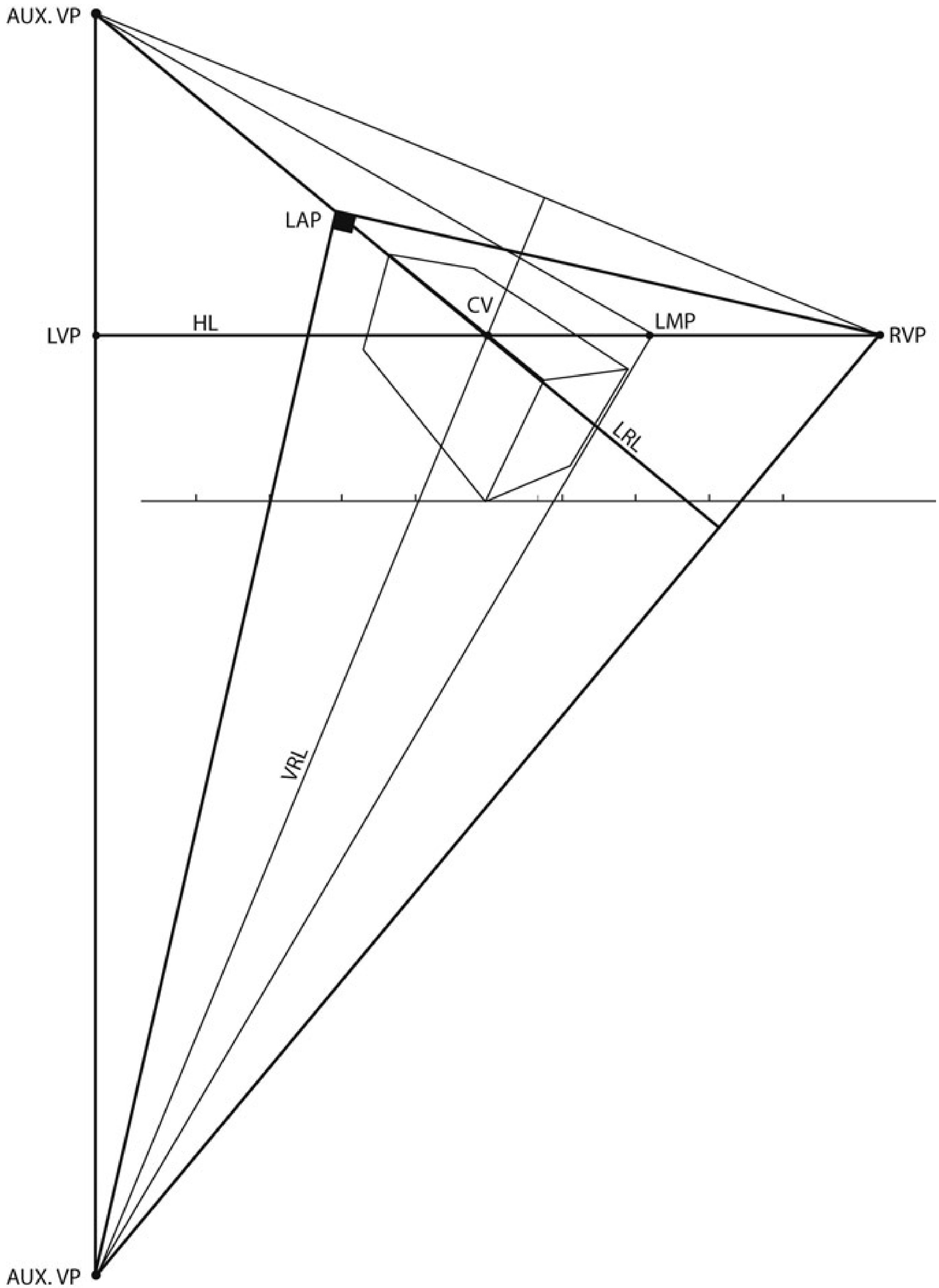


Figure 22.3 **Create three reference lines, each intersecting the center of vision, and each being at a right angle to the lines connecting the three vanishing points. This**

diagram is the same as a three-point diagram. The labels are different, but the geometry is the same.

Axis Point

Since the right and vertical vanishing points change positions, the box rotates along a left axis. Thus, a left axis point is needed. This point is a true 90° between the right and the lower auxiliary vanishing points. Place the left axis point on the left reference line (Figure 22.4).



[Figure 22.4](#) **Establish the left axis point along the left reference line.**

Complete the Box

Create new right and left vanishing points, rotated 30° counterclockwise (Figure 22.5). Plot corresponding measuring points (Figure 22.6). Be sure the measuring line is at the proper angle (Figure 22.7). Measure the box (Figure 22.8) and connect the corners to the vanishing points to complete the drawing (Figure 22.9). Review Chapter 19 for a detailed description of this procedure.

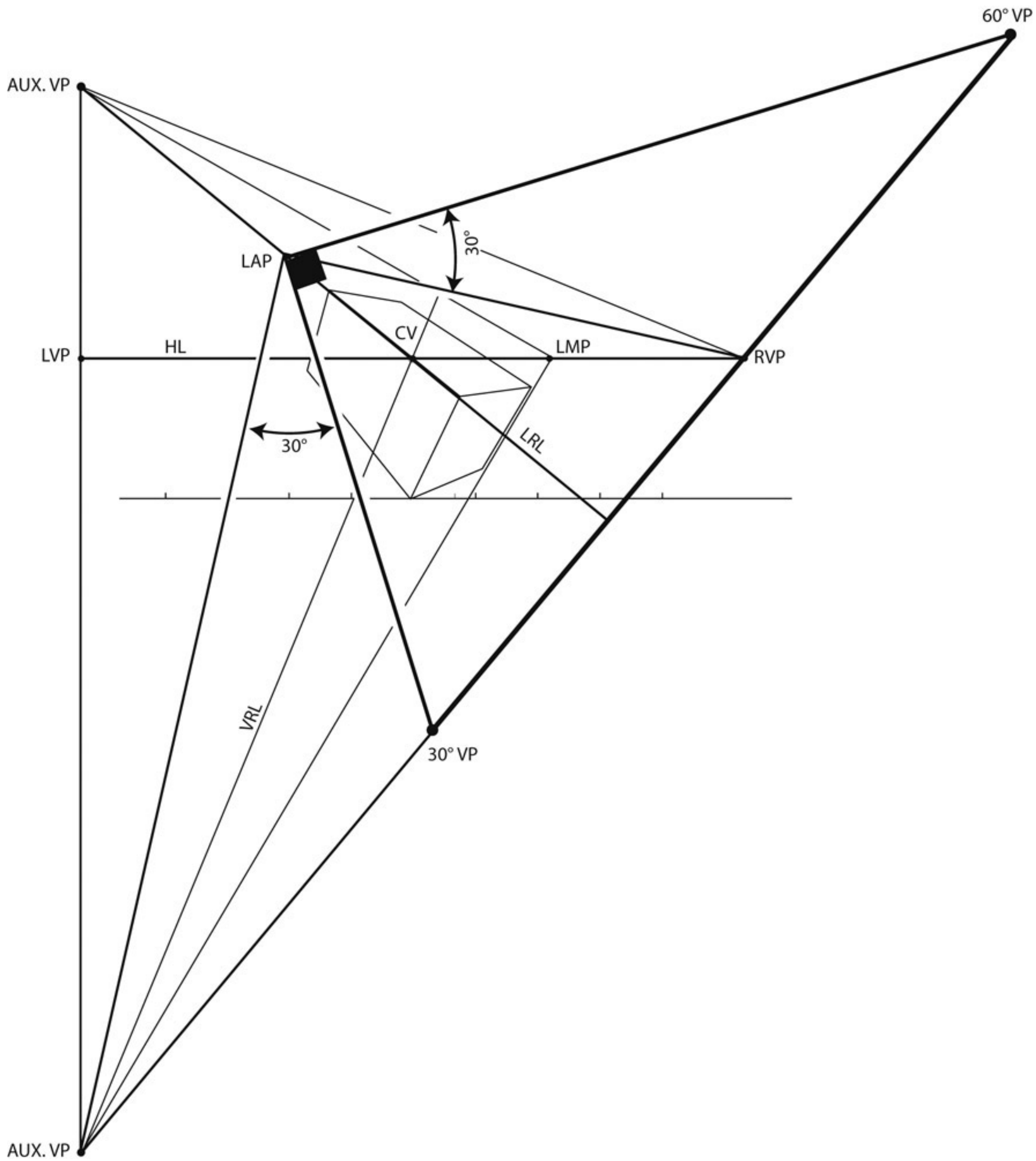


Figure 22.5 Plot two new vanishing points, keeping a 90° angle at the left axis point.

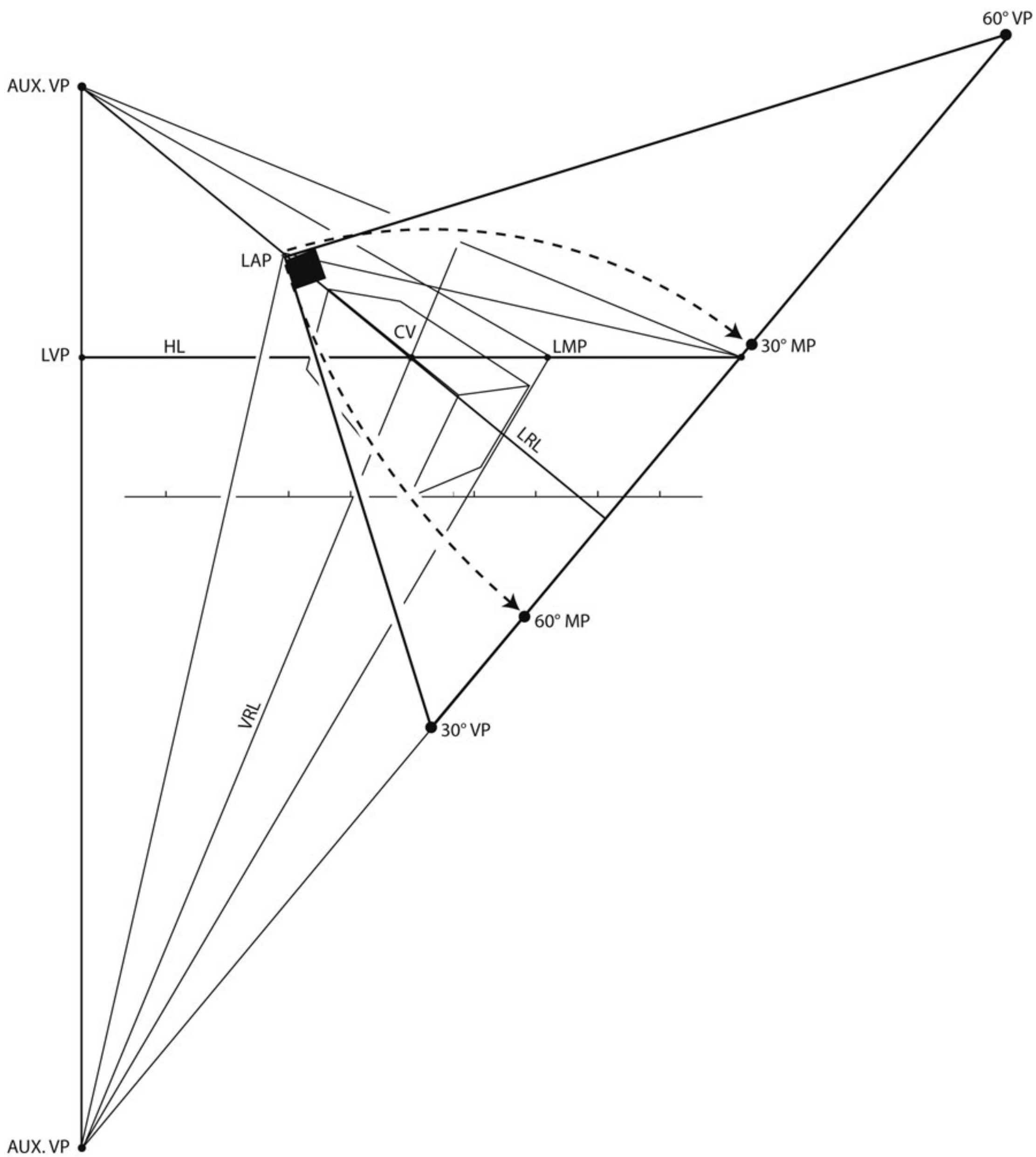


Figure 22.6 Plot measuring points for each of the new vanishing points.

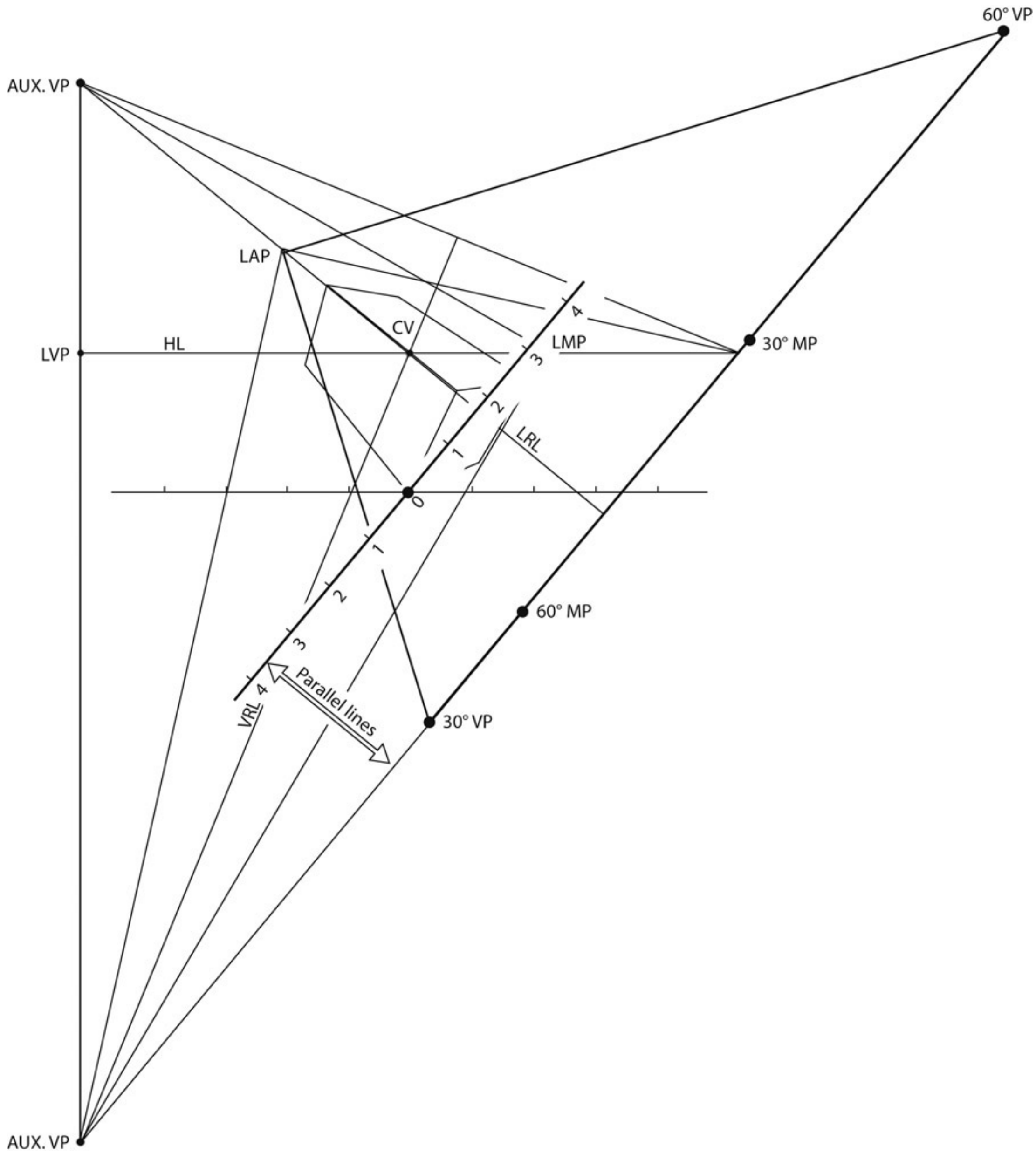


Figure 22.7 The measuring line must be parallel with the line the measuring points are on. Place the measuring line at the corner of the box to ensure it is on the same plane as the line being measured.

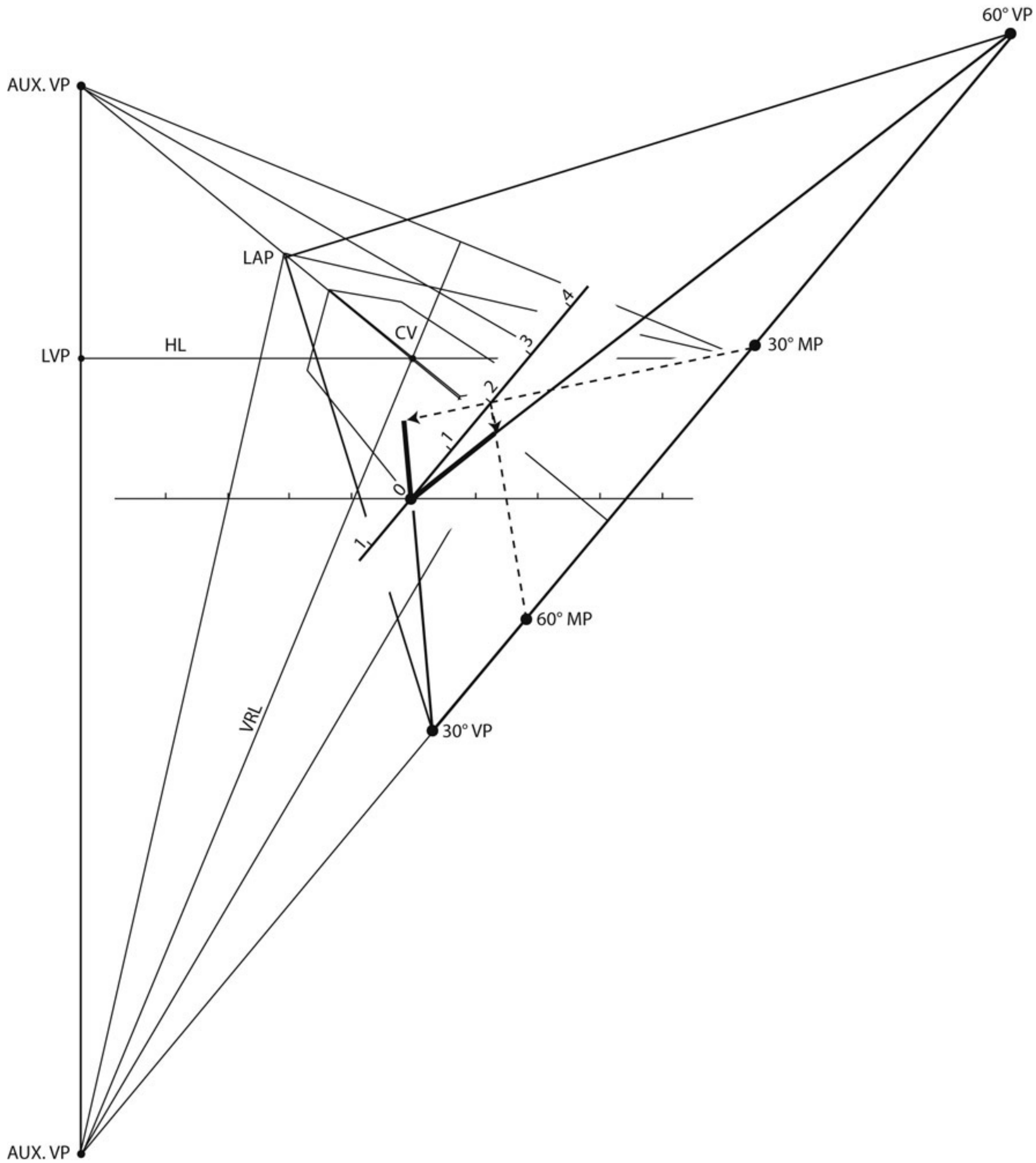
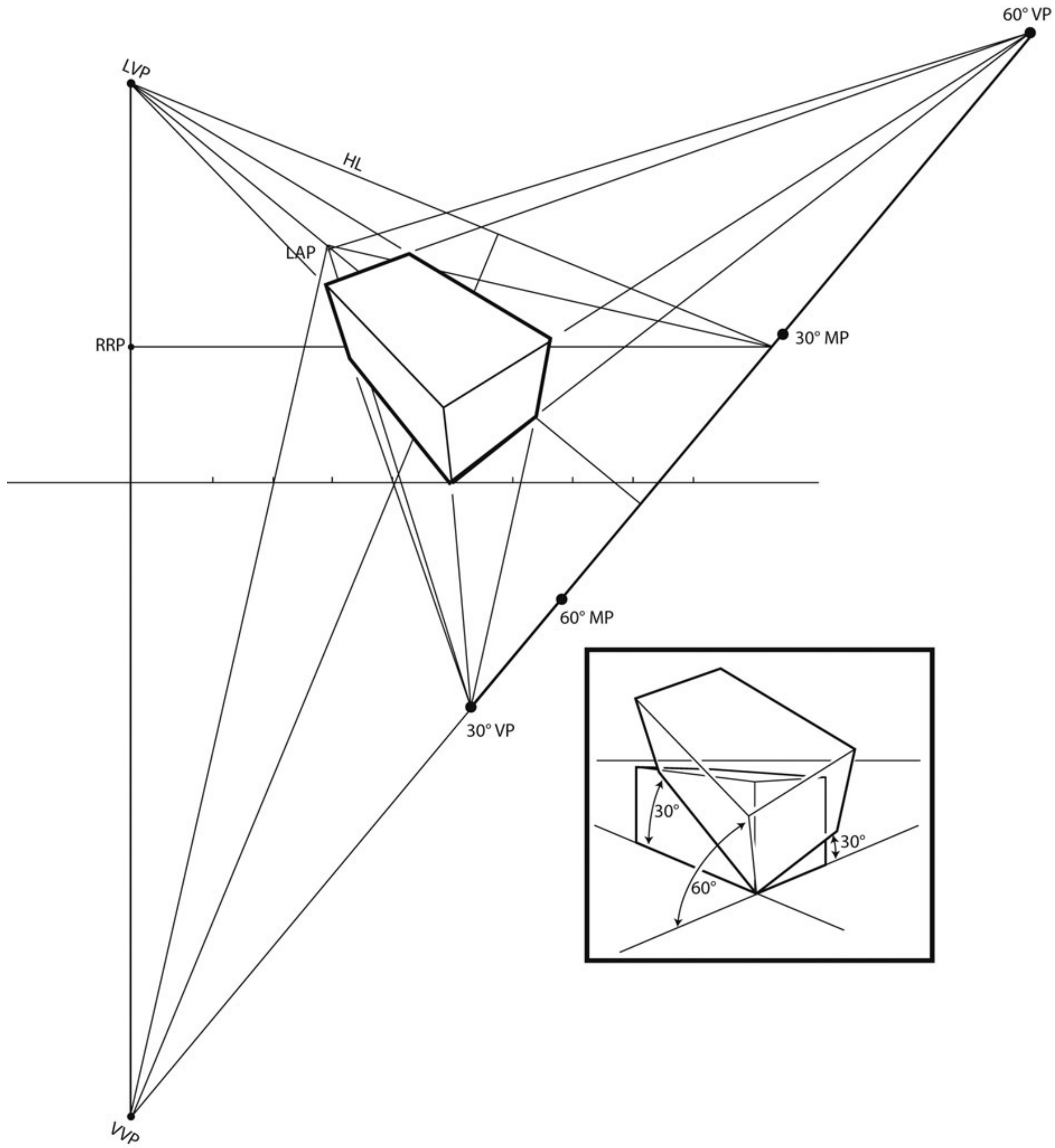


Figure 22.8 Measure the lines connecting to the 30° and 60° vanishing points.



[Figure 22.9](#) **Connect lines to vanishing points to complete the box. This box is tilting up 30° from the ground plane, and rotating 30° counterclockwise.**

23
Shadows

There are two types of light sources: natural and artificial. In the world of perspective, **natural light** emanates from the sun (including that which is reflected by the moon). Any other light source is considered artificial. Fire sources such as candles, and electrical sources such as light bulbs, are all considered artificial.

Shadow Components

Natural Light Shadows

Shadows from natural light can be placed into three categories: positive shadows, **negative shadows**, and parallel shadows. These categories are based on whether the light source is in front, behind, or to the side of the viewer.

Artificial Light

Shadows from **artificial light** sources are called converging shadows (sometimes the terms concentric or radiating shadows are used when discussing artificial light).

Ground Lines and Light Angles

Whether natural or artificial, all shadows are plotted using a ground line (GL) and a **light angle (LA)**. A light angle is the angle of the light ray to the ground plane, and is used to determine the length of the shadow. The ground line is a directional line parallel with the ground plane, and is used to determine the angle of the shadow.

Shadow Rules

There are rules that describe the behavior of shadows. These rules may seem abstract at first, but with practice they become intuitive.

Shadows on Horizontal Surfaces

Shadows on horizontal surfaces follow these rules:

Rule One. Shadows of vertical lines follow the ground line angle.
Rule Two. Shadows of horizontal lines are parallel with the lines casting them.
Rule Three. Shadows of angled lines are found by plotting the line's end points.

Shadows on Vertical Surfaces

When a shadow falls on a vertical surface, these rules must be followed:

Rule Four. Vertical lines cast vertical shadows.
Rule Five. Shadows of horizontal lines are found by plotting the line's end points.
Rule Six. Shadows of angled lines are found by plotting the line's end points.

Shadows on Angled Surfaces

On angled surfaces, there is one rule:

Rule Seven. Shadows on all angled surfaces are found by plotting the line's end points.

Parallel Shadows

Parallel shadows occur when the sun is directly to the right or left of the viewer, 90° from the line of sight. Imagine an arc overhead, beginning at the viewer's left and ending 180° to the viewer's right. If the sun is anywhere along this arc, the light angle and the shadows (the ground line) have no vanishing points, they are parallel with the picture plane (Figure 23.1). The ground line is drawn horizontally, directly to the left or right of the object.

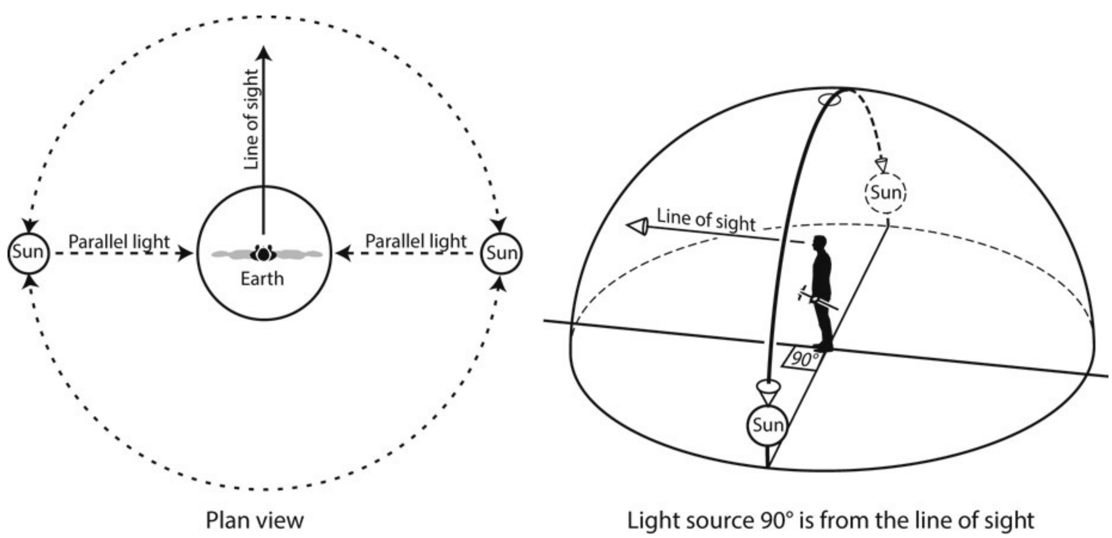


Figure 23.1 When the position of the sun is 90° from the line of sight, the shadows are parallel with the picture plane.

If the sun is closer to the horizon line, the light angle is more oblique, and the shadows are longer. The higher the sun, the steeper the light angle, and the shorter the shadows. The light angle determines the length of the shadow. The light angle—being parallel with the picture plane—is a true angle. For example, if the sun is 45° above the horizon line, all light angles are drawn at a true 45°. If the sun is directly overhead, all light angles are true verticals.

Below are examples of parallel shadows, each following one of the rules listed above.

Rule One

Shadows of vertical lines follow the ground line angle. When the light source is 90° from the line of sight, the ground line is drawn parallel with the picture plane, being a true horizontal line. If the light source is to the right, the shadows are to the left. If the light source is to the left, the shadows are to the right (Figure 23.2).

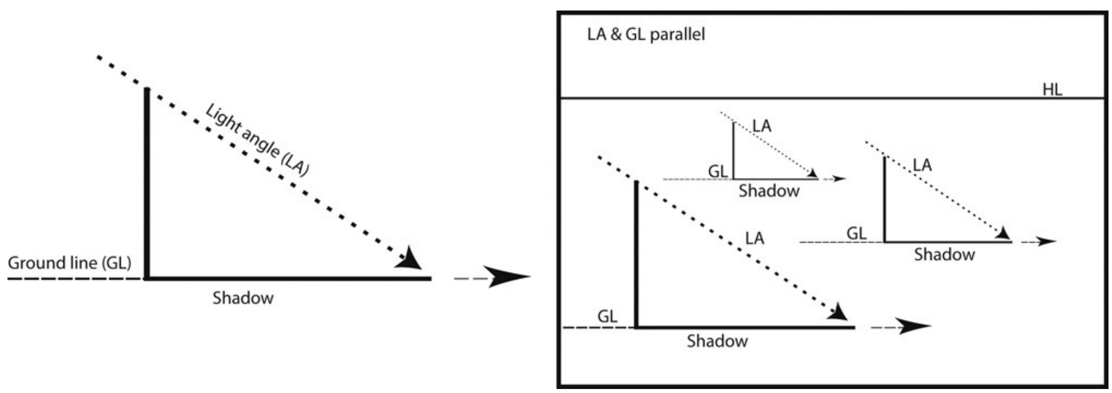


Figure 23.2 Rule One: shadows of vertical lines follow the ground line angle. They are parallel with the horizon line.

Rule Two

The shadows of horizontal lines are parallel with the lines casting them. The shadow, and the line casting the shadow, connect to the same vanishing point. To plot the shadow of a horizontal line, first project a vertical line to the ground. This creates a "flagpole." Draw the shadow of the flagpole using rule number one. After finding the end of the flagpole's shadow, draw a line to the vanishing point. The flagpole is used to find the location of the shadow. The vanishing point is used to establish the angle of the shadow (Figure 23.3).

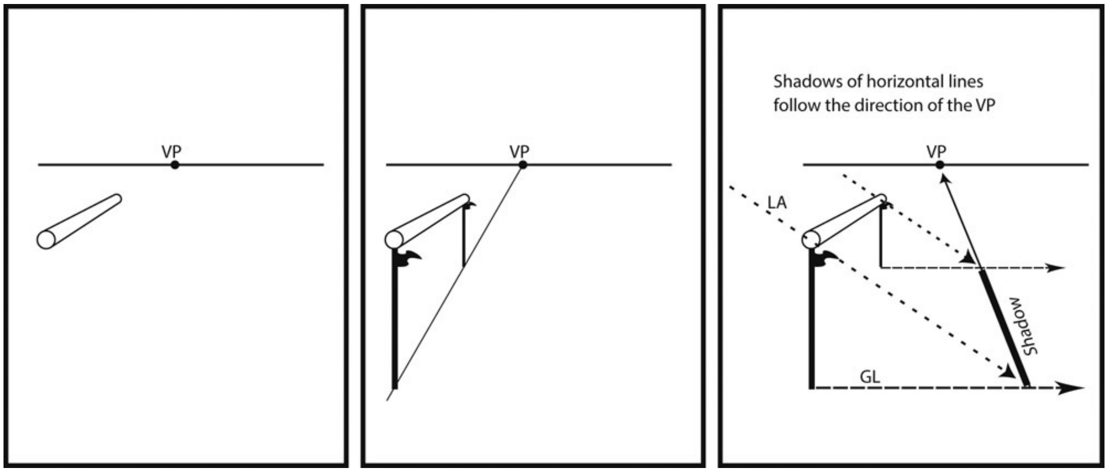


Figure 23.3 Rule Two: shadows of horizontal lines, parallel with the ground, connect to the same vanishing point as the line casting the shadow.

Rule Three

Shadows of angled lines are found by locating end points (finding the beginning and end of the shadow) and then connecting the dots. Angled lines can be difficult. They do not follow the ground line and they do not go to any easily-found vanishing point. Again, use the flagpole technique and rule number one to find the end points (Figures 23.4–23.5).

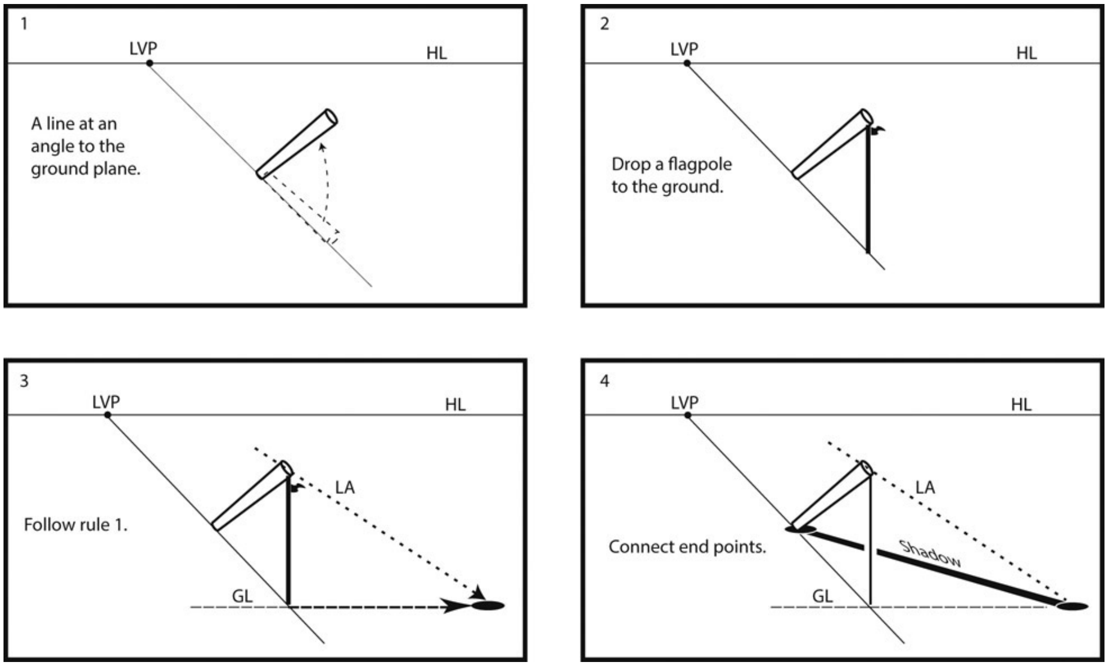


Figure 23.4 Rule Three: plot the end points, and connect the dots to find the shadow of angled lines.

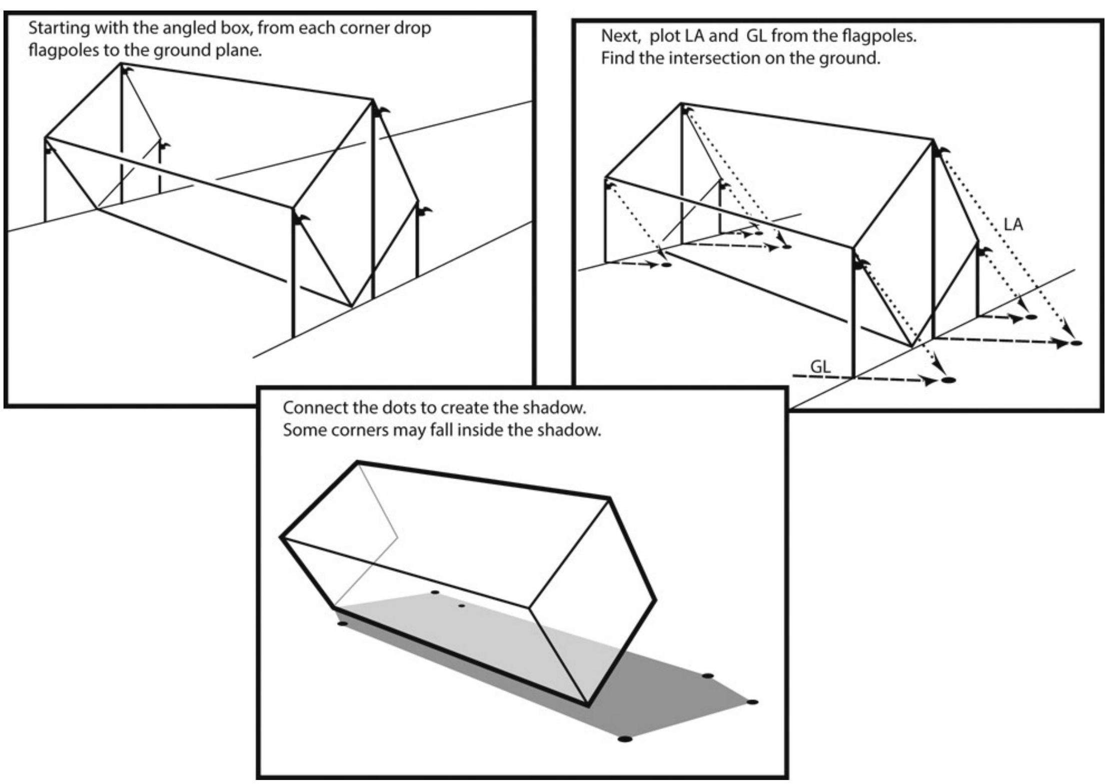


Figure 23.5 **This example uses the flagpole technique to plot the shadow of a cuboid angled to the ground plane**.

Rule Four

Vertical lines cast vertical shadows. The pole and the wall are parallel, so the shadow is parallel with the pole. A vertical line casting a shadow on a vertical wall creates a vertical shadow (Figure 23.6).

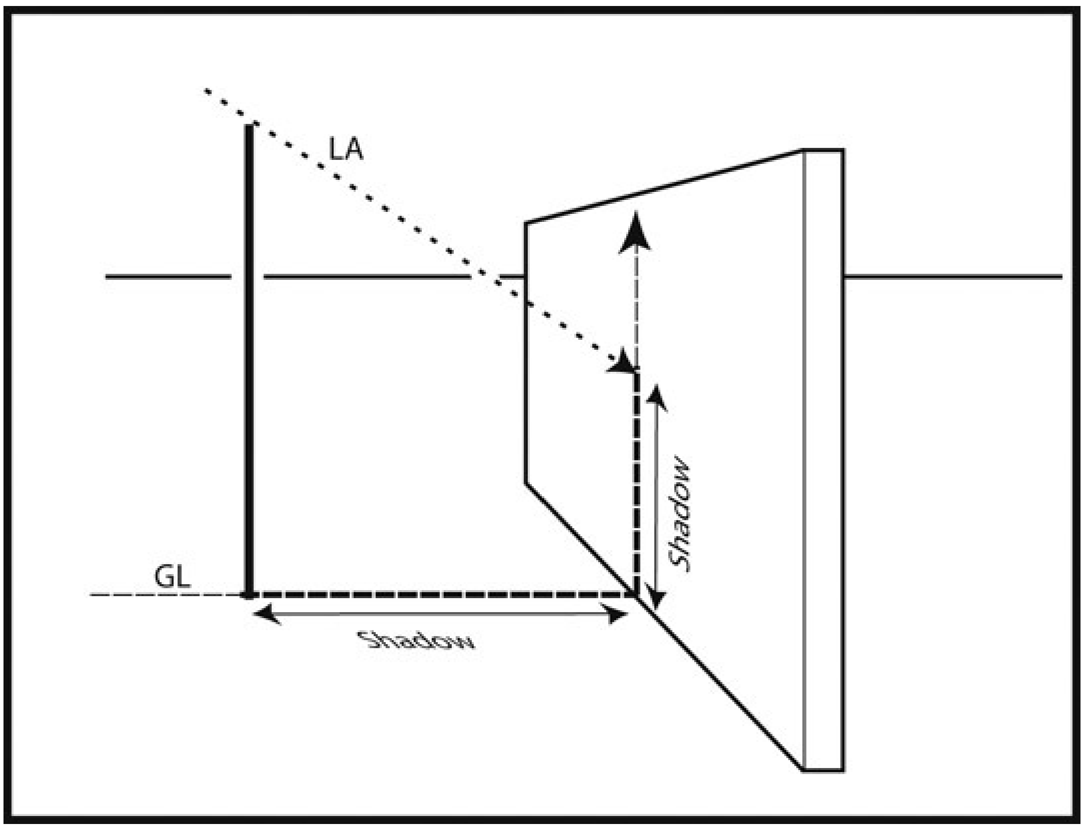


Figure 23.6 **Rule Four: vertical lines cast vertical shadows on vertical surfaces. If the pole and the wall are parallel, the pole and the shadow will also be parallel.**

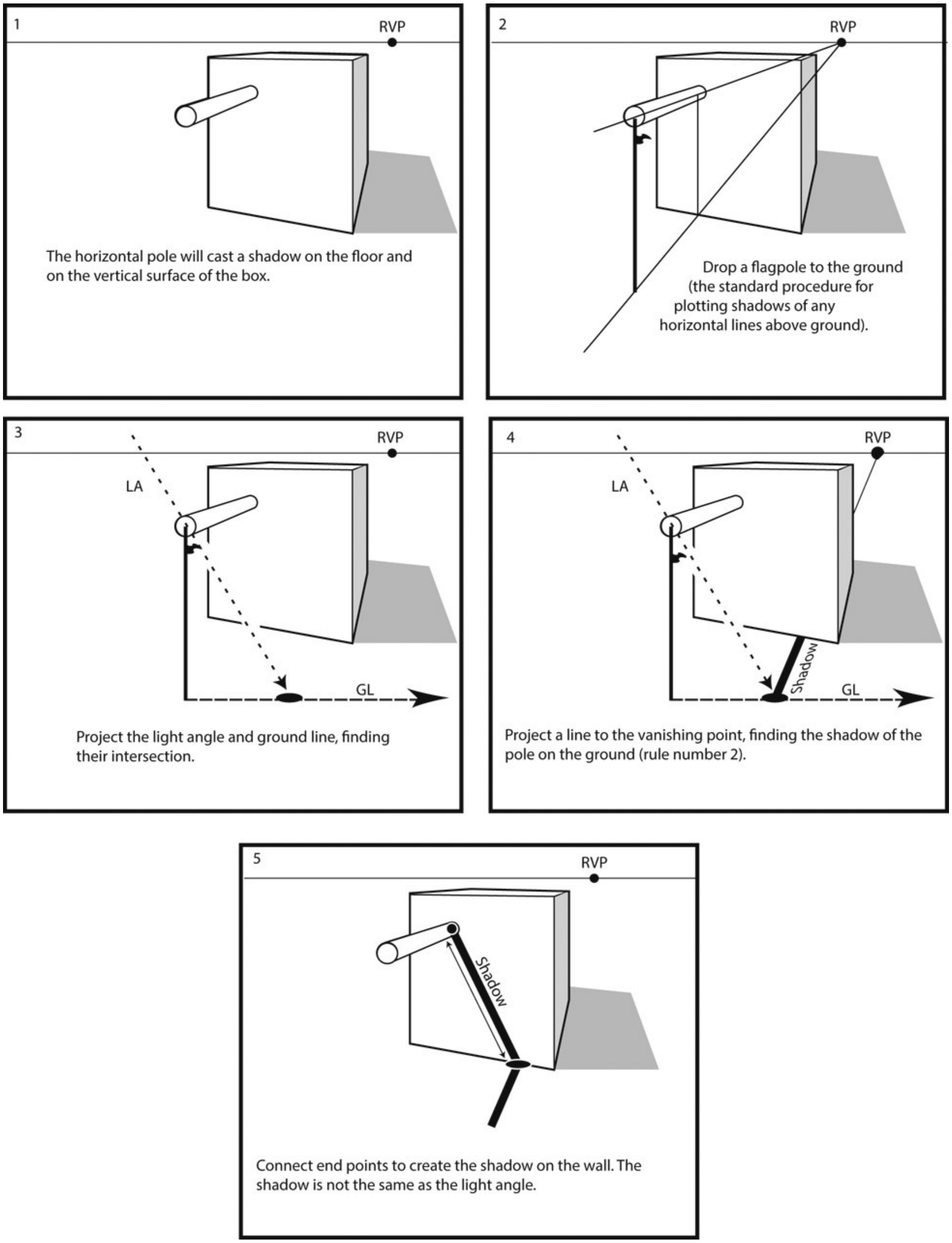


Figure 23.7 The shadow cast from a horizontal line on a vertical surface is found by plotting the end points.

Rule Five

A horizontal line casting a shadow on a vertical surface is a little more challenging than the previous examples. The shadow is typically at an angle, but not necessarily the same as the light angle. To plot this angle, find the shadow of the end points, then connect the dots. Finding the end points can be straightforward (Figure 23.7) or a little more difficult (Figure 23.8). Sometimes these shadows fall across several surfaces. Finding the end points in these situations can be very difficult indeed, as there are many possible scenarios.

Begin by plotting the shadow of a horizontal line that falls across two surfaces (a horizontal and a vertical surface). First, use rule number two to find the horizontal shadow. Then locate the intersection of the horizontal shadow and the vertical surface. Connect the end points (Figure 23.7).

If the shadow does not hit the vertical surface, then there is no intersection at the wall. With no intersection, how is the second end point found? There are two solutions to this problem: extend the wall to the shadow (Figure 23.8), or project the shadow to the wall (Figure 23.9).

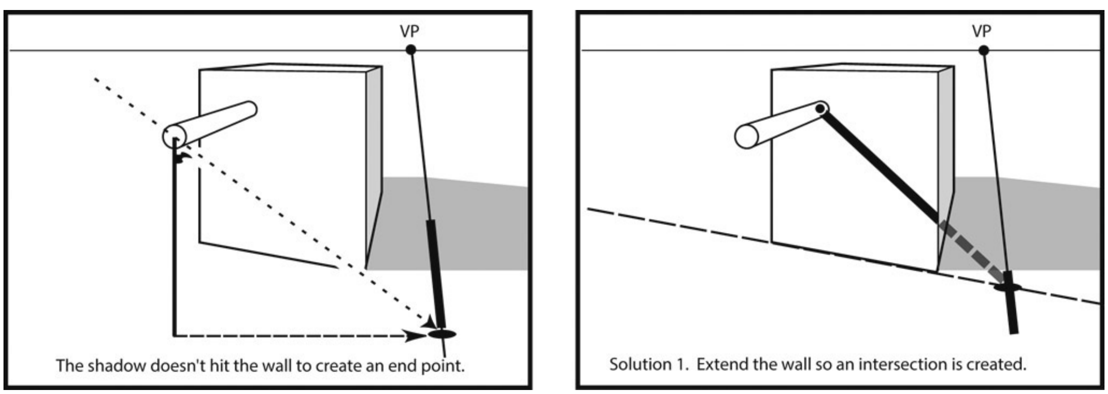


Figure 23.8 Solution 1. If the shadow does not intersect the box, the box can be extended. Draw a phantom wall long enough to create an intersection.

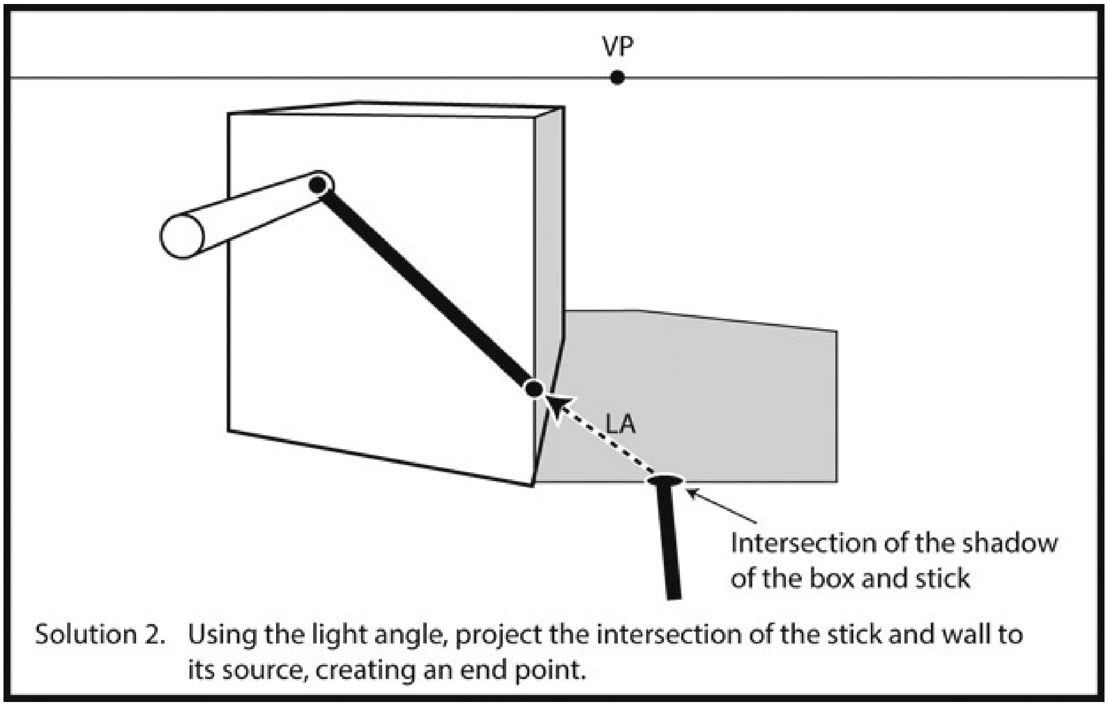


Figure 23.9 **Solution 2. Draw the shadow of the box and pole. Find the intersection of the two shadows. Using the light angle, project the intersection of the shadows on the ground, to the side of the box, creating an end point.**

Rule Six

Angled lines are found by plotting the end points. Draw flagpoles from the end points, plot the shadow of the flagpoles, and connect the end points (Figure 23.10).

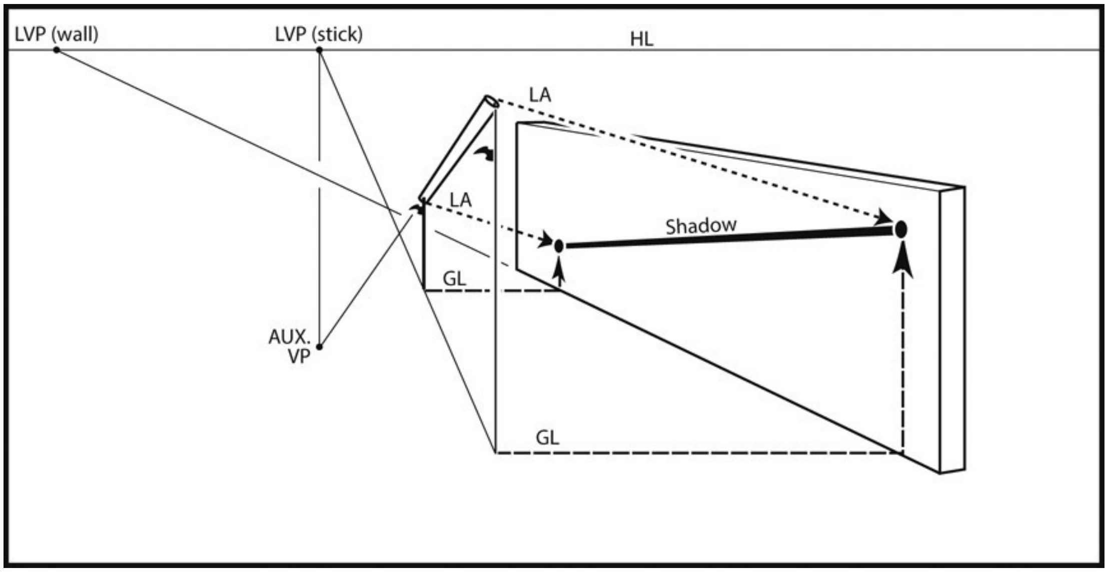


Figure 23.10 Rule Six: shadows of angled lines (this line is angled to both the ground and the wall) are found by plotting, and then connecting, the end points.

Rule Seven

Plotting shadows on angled surfaces is no different than plotting shadows of angled lines. Find, and then connect, the end points. Anything that is not parallel or perpendicular to the ground plane will require plotting end points. Whether it is the line casting the shadow, or the surface the shadow is cast on, end points will need to be found (Figure 23.11).

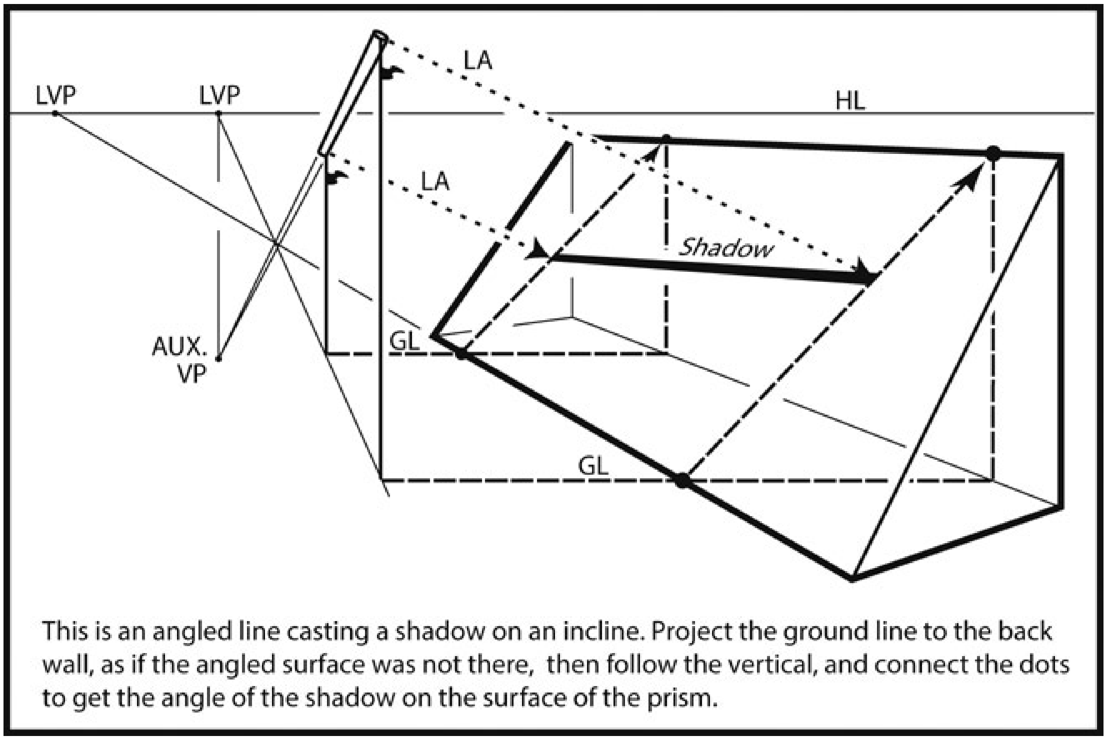


Figure 23.11 Rule Seven: shadows on angled surfaces are found by plotting and then connecting the end points.

24
Shadows of Round, Spherical, and Curved Objects

Shadows of round objects, or any curved object, are found by placing flagpoles at various points along the curve (Figure 24.1), plotting shadows for each flagpole (Figure 24.2), then connecting the dots to create the shadow (Figure 24.3). All curved lines, whether vertical, horizontal, or angled use the same procedure. More challenging shapes are created when shadows fall across multiple surfaces. Find and connect end points to resolve the shape (Figure 24.4).

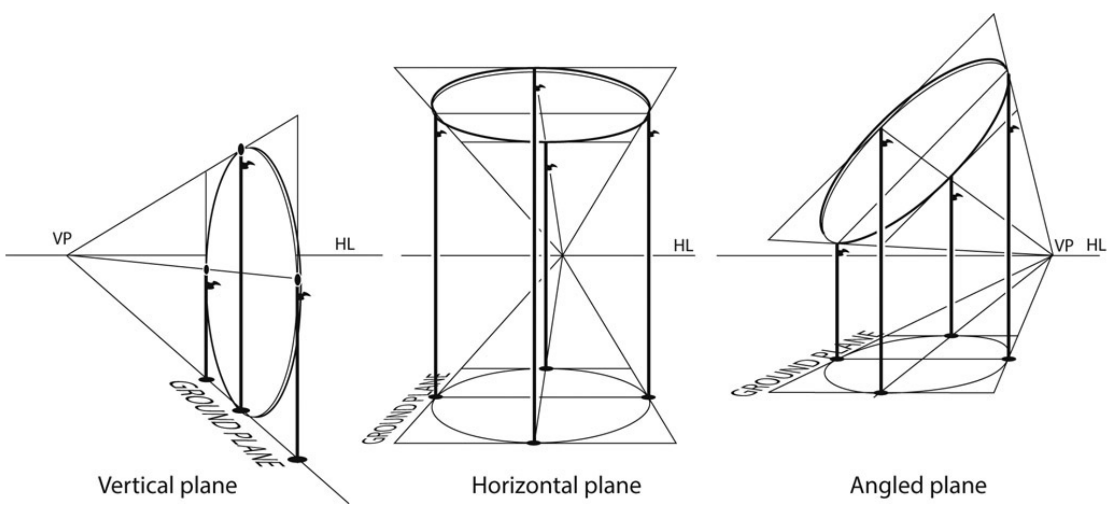


Figure 24.1 **Draw flagpoles from points along the curve, stopping at the ground plane. The more flagpoles that are drawn, the more accurate the shadow will be.**

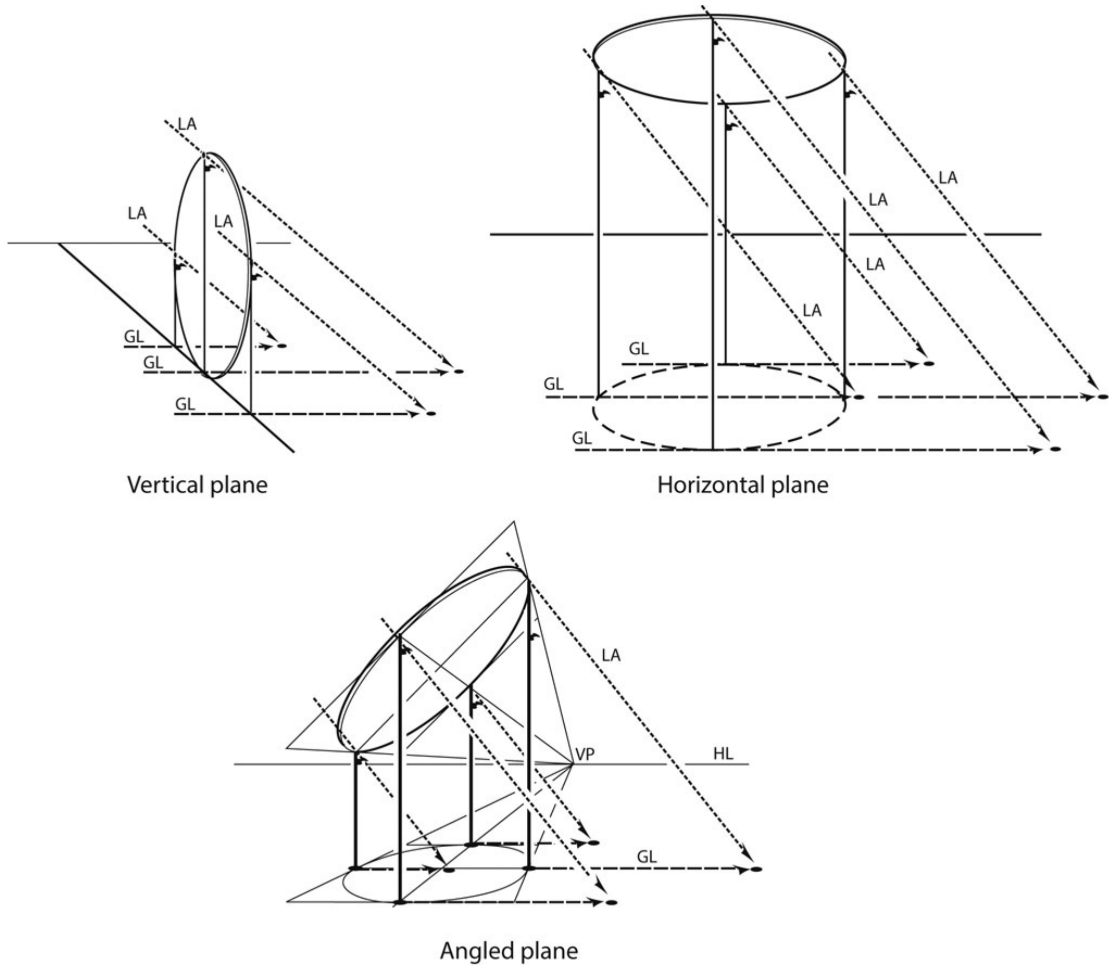


Figure 24.2 Plot the intersection of the light angle and the ground line. Plot as many points as necessary to create an accurate shadow.

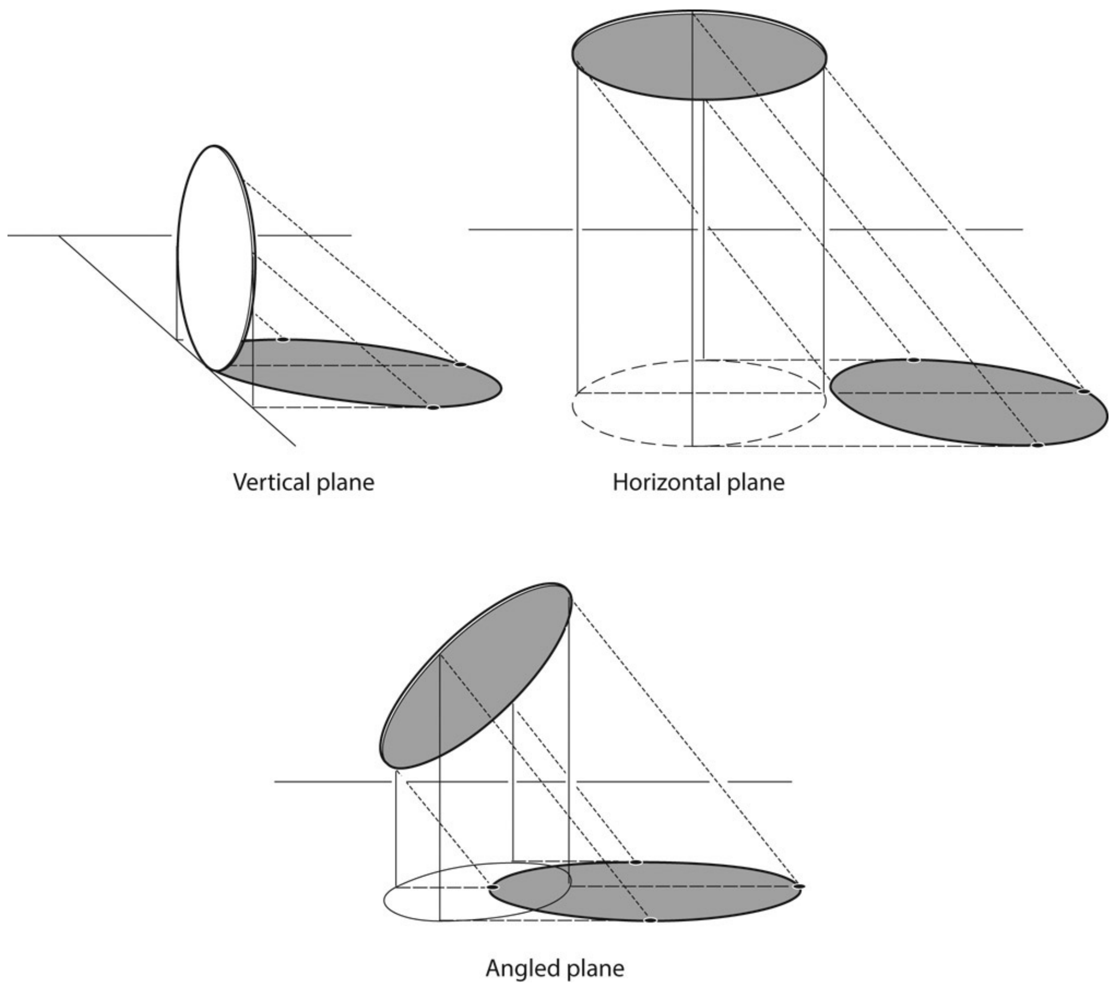
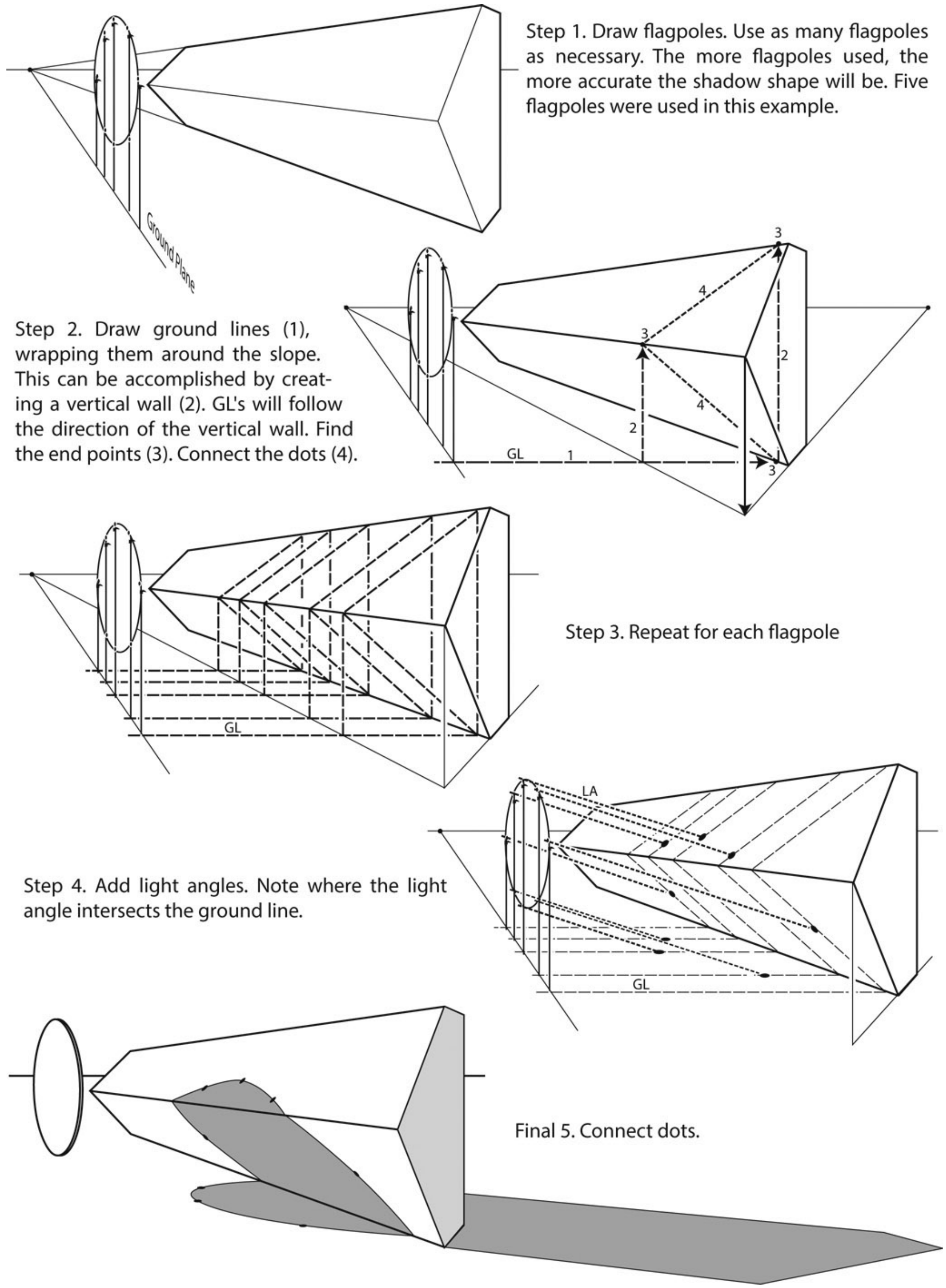


Figure 24.3 **Connect the dots from the intersection of the light angle and ground line to create the shadow.**



[Figure 24.4](#) **Plotting shadows that fall across multiple surfaces can be challenging**.

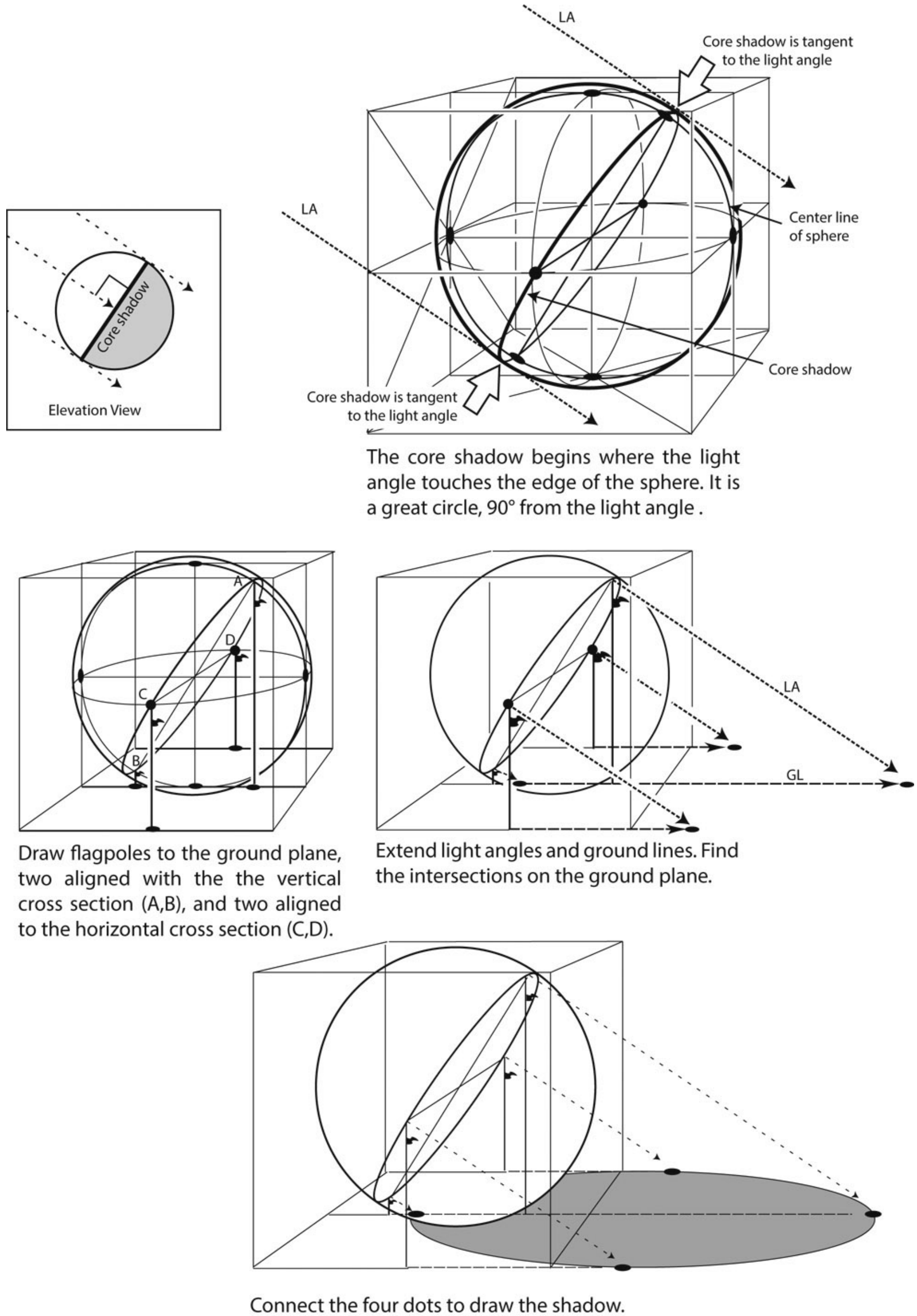


Figure 24.5 Plotting the shadow of a sphere is the same as plotting the shadow of an ellipse.

Shadows of Spheres

It may be useful to review the chapter on drawing spheres, as it is helpful to know how to draw a sphere before drawing its shadow. Because light angles originate from a point at infinity when natural light is used, the core shadow on a sphere is a **great circle**. Light angles touch the sphere at its widest point (Figure 24.5, top). With artificial light, the core shadow becomes progressively smaller as the light angle moves closer to the sphere. The sphere's core shadow is tangent to the intersection of the light angle and the edge of the sphere.

The cast shadow of a sphere is plotted by drawing flagpoles along the core shadow (Figure 24.5, middle left). Plot the shadow of each flagpole (Figure 24.5, middle right). Then connect the dots (Figure 24.5, bottom). Plotting the shadow of a sphere is the same as plotting the shadow of an ellipse, as the core shadow is elliptical.

Shadows on Spheres

The shadow of a straight line, cast on the surface of a sphere, is always circular. It follows the shape of the sphere. **Parallel light** creates cast shadows parallel with the picture plane. These shadows are a true half circle, stopping at the core shadow (Figure 24.6). Shadows not parallel with the picture plane strike the sphere at an angle and are elliptical.

One of the easiest ways to locate a cast shadow falling across a sphere is to find the intersection of the cast shadow and the core shadow on the ground plane (Figure 24.6, A). Then, using the light angle, project the intersection back to the sphere (Figure 24.6, B).

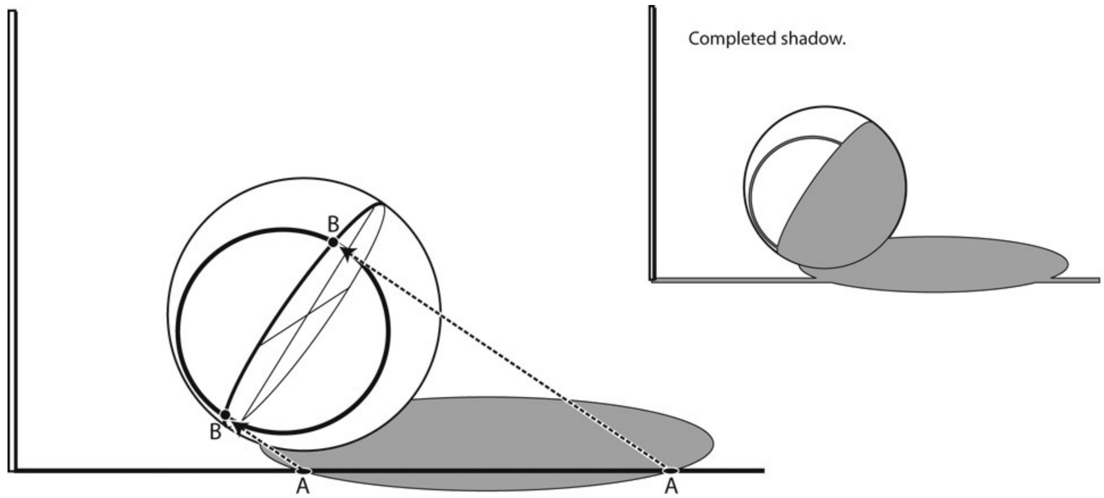
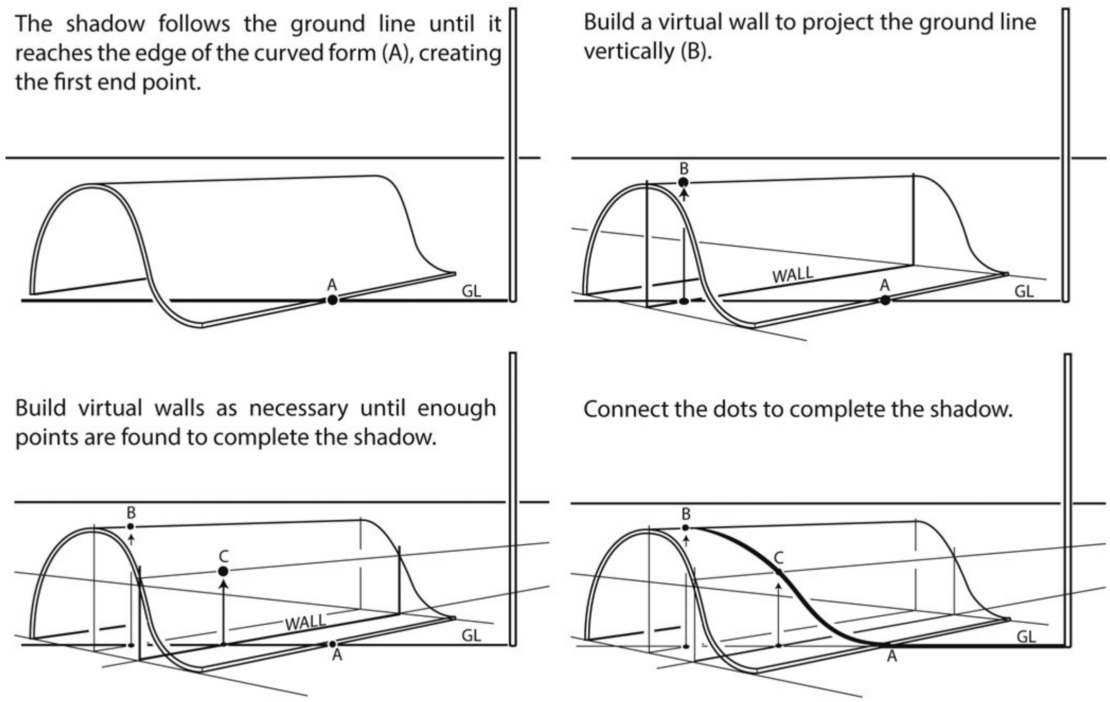


Figure 24.6 **A cast shadow follows the shape of the sphere.**

Shadows on Curved Surfaces

Plotting shadows that fall across curved surfaces requires the same creative and analytical problem-solving skills. As with all curved forms, end points are plotted and then connected. The more points that are plotted, the more accurate the shadow. To find these points, it is often helpful to draw the shadow first on a flat plane, then wrap the shadow around the curved surface by finding intersections. This can require some clever maneuvering (Figure 24.7).



[Figure 24.7](#) **The shadow was first drawn on the floor then projected to the curved surface.**

25
Positive Shadows

When the sun is in front of the viewer, objects are backlit and shadows are angled forward, away from the horizon line. As the position of the sun moves farther to the left or right, the shadows become more horizontal, closer to the angle of the horizon line, and closer to being parallel. Positive shadows appear when the light source is anywhere within 90° of the center of vision (Figure 25.1).

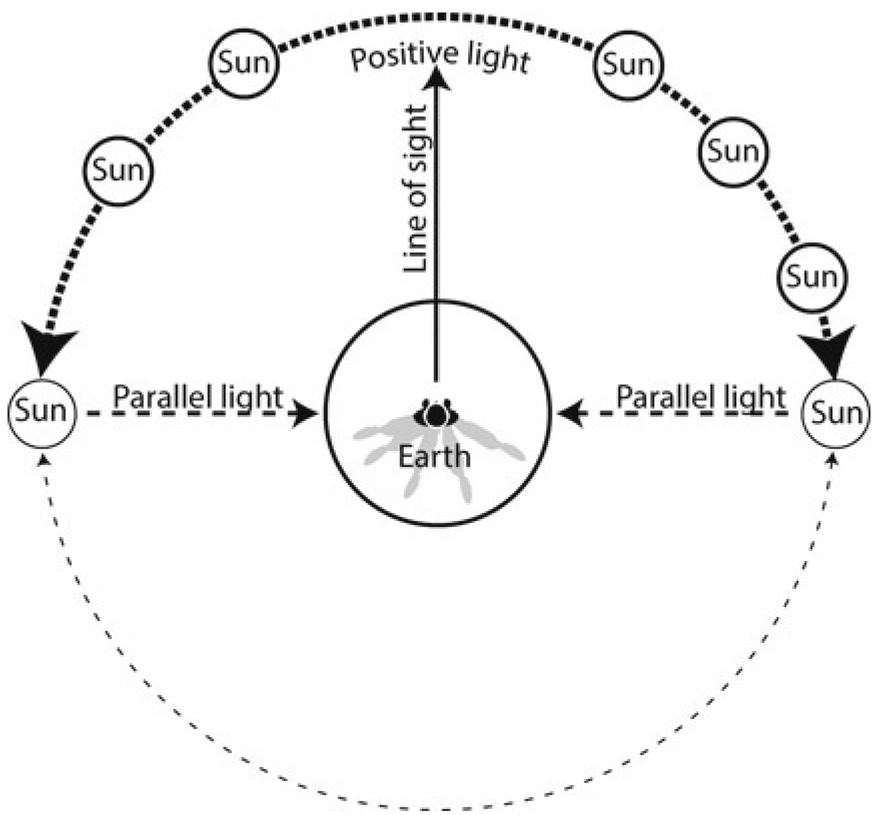


Figure 25.1 Positive light is from a natural light source in front of the viewer.

Positive Shadow Components

Ground Line Vanishing Point

The ground line is now foreshortened and has a vanishing point. The ground line vanishing point (GLVP) is always placed on the horizon line (Figures 25.2–25.3).

Light Angle Vanishing Point

The light angle vanishing point (LAVP) represents the light source. It is aligned with and placed directly above the ground line vanishing point (Figures 25.2–25.3).

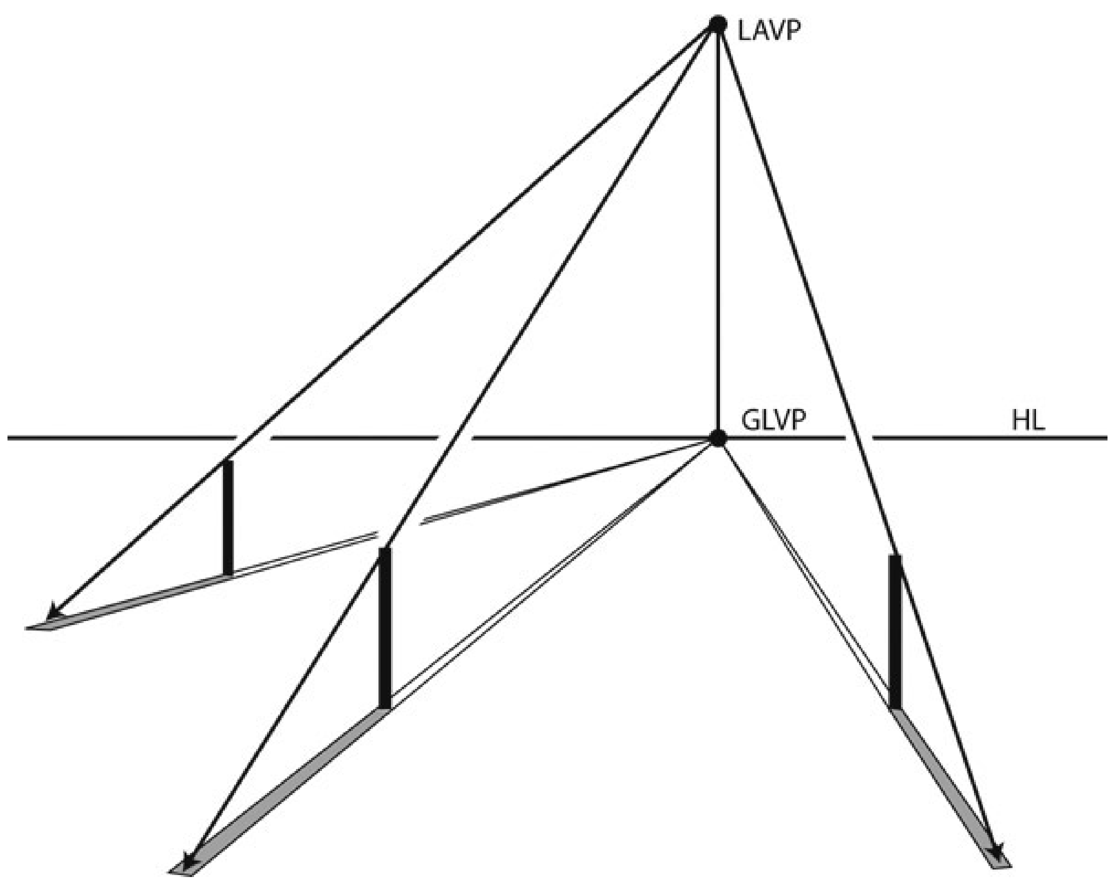


Figure 25.2 Align the light angle vanishing point with the ground line vanishing point.

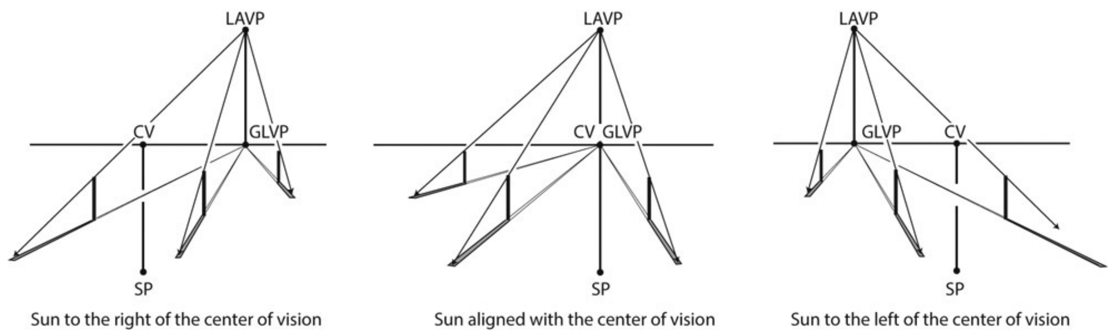


Figure 25.3 The ground line vanishing point can be located anywhere along the horizon line.

Angle of Shadow

The location of the ground line vanishing point determines the angle of the shadow. Approach the ground line vanishing point like any other vanishing point. A shadow is nothing more than a horizontal line. Drawing a specific ground line angle is the same as drawing any horizontal angle. For example, if the sun is 30° to the right of the center of vision, make a 30° angle from the station point to the horizon line. This ground line vanishing point will draw shadows at a 30° angle to the line of sight, or 60° from horizontal (Figure 25.4).

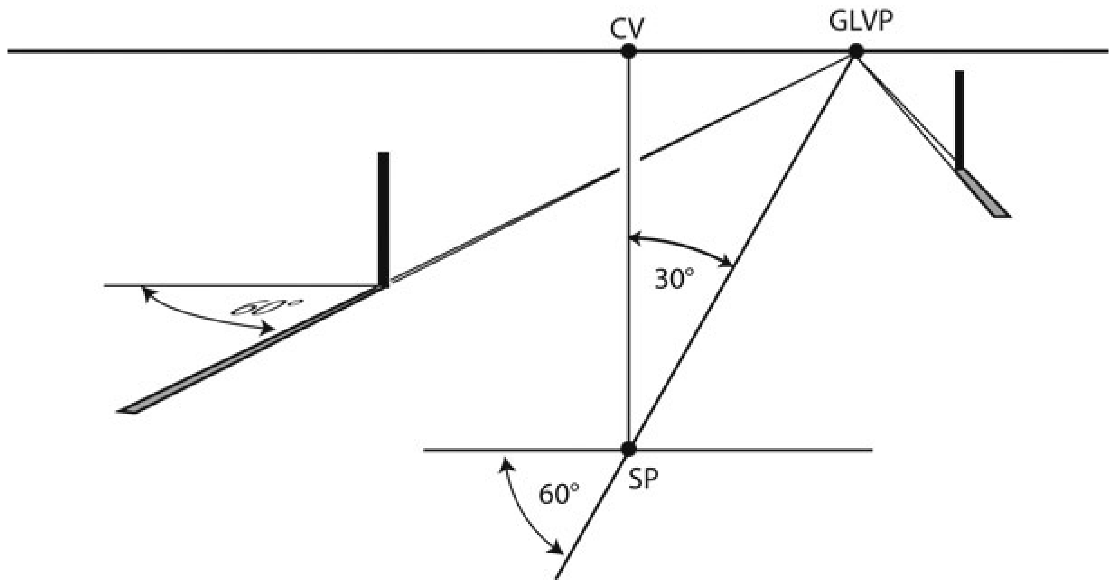


Figure 25.4 Create specific ground line angles by projecting the desired angle from the station point. In this example, the sun is 30° to the right of the center of vision. This creates shadows 60° from horizontal.

Angle of Light

The location of the light angle vanishing point determines the length of the shadow. A light angle vanishing point closer to the horizon line produces a longer shadow. A light angle vanishing point farther from the horizon line produces a shorter shadow. To illustrate a specific time of day, the light angle vanishing point must be properly positioned. Early morning or late evening light requires an oblique light angle, while mid-afternoon suggests a steeper light angle. Drawing a specific light angle is the same as drawing any incline. The light angle vanishing point can be treated like any auxiliary vanishing point. For example, if the sun is 45° above the horizon line, first make a ground line measuring point (true angles for inclines are found at measuring points). Then draw a 45° angle, intersecting a point directly above the ground line vanishing point (Figure 25.5). Review Chapter 9 for more information about inclines.

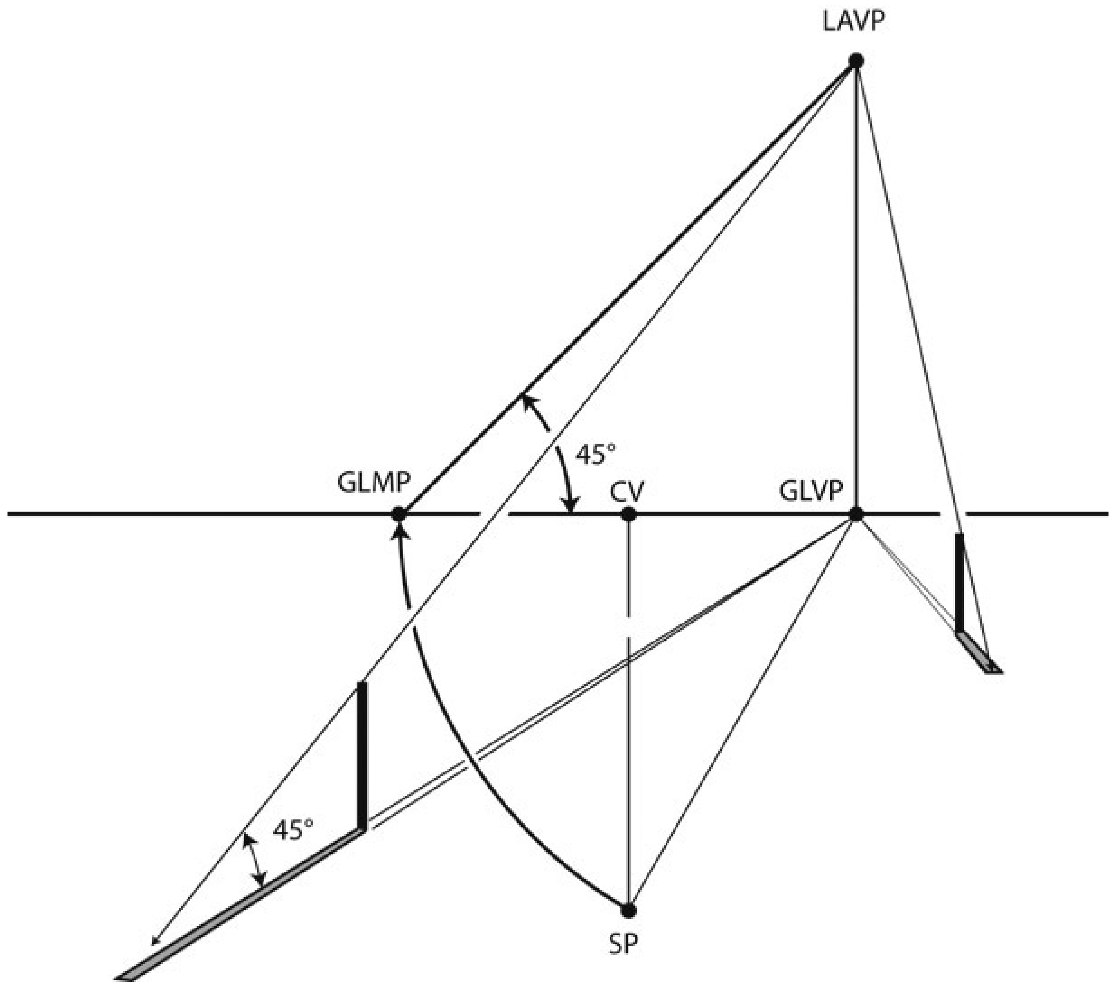


Figure 25.5 **To create a specific light angle, first plot a ground line measuring point (GLMP). True angles drawn from the ground line measuring point create a light angle vanishing point, which draws those same angles in perspective.**

26
Negative Shadows

Negative shadows appear when the sun is behind the viewer. Shadows are angled backward, toward the horizon line (Figure 26.1).

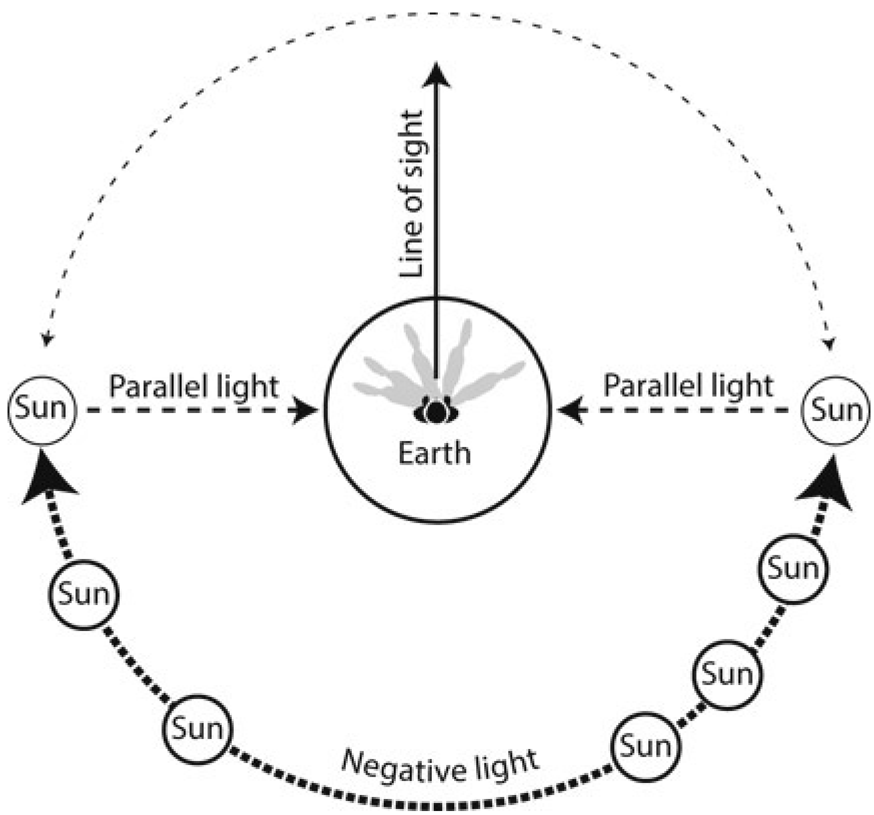


Figure 26.1 Negative shadows occur when the light source is behind the viewer.

Negative shadows create an interesting problem. Only things in front of the viewer can be drawn. Points that are behind the viewer's head can't be drawn on the paper (the exception being curvilinear perspective). So how can the light source be represented when the light source is behind the viewer? Using some creative thinking and an understanding of geometry—mainly an understanding of geometry—can unlock this problem.

Negative shadows are caused by light rays coming over the viewer's shoulder. These rays are at a specific angle to the ground. Imagine the light rays continuing underground, beyond the ground plane, continuing to infinity, and leading to a vanishing point below the horizon line. This is the light angle vanishing point. This vanishing point creates **congruent** angles to the light source. In this scenario, the light angle vanishing point is not the light source itself but a point below, which draws the same angle as the light source (Figure 26.2).

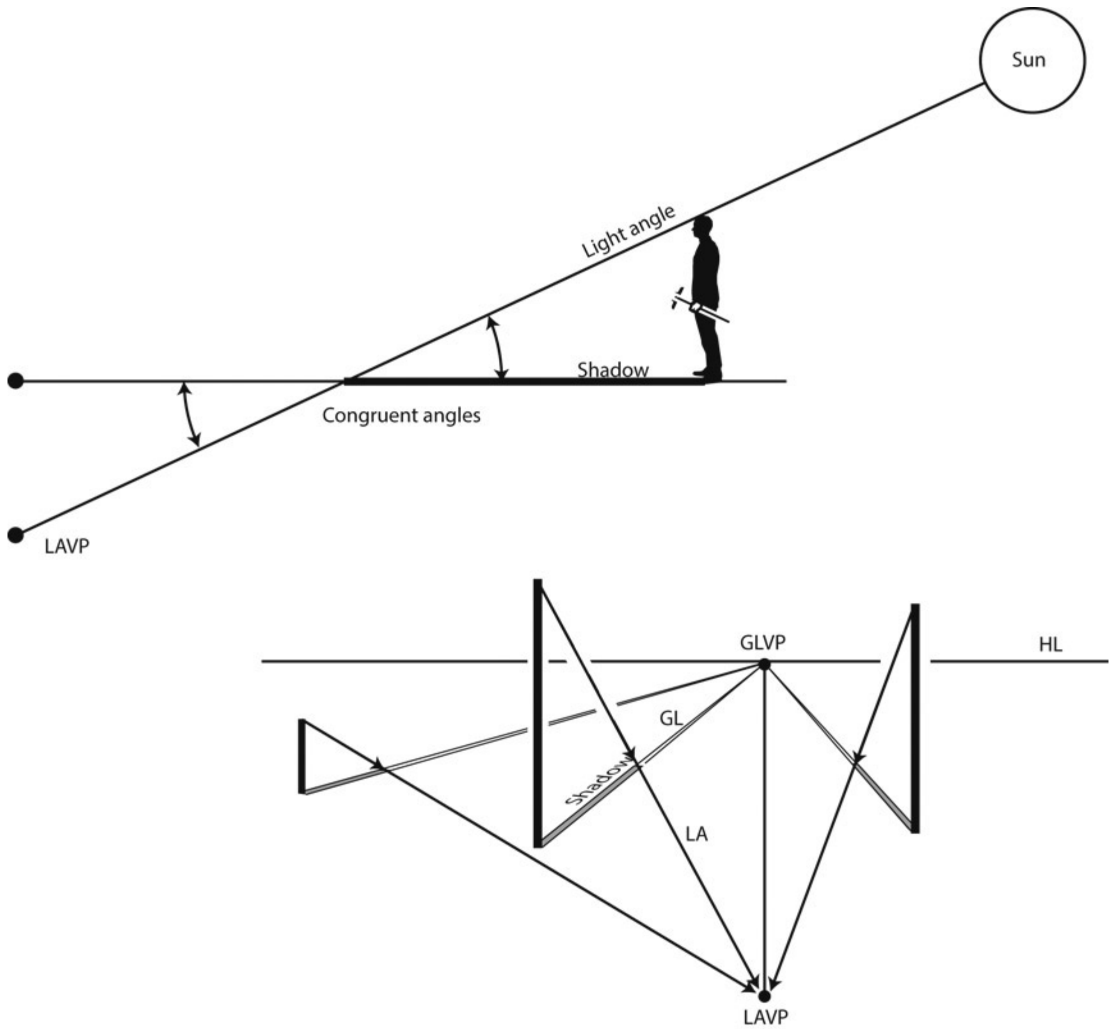


Figure 26.2 Negative shadows require the light angle vanishing point to be below the horizon line, creating congruent angles to the light source located above and behind the viewer.

Creating Negative Shadows

Angle of the Shadow

Use the station point to determine the angle of the shadow (this is the same technique used when drawing positive shadows). Angles projected from the station point create vanishing points that draw the same angle in perspective (Figure 26.3).

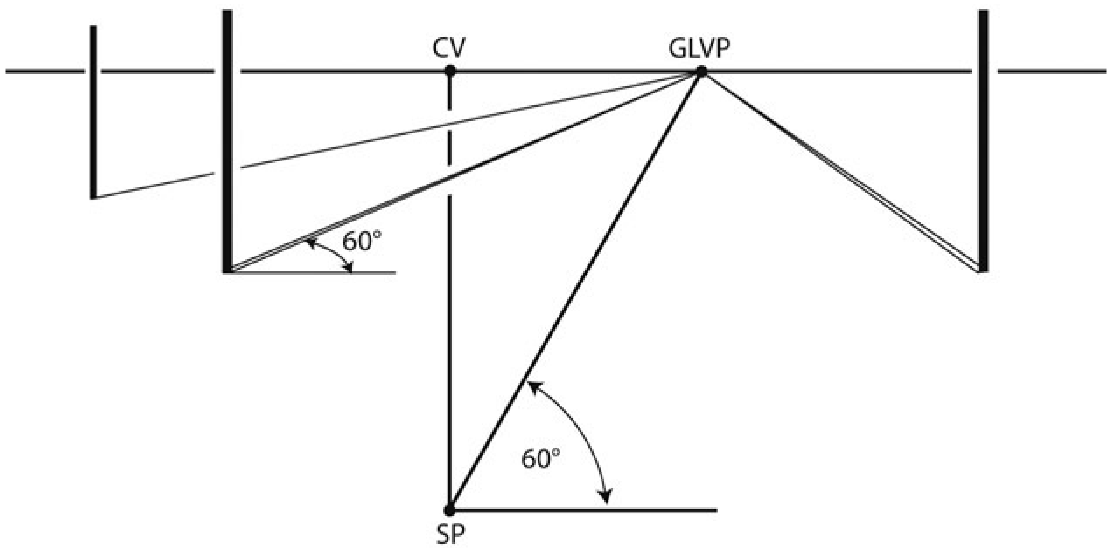
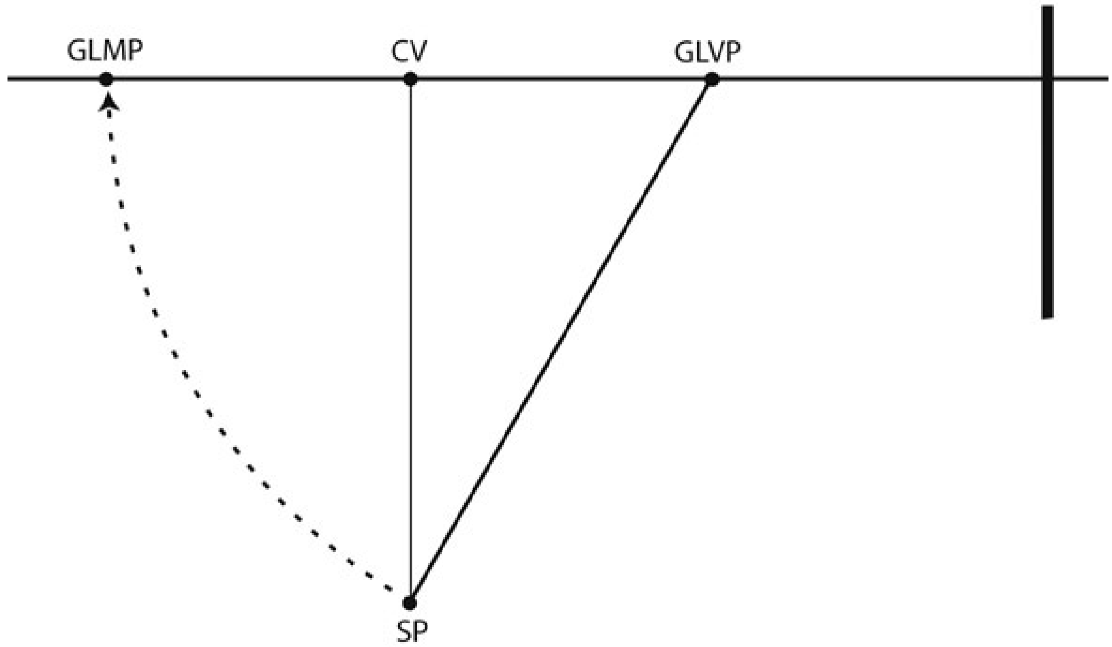


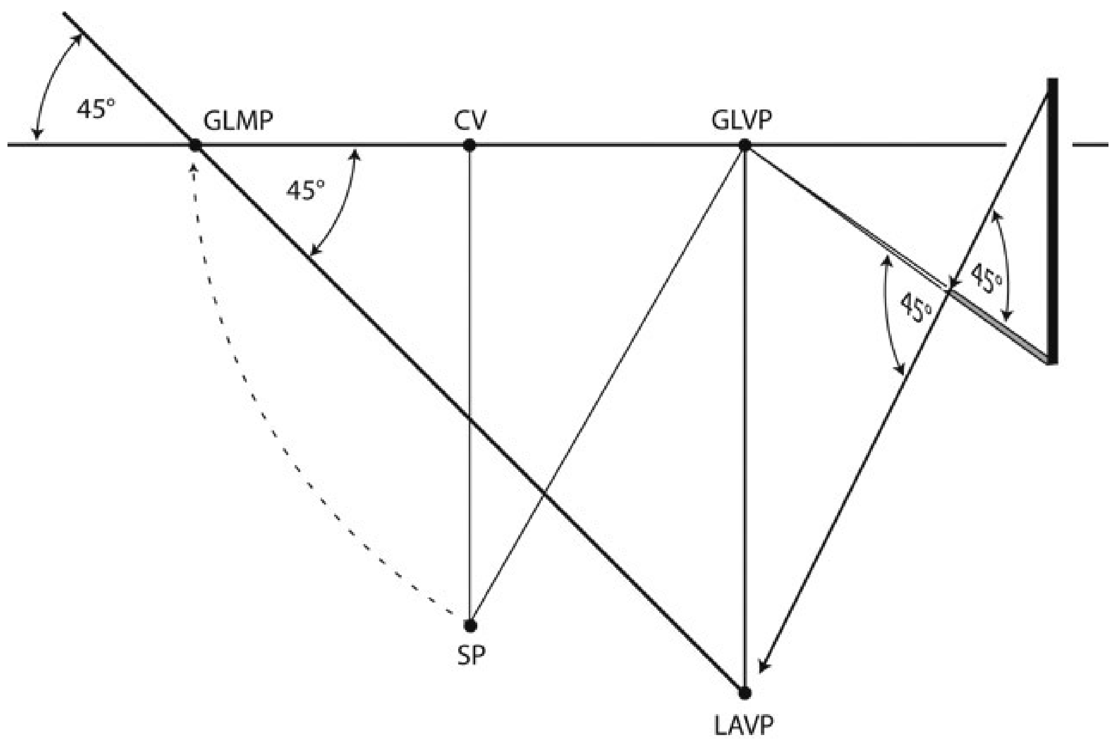
Figure 26.3 Ground line angles are found from the station point.

Angle of the Light

The first step in creating a specific light angle is to create a ground line measuring point (Figure 26.4). Finding a specific light angle vanishing point follows the same procedures as finding an auxiliary vanishing point. For example, if the light source is to be 45° above the horizon, project that angle from the ground line measuring point. Negative shadows require the light angle vanishing point to be below the horizon line (Figure 26.5).



[Figure 26.4](#) **A ground line measuring point is needed to locate the light angle vanishing point.**



[Figure 26.5](#) **Angles drawn from the ground line measuring point create a light angle vanishing point that draws those same angles in perspective.**

27
Shadows from Artificial Light Sources

Natural light is from a source at such a distance that the shadows cast are considered parallel. All lines drawn from the same point on the horizon line are parallel in perspective. Thus, all shadows from a natural light source are likewise parallel in perspective.

An artificial light source is much closer. The shadows are not parallel but radiate around a point. The shadows become wider as they get farther away from the light source. This type of light source is called **converging light**.

Components of Shadows Created From Artificial Light

Ground Line Location

The ground line is located 90° from the light source. It is most frequently placed on the surface the shadow is cast upon, typically the ground plane (Figure 27.1). In contrast to natural light, artificial light can have multiple ground line vanishing points. They can be on walls, floors, and ceilings (Figures 28.2–28.5). Shadows at different heights (e.g., shadows on a floor and shadows on a table) have ground line vanishing points at the same height as the shadow (Figure 27.2).

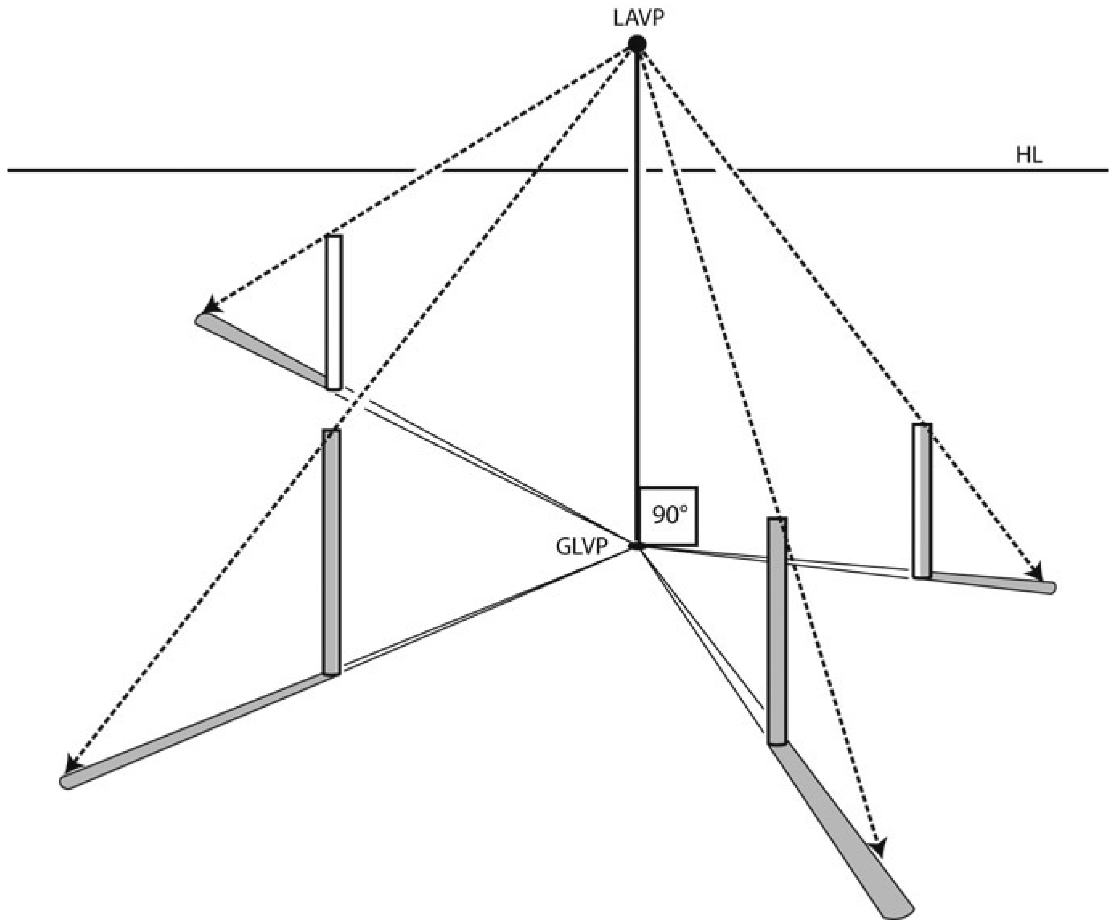


Figure 27.1 Converging shadows radiate around the ground line vanishing point. The ground line vanishing point is always 90° from the light source.

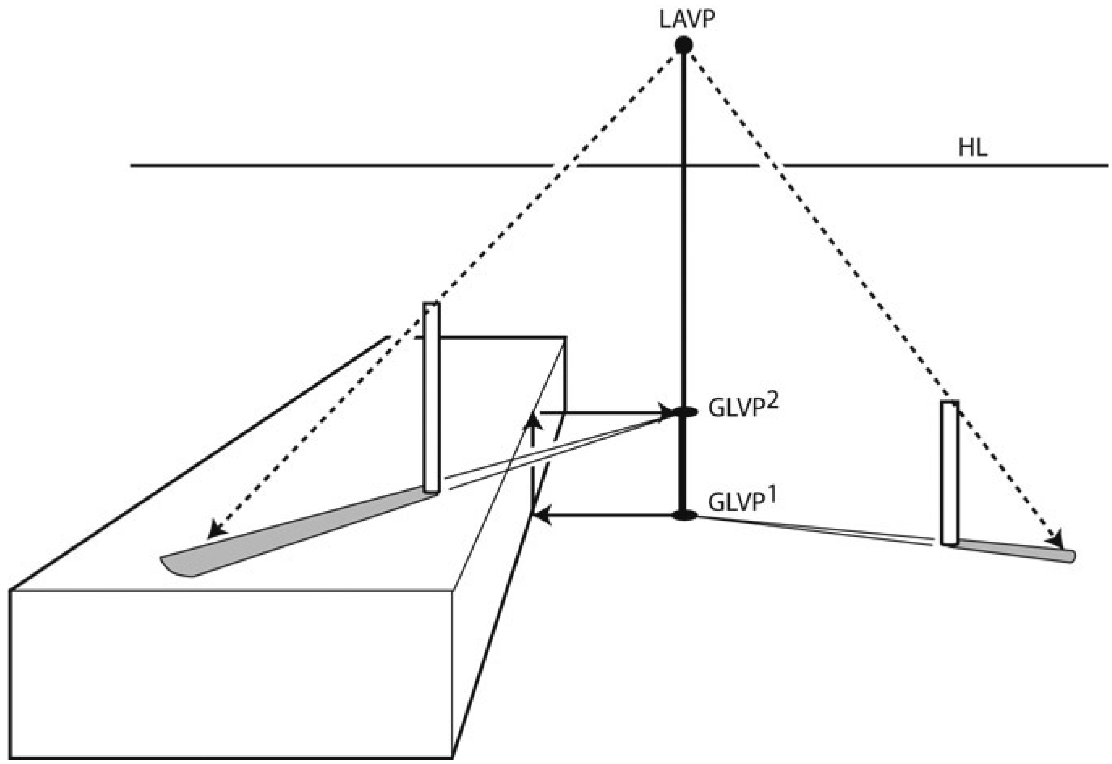


Figure 27.2 The ground line vanishing point must be at the same level as the surface the shadow is on.

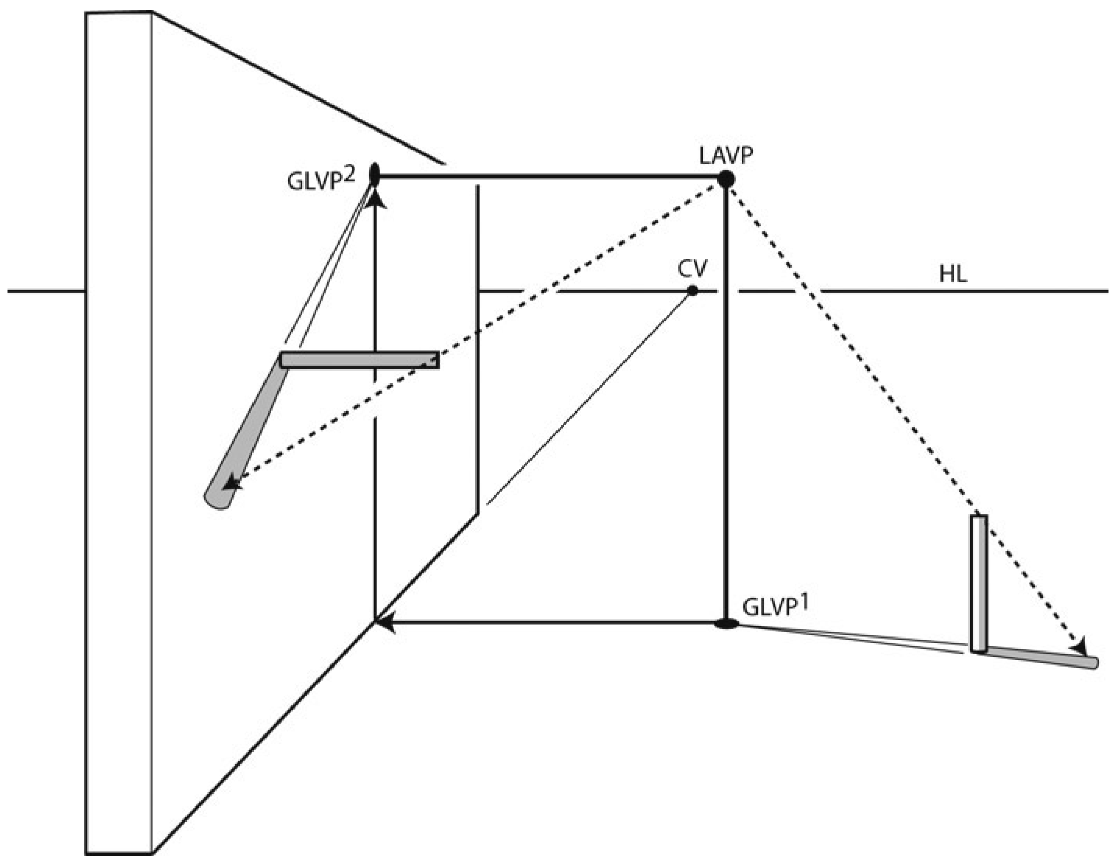


Figure 27.3 Ground line vanishing points can be on vertical surfaces.

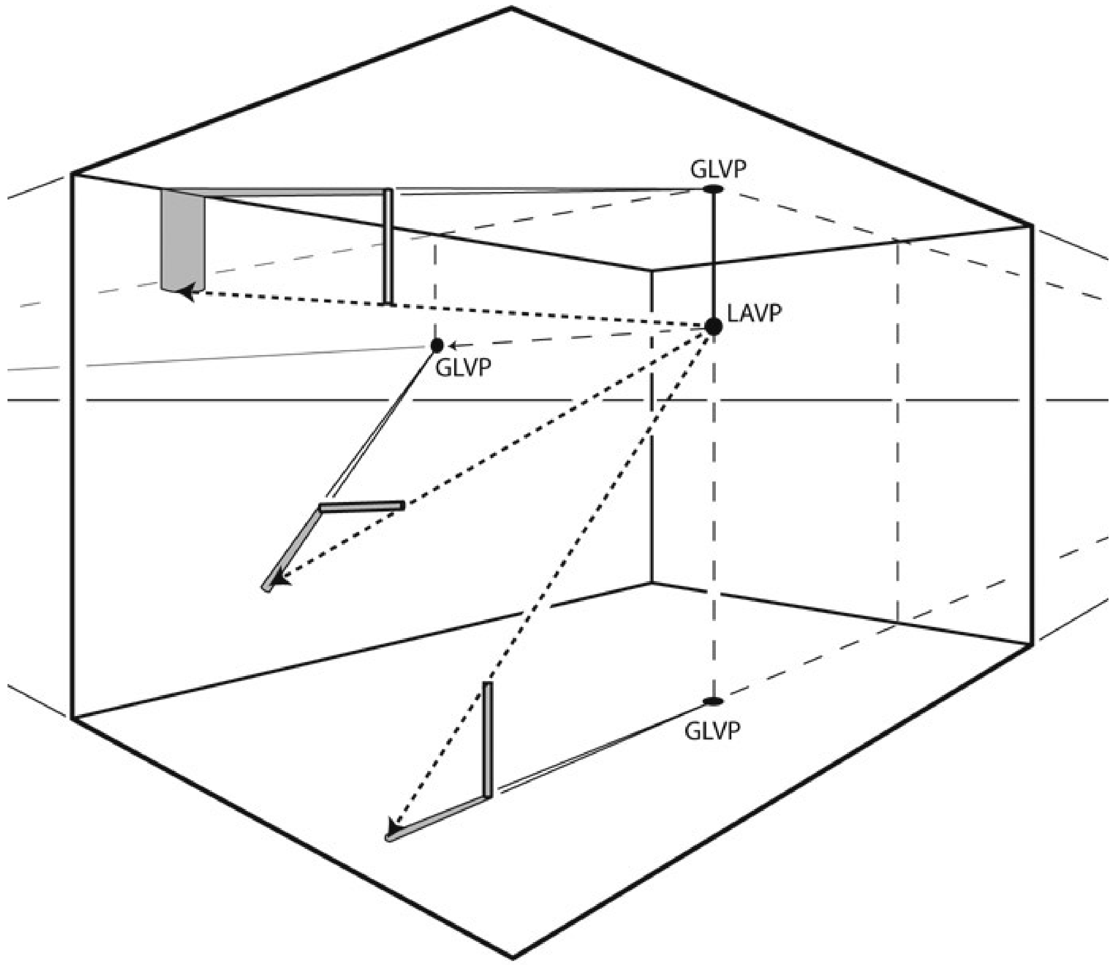


Figure 27.4 Ground line vanishing points are located 90° from the light source and on the same surface as the object casting the shadow.

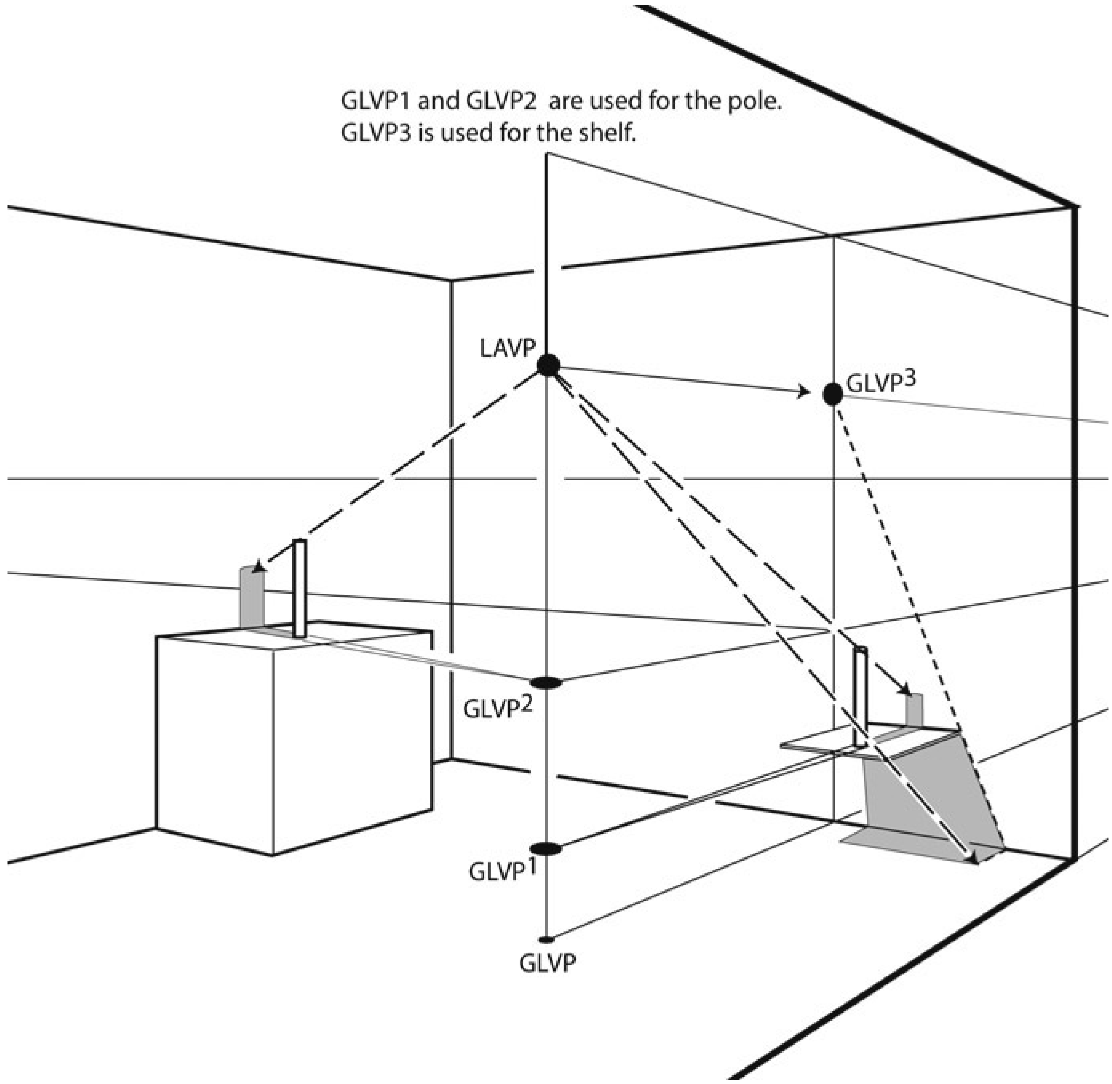


Figure 27.5 Each ground line vanishing point is placed on the same plane as the shadow.

Light Angle Location

The light angle vanishing point represents the light source. It can be anywhere. However its position must be known, as the placement of the ground line vanishing point depends on the light source's location.

28
Three-Point Shadows

All the previous shadow rules apply to three-point perspective. But, in three-point perspective, due to the vertical lines being foreshortened, a more complicated situation is created when faced with the task of locating the light angle and ground line vanishing points. The following pages illustrate how to apply natural and artificial light systems in three-point perspective.

Parallel Shadows, Bird's-Eye View

Ground Line

As with one- and two-point perspective, when the sun is 90° from the viewer's line of sight, the ground line is parallel with the picture plane and there is no ground line vanishing point.

Light Angle

Unlike one- and two-point perspective, the light angle is not parallel with the picture plane. In three-point perspective, due to the picture plane being foreshortened, the light angle has a vanishing point. If the sun is directly overhead, light rays are vertical (90° to the ground plane). Since all vertical lines connect to the vertical vanishing point, the vertical vanishing point becomes the light angle vanishing point. If the sun is to the right or left, the light angle vanishing point moves horizontally to the left or right of the vertical vanishing point (Figures 28.1–28.2).

Location

To place the light source in a specific location, find the point of true angles (this is the same procedure outlined in Figure 18.21). Measure the distance from the vertical vanishing point to the vertical station point, then transfer that distance to the vertical reference line, establishing the *x*-axis point (Figure 28.3, top). From the *x*-axis point, project the desired angle, locating the light angle vanishing point. For example, if the light source is to be at a 60° angle from the ground plane, project a true 60° angle from the point of true angles (Figure 28.3, bottom).

The light angle vanishing point does not represent the light source, but it does create angles congruent with the light source (review Chapter 26 on Negative Shadows). If the light angle vanishing point is to the right of the vertical vanishing point, the sun is to the viewer's left. Conversely, if the light angle vanishing point is to the left of the vertical vanishing point, the sun is to the viewer's right (Figure 28.1).

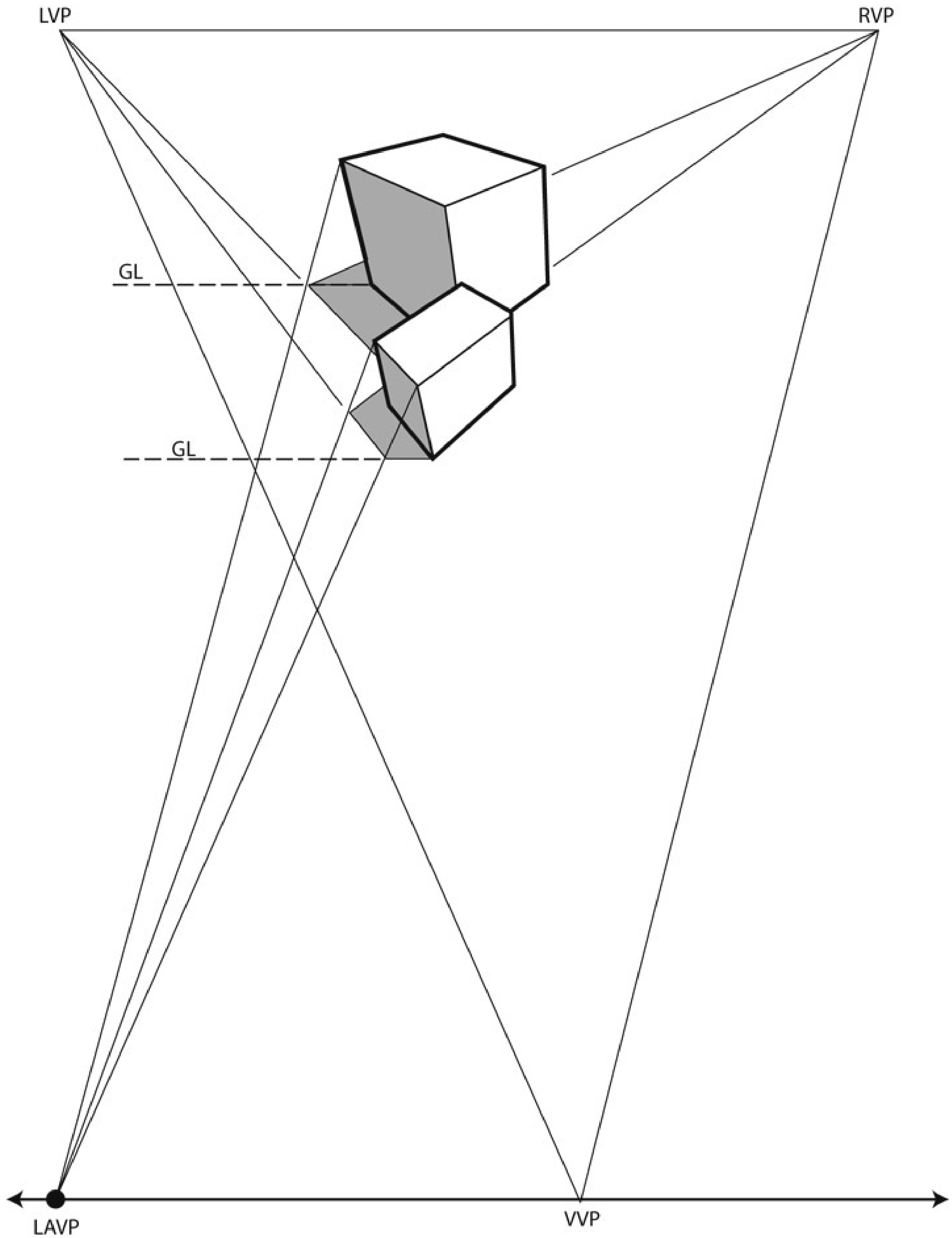


Figure 28.1 **When the light angle vanishing point is to the left of the vertical vanishing point, the sun is above and to the right of the viewer.**

Parallel Shadows, Worm's-eye view

A worm's-eye view is approached the same as a bird's-eye view, only upside-down. The procedure is the same. However, in a worm's-eye view, the light angle vanishing point is above the horizon line, and does represent the sun's actual location (Figure 28.2).

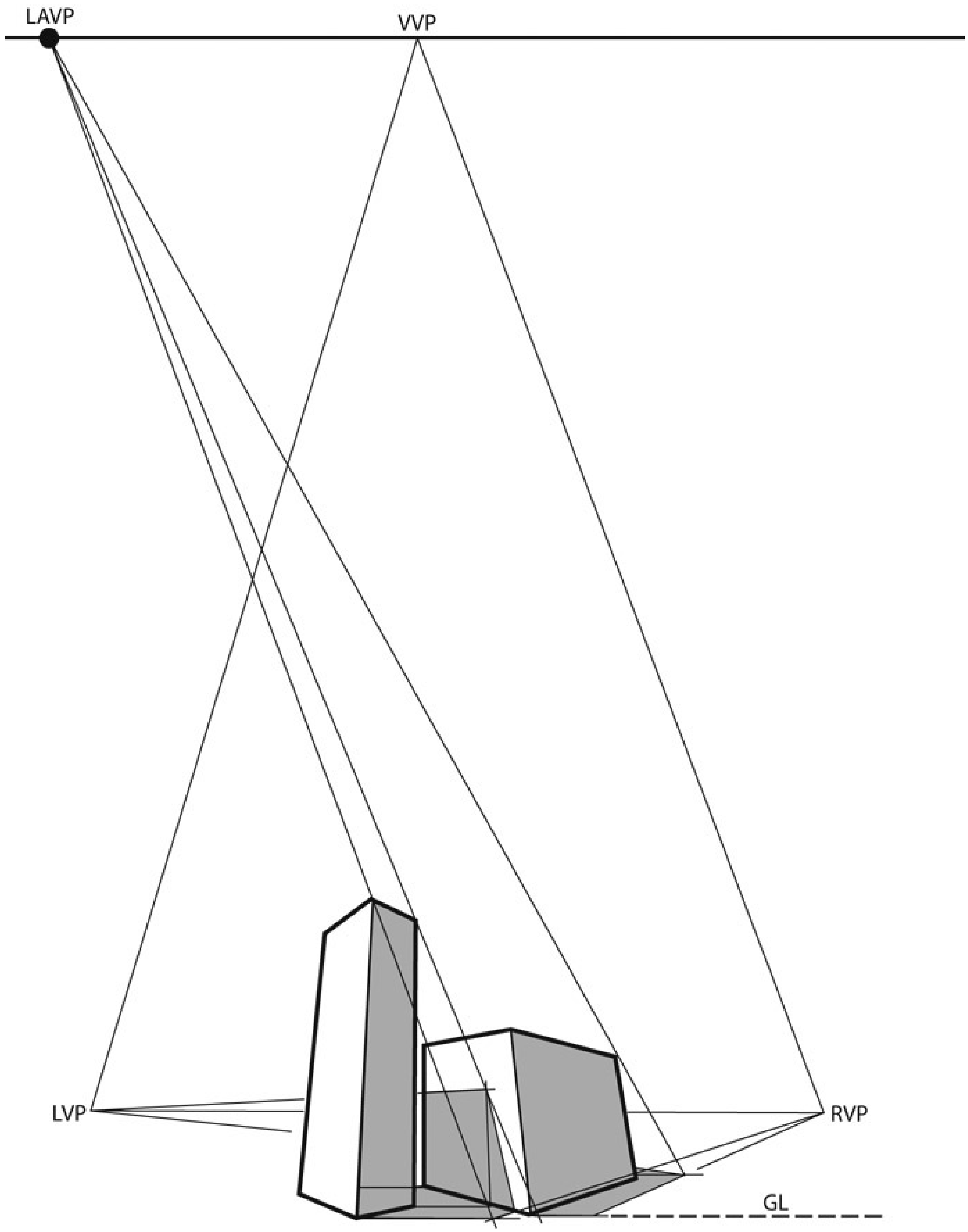


Figure 28.2 **Unlike the bird's-eye-view, in a worm's-eye view the light angle vanishing point represents the location of the sun.**

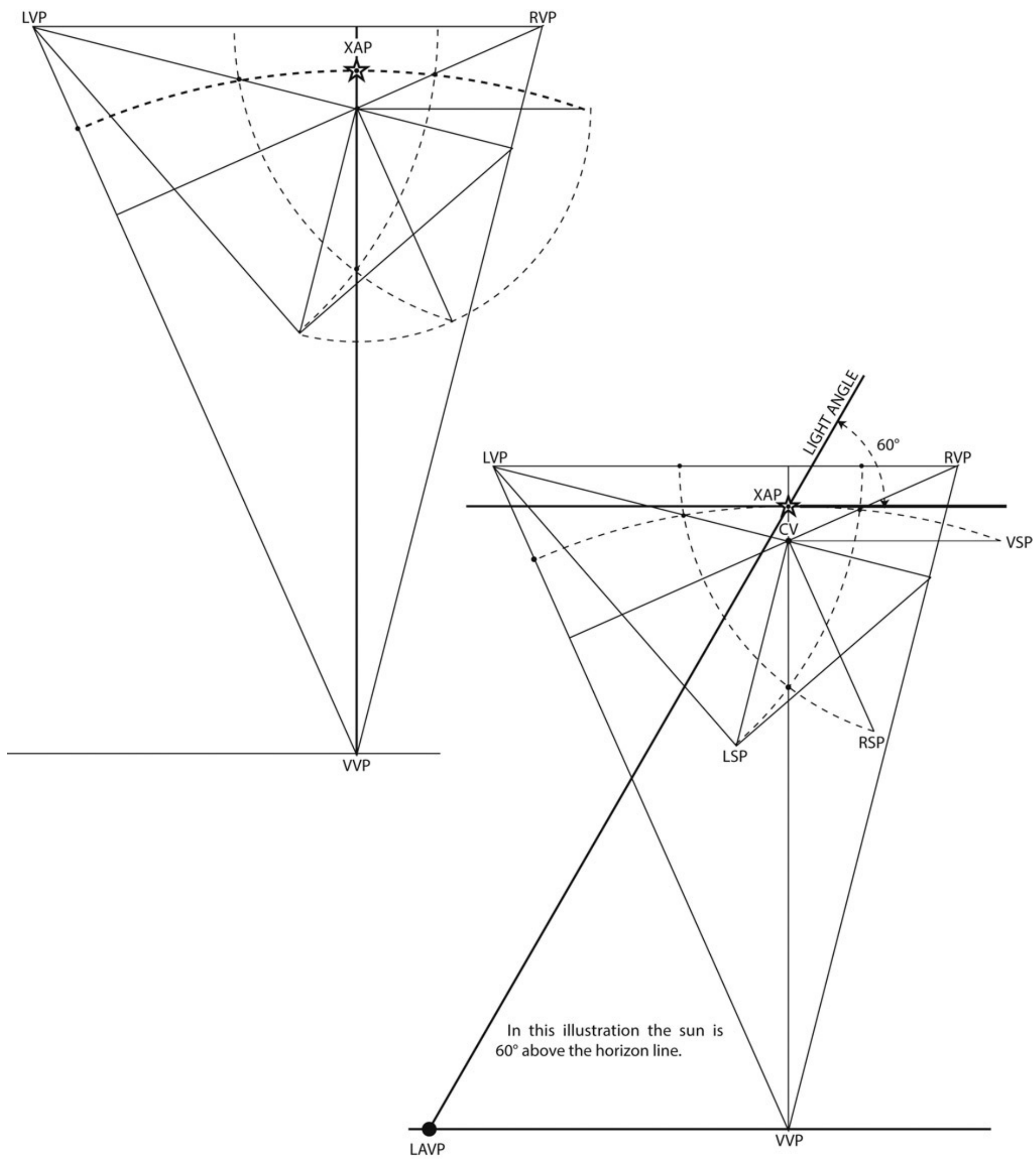


Figure 28.3 Use the *x*-axis point (the point of true angles) to establish a light source at a specific angle.

Positive Shadows

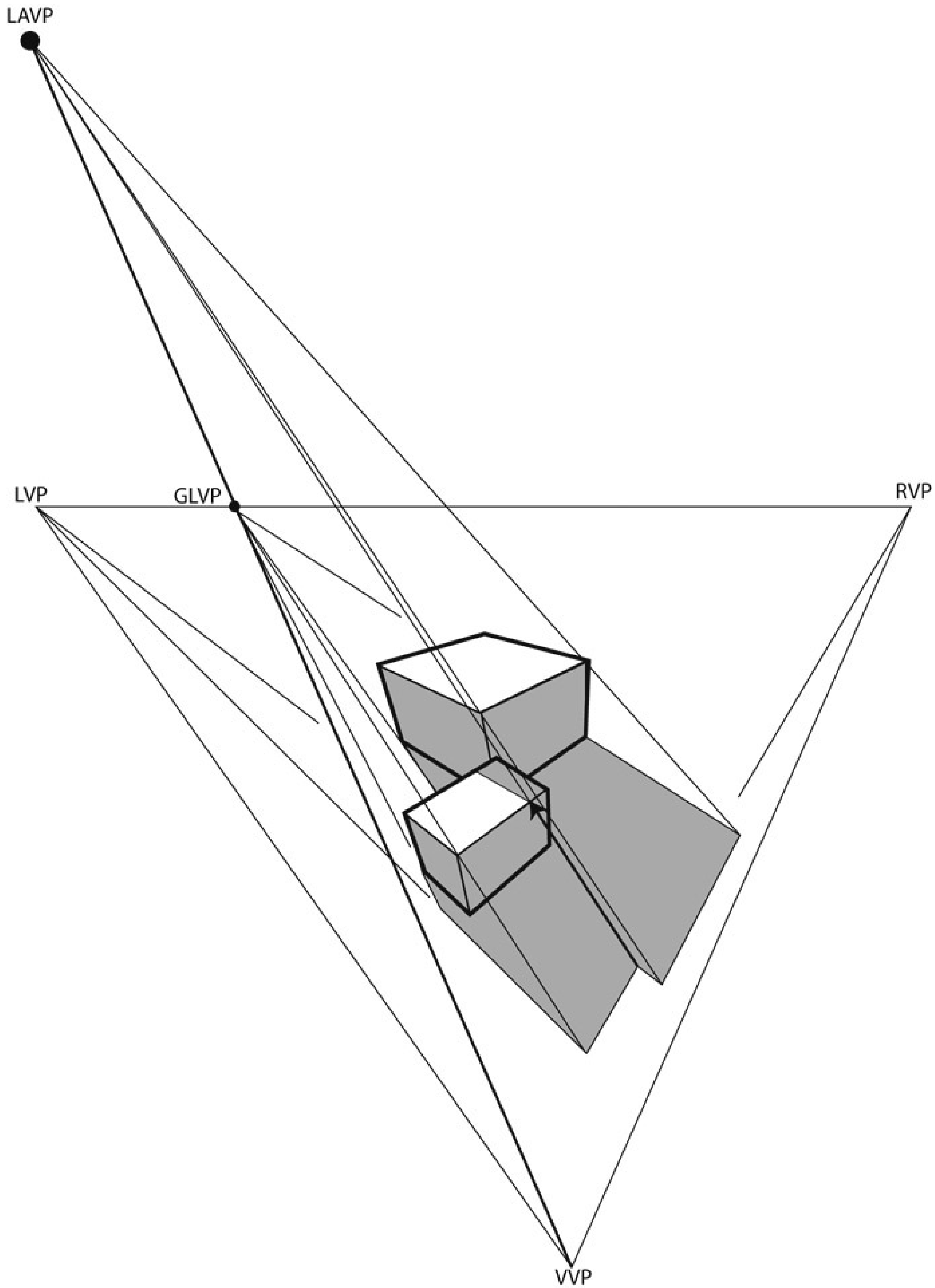
The sun is in front of the viewer. Objects are backlit, and shadows angle toward the viewer.

Ground Line

The ground line vanishing point is located on the horizon line, as it is for all natural light situations.

Light Angle

The light angle vanishing point is directly above the ground line vanishing point. In three-point perspective, all vertical lines connect to the vertical vanishing point. Thus, the ground line vanishing point and the light angle vanishing point align with the vertical vanishing point (Figures 28.4–28.5).



[Figure 28.4](#) **When drawing positive shadows in a bird's-eye view, the light angle vanishing point is above the horizon line, and aligned to the vertical vanishing point.**

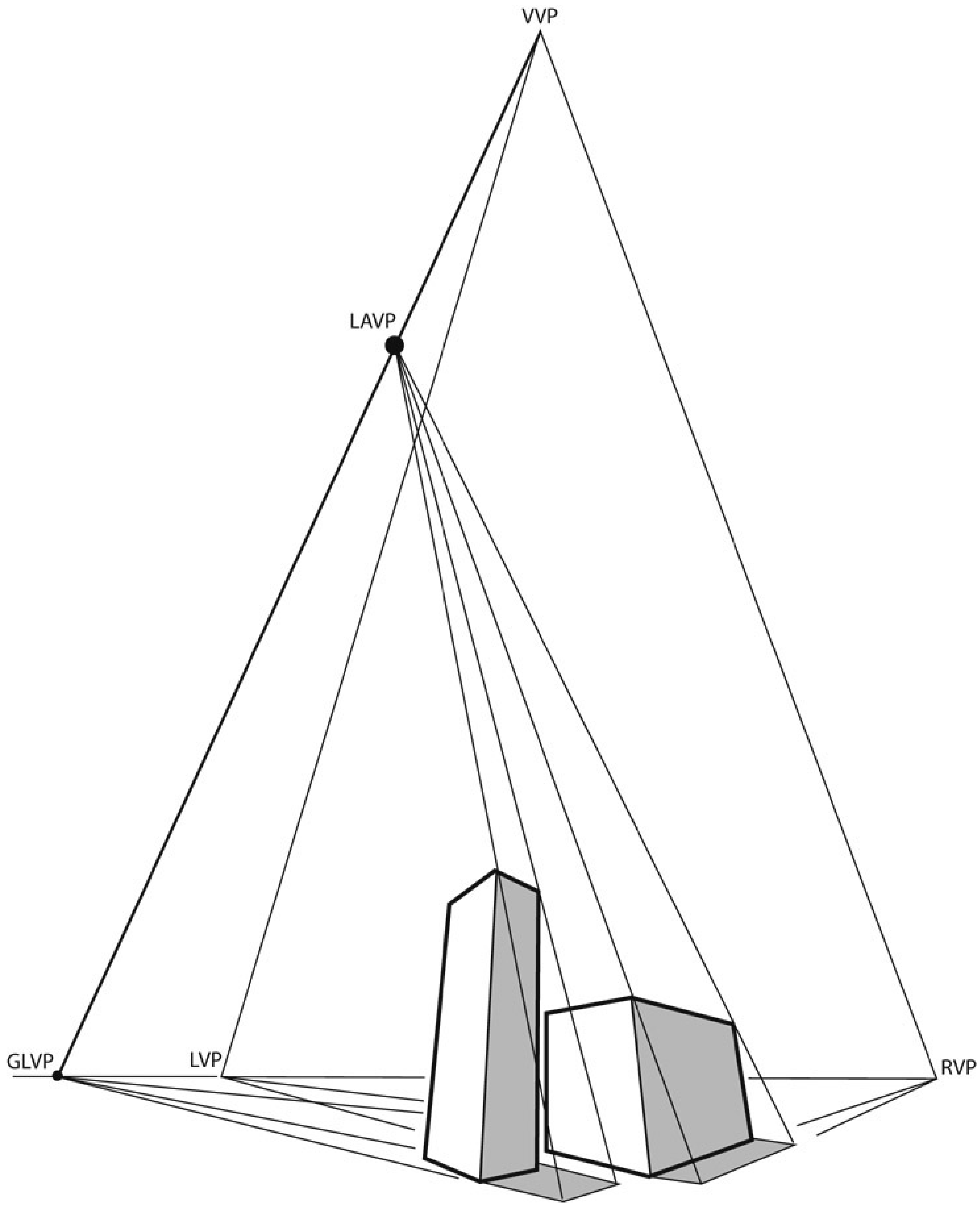


Figure 28.5 Positive shadows in a worm's-eye view. The light angle vanishing point and the ground line vanishing point connect to the vertical vanishing point.

Negative Shadows

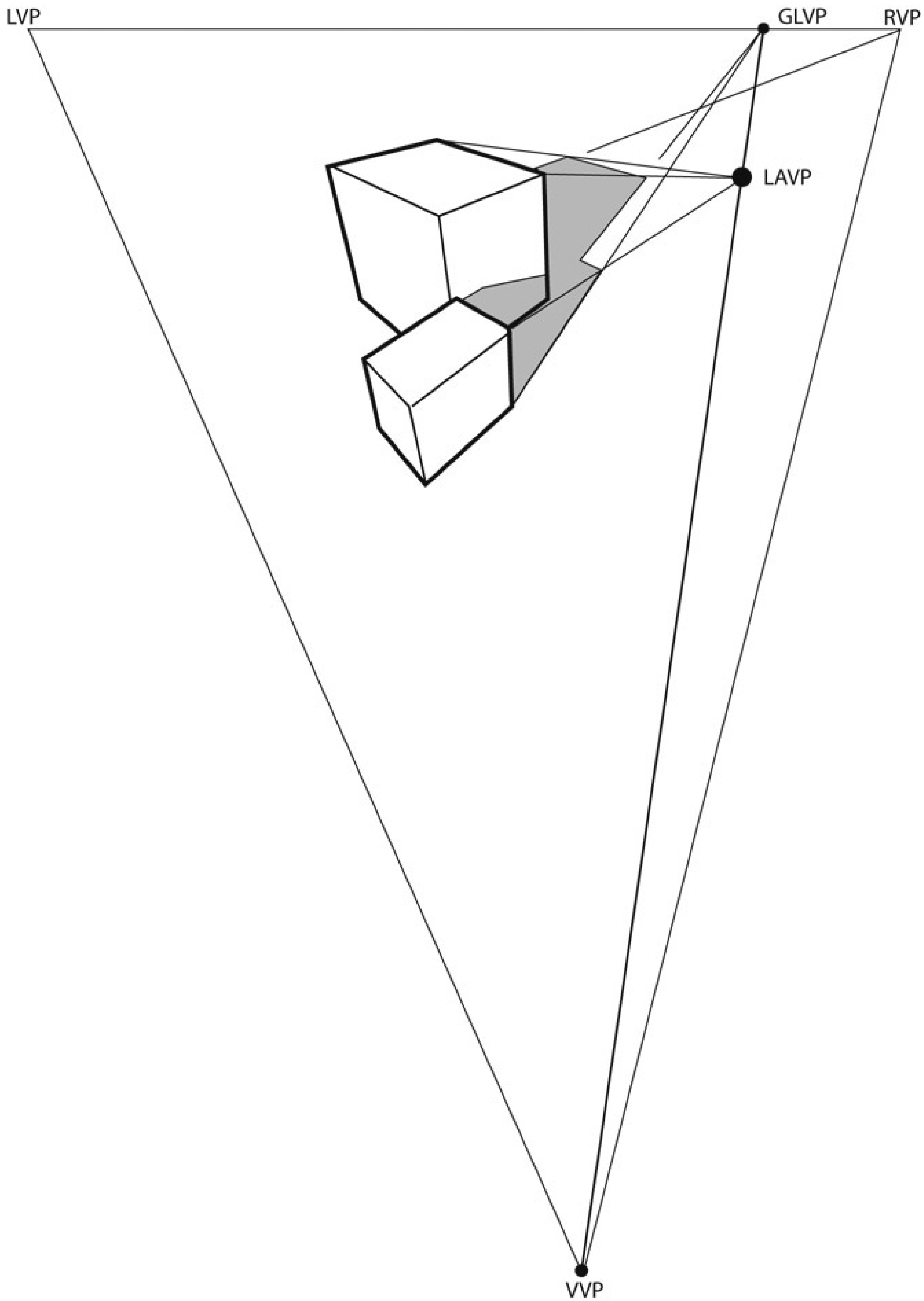
Objects that are behind the station point can't be drawn on the paper. When the sun is behind the viewer, geometry must be used to find a point that draws angles congruent with the sun (refer to Figure 26.2).

Ground Line

The ground line vanishing point is located at infinity and placed on the horizon line.

Light Angle

The light angle vanishing point is placed below the horizon line, aligned with the vertical vanishing point (Figures 28.6–28.7).



[Figure 28.6](#) **When drawing negative shadows in a bird's-eye view, the light angle vanishing point and the ground line vanishing point are aligned with the vertical**

vanishing point.

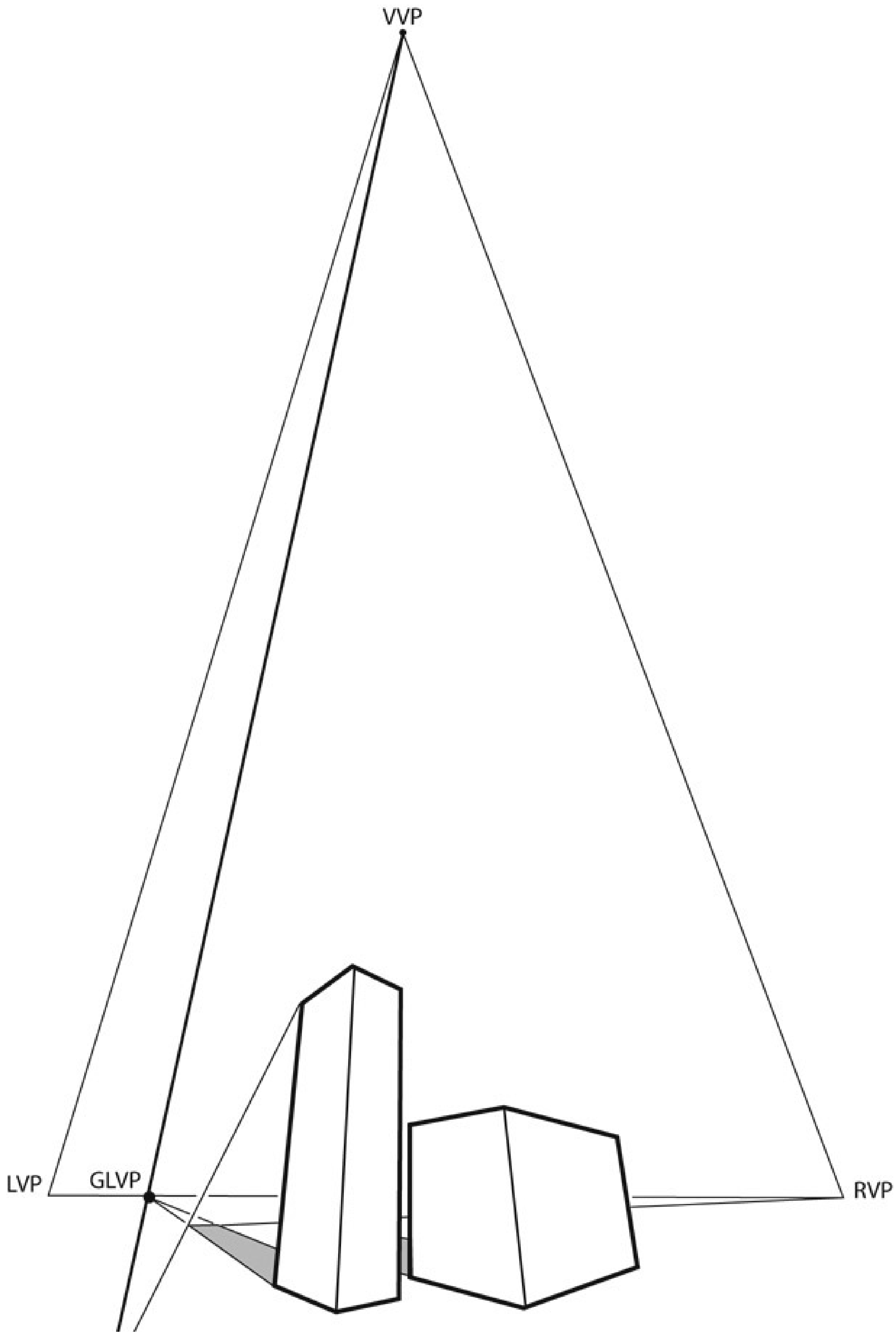


Figure 28.7 When drawing negative shadows in a worm's-eye view, the light angle vanishing point and the ground line vanishing point are aligned with the vertical vanishing point.

Light Source Location

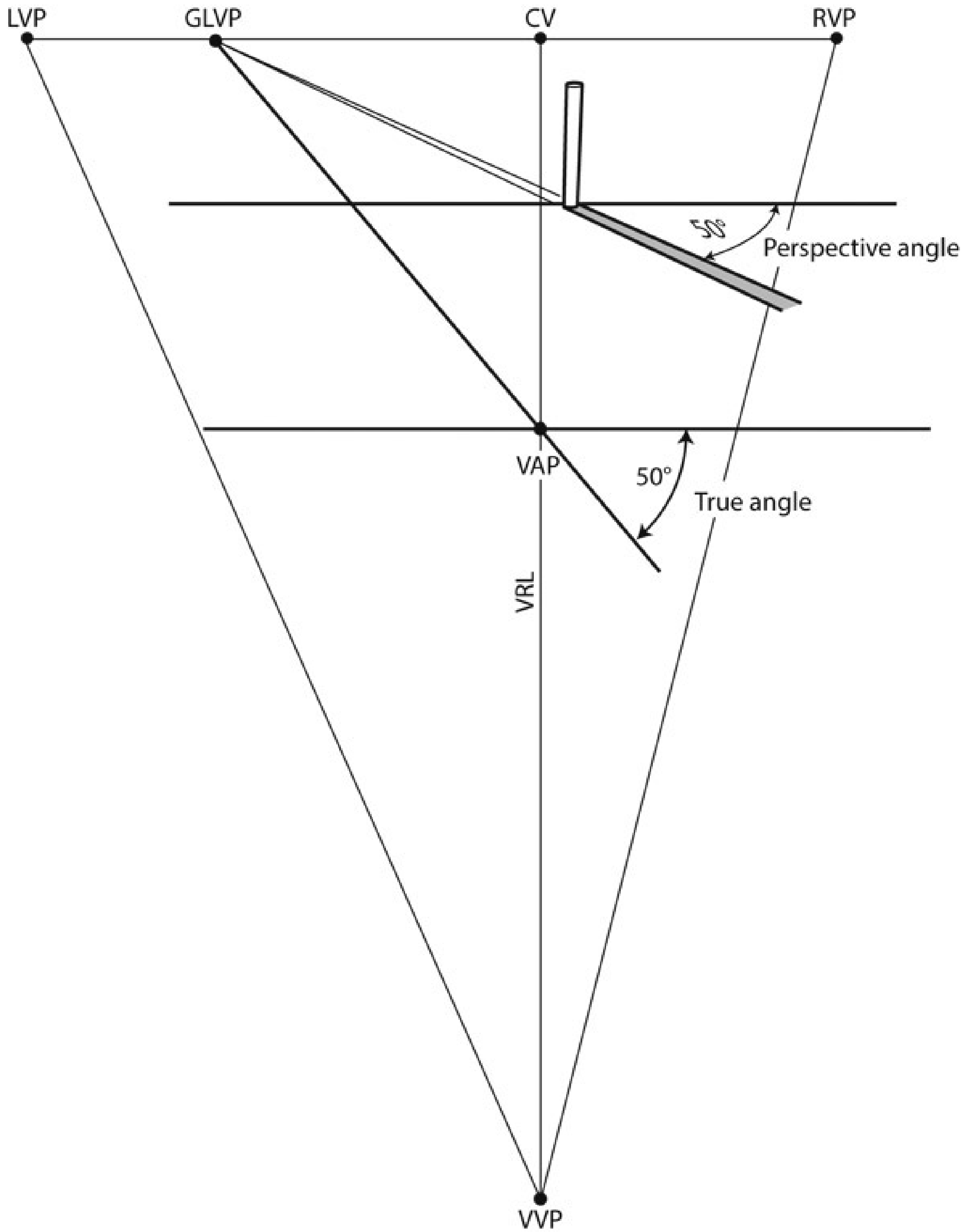
Placing the light source in a specific location requires a solid understanding of three-point angles.

Ground Line

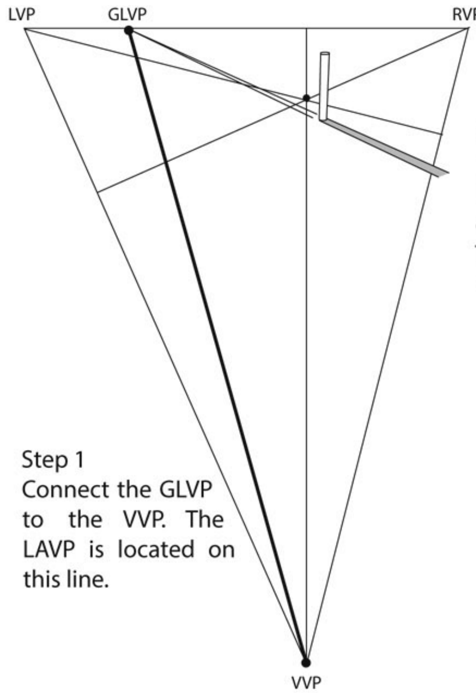
The ground line angle is established using the vertical axis point. For example, to draw a shadow that is 50° from horizontal, draw a true 50° angle at the vertical axis point. Then project that angle to the horizon line (Figure 28.8).

Light Angle

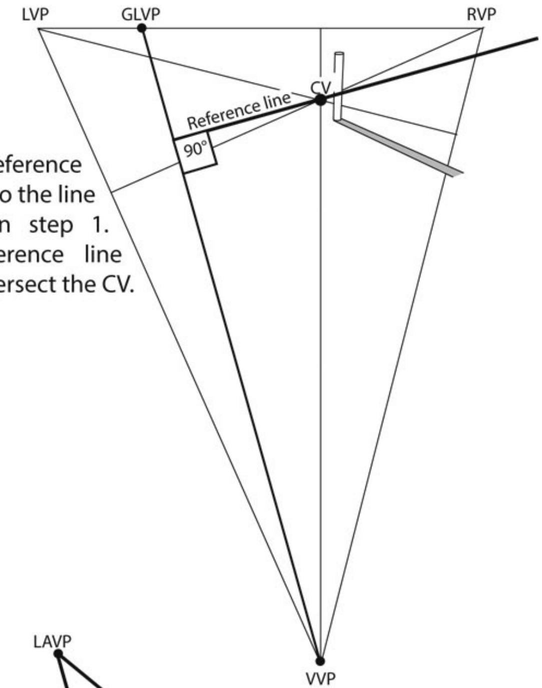
The light angle is a bit more complicated. It involves making a new reference line and a new axis point. Use this procedure for both positive and negative shadows (Figures 28.9–28.10).



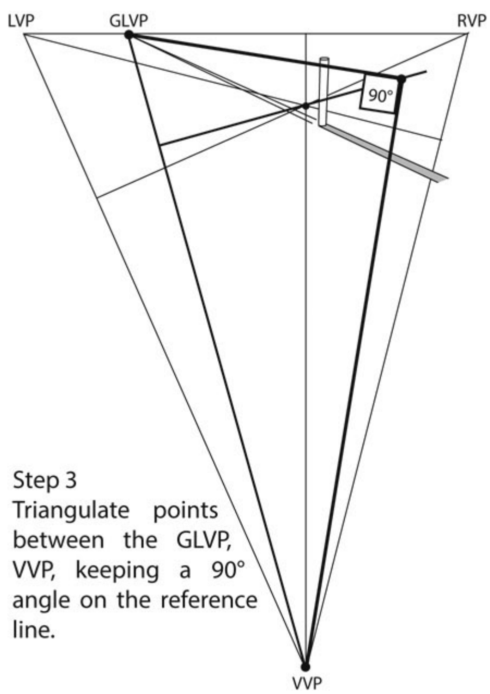
[Figure 28.8](#) **Use the vertical axis point to establish a specific ground line angle.**



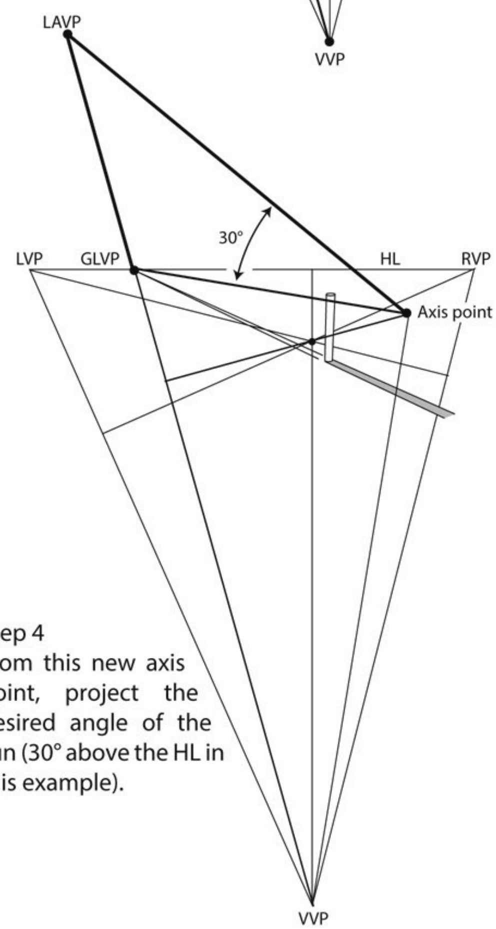
Step 1
Connect the GLVP to the VVP. The LAVP is located on this line.



Step 2
Draw a reference line 90° to the line drawn in step 1. The reference line must intersect the CV.



Step 3
Triangulate points between the GLVP, VVP, keeping a 90° angle on the reference line.



Step 4
From this new axis point, project the desired angle of the sun (30° above the HL in this example).

[Figure 28.9](#) **Positioning the light angle vanishing point in a specific location requires a new axis point.**

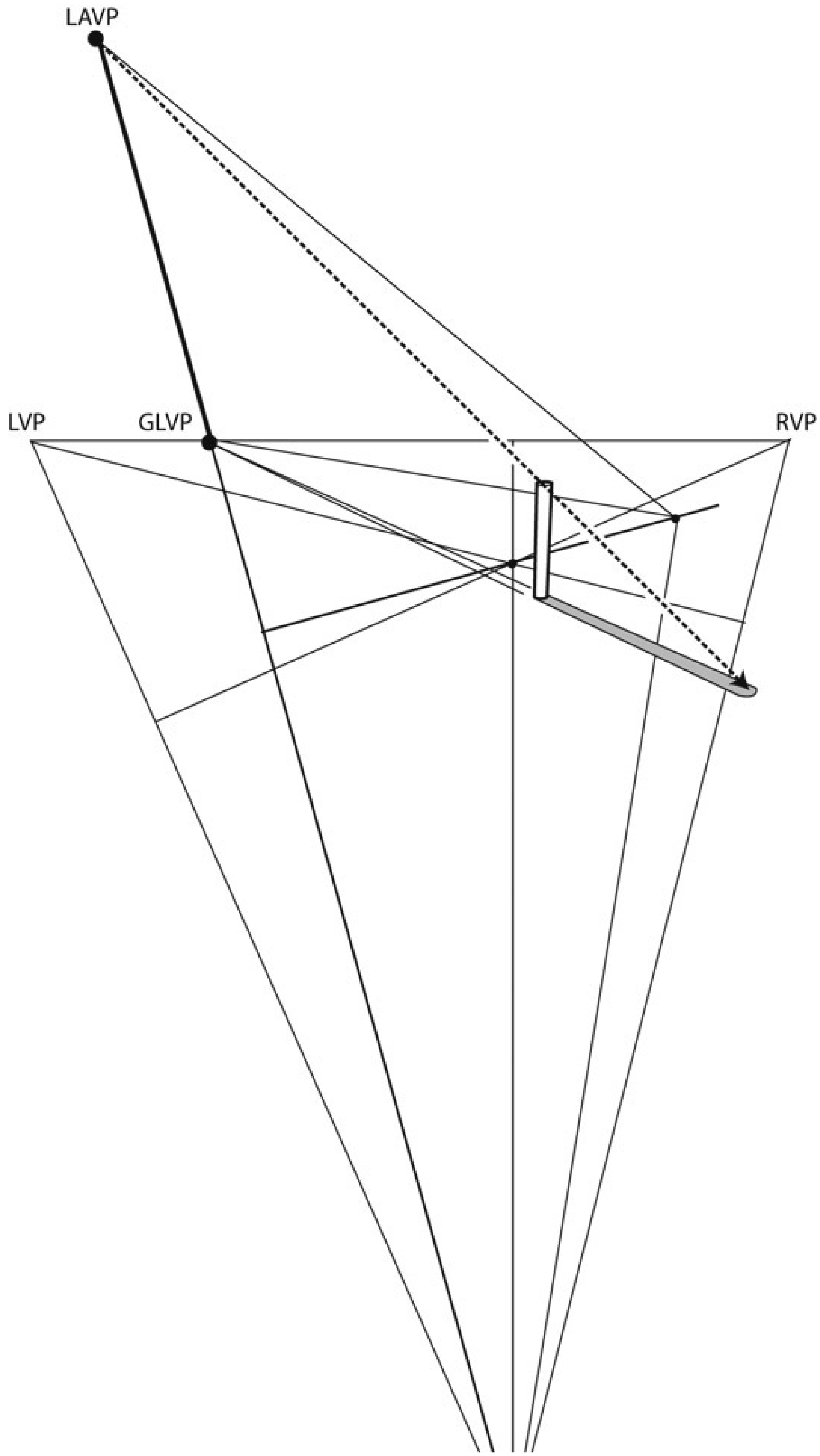
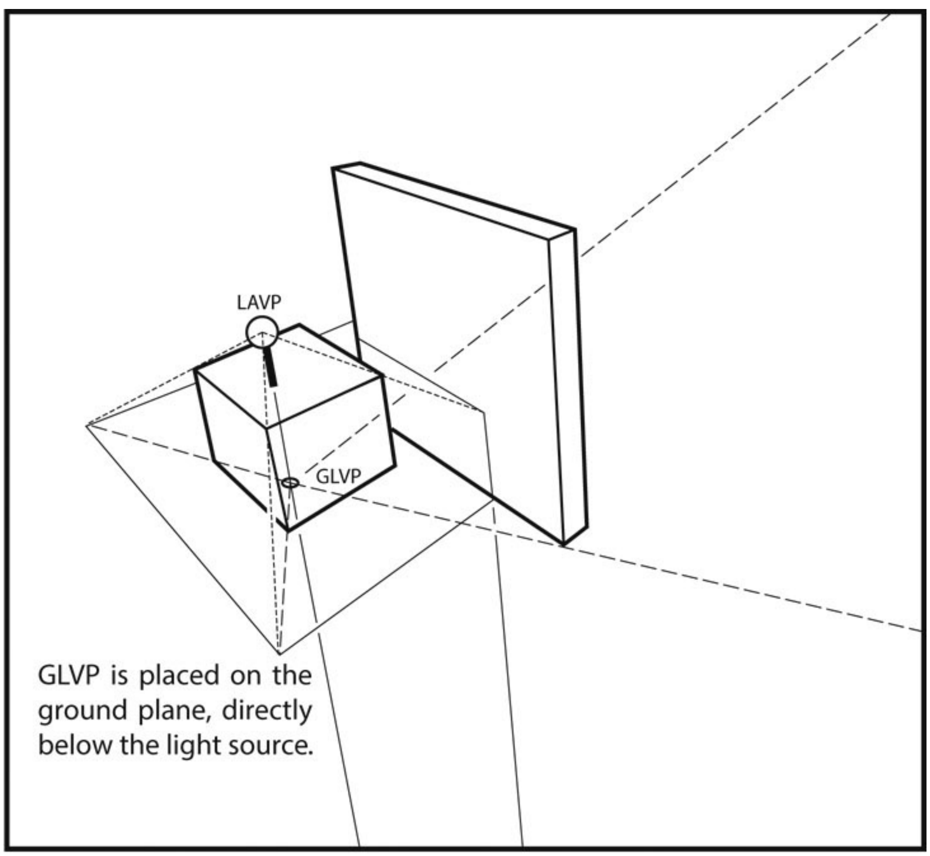
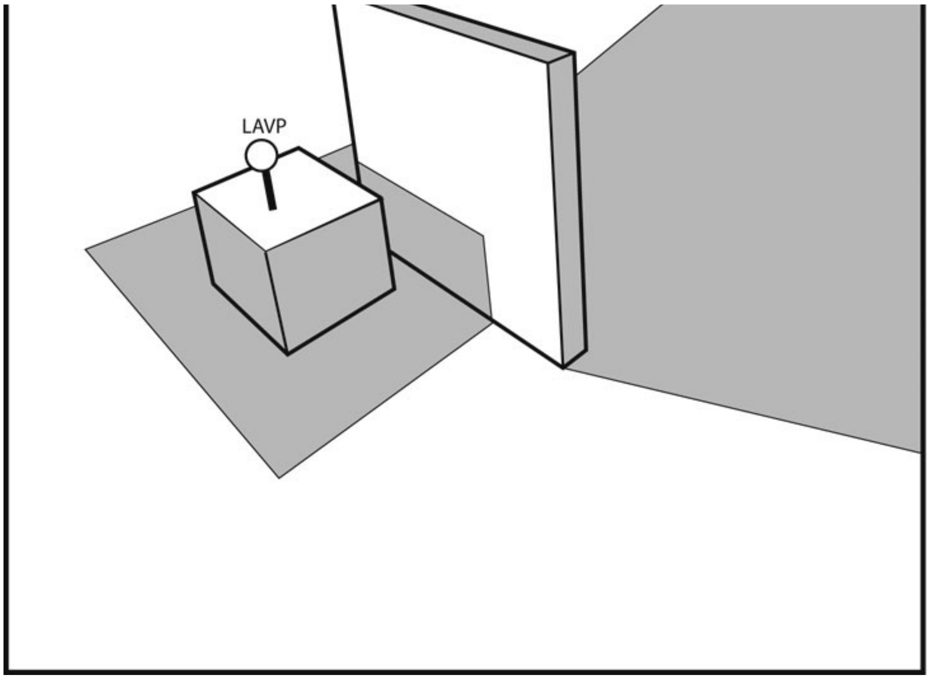


Figure 28.10 **The finished shadow. The sun is 40° to the left and 30° above the horizon line.**

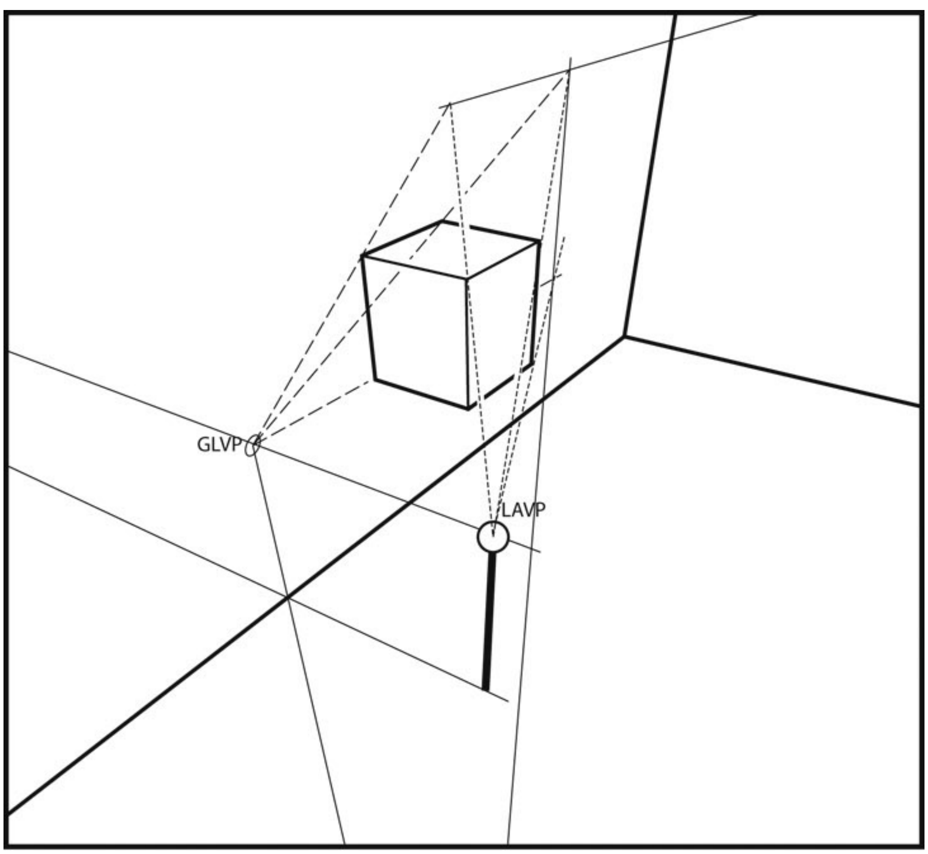
Converging Light in Three-Point Perspective

Artificial light in three-point perspective is approached in the same way as one- or two-point perspective. The only difference is that vertical lines are now foreshortened. Otherwise, the same rules apply (Figures 28.11–28.12).





[Figure 28.11](#) **When drawing shadows from artificial light sources, the ground line vanishing point is located 90° from the light angle vanishing point.**



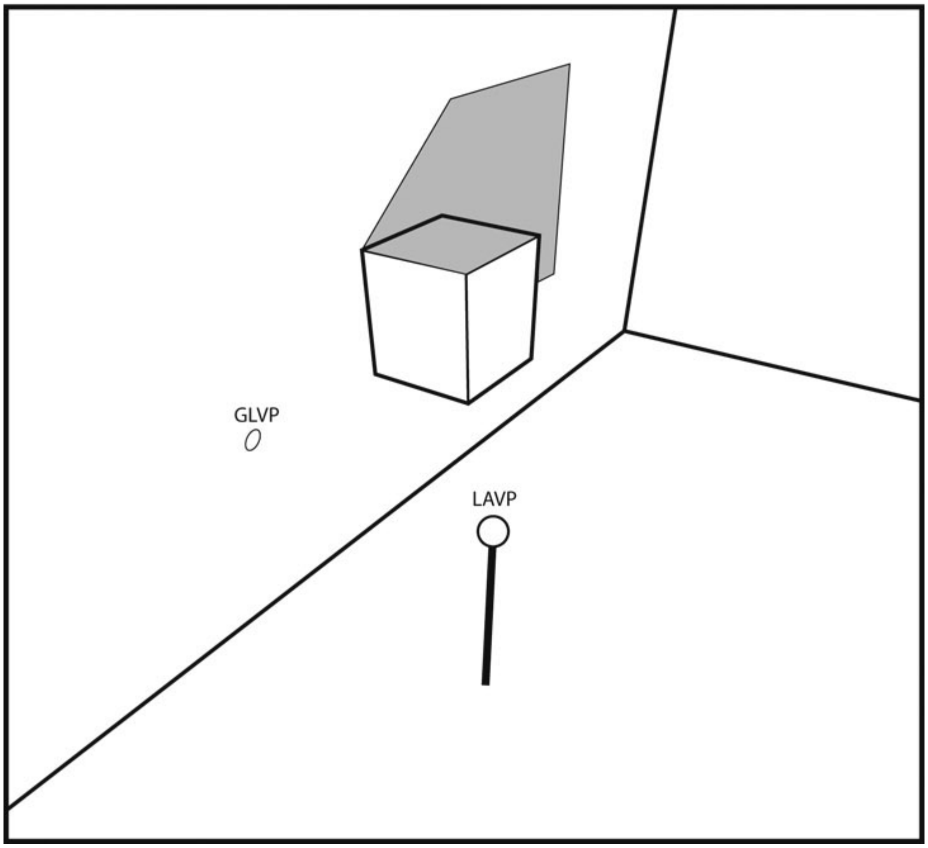


Figure 28.12 Shadows on vertical surfaces have ground line vanishing points on the same surface as the shadow.

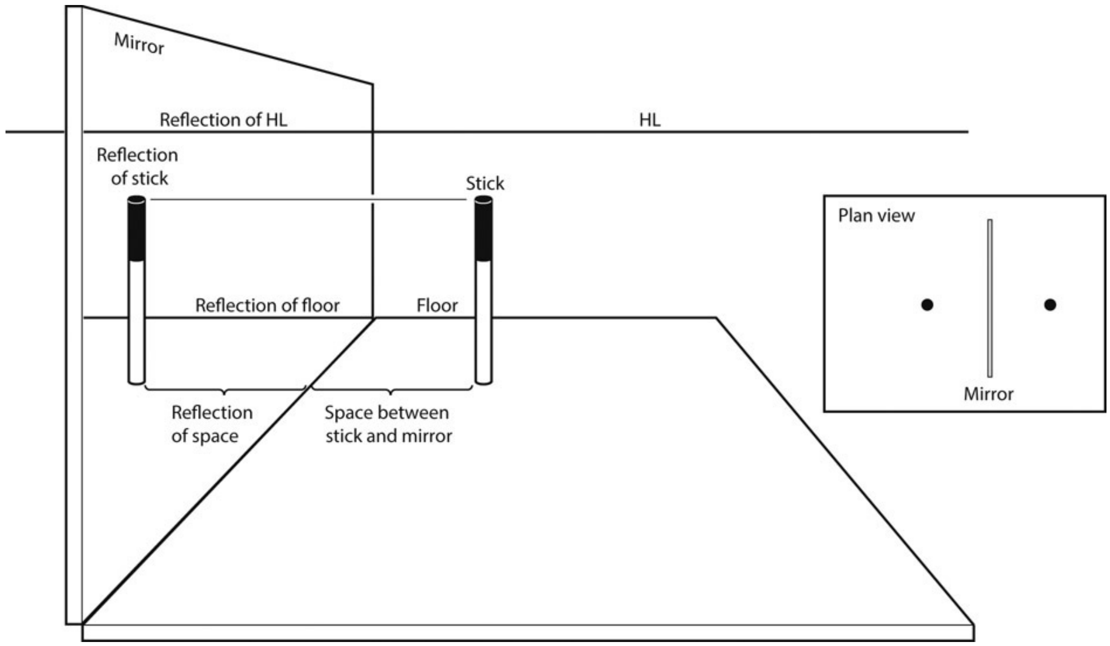
29
Reflections

While shadows are *on* surfaces, reflections appear *inside* surfaces and appear as a virtual object. Reflections are aligned with the real object, at a right angle to the mirror's surface. The distance from the virtual object to the mirror is the same as the distance from the real object to the mirror. Various methods can be used to plot reflections. The following pages outline a few of the options.

One-Point Perspective

Vertical Mirror (reflections along the *y*-axis)

These reflections have no foreshortening. Measure the distance from the object to the mirror and duplicate that distance inside the mirror. The height of the object does not change (Figure 29.1).



[Figure 29.1](#) **Horizontal reflections are not foreshortened.**

Horizontal Mirror (reflections along the *x*-axis)

Likewise, reflections in a horizontal mirror have no foreshortening. Simply duplicate the object in the reflective surface. If there is distance between the object and the reflective surface that distance must also be duplicated ([Figure 29.2](#)).

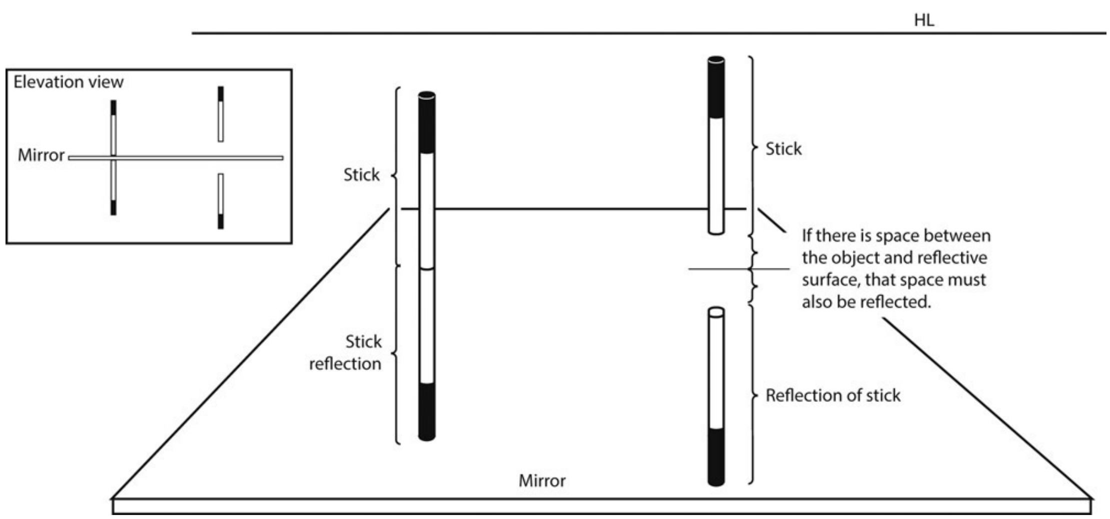


Figure 29.2 Vertical reflections are not foreshortened.

Vertical Mirror (reflections along the *z*-axis)

In this scenario, the distance from the object to the mirror is foreshortened. There are several ways to find a foreshortened distance. A measuring point can be used (Figure 29.3), or a reference point (Figure 29.4). A third method involves using geometry to create two lines of equal length (Figure 29.5). See Figure 36.3 (top) for step-by-step instructions for this procedure.

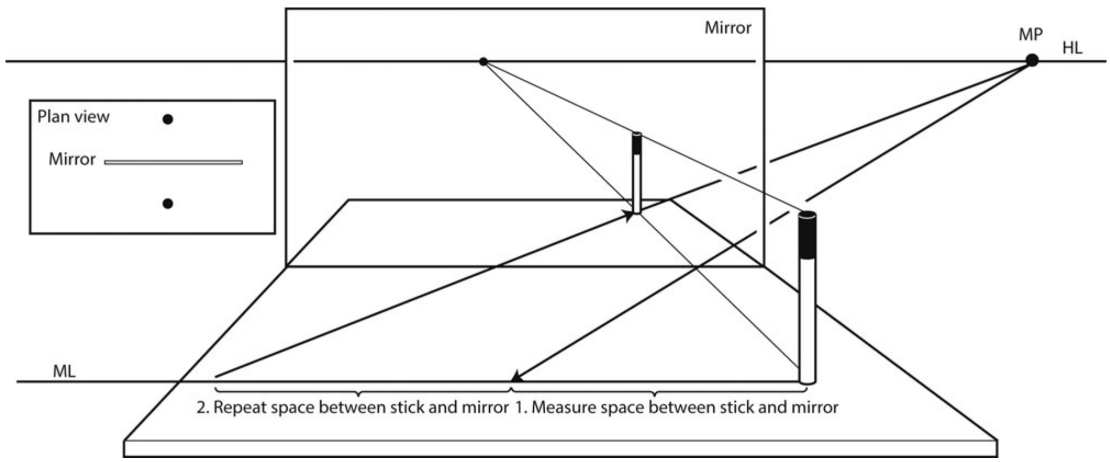


Figure 29.3 The distance between the object and mirror is foreshortened. Use a measuring point to reflect the distance.

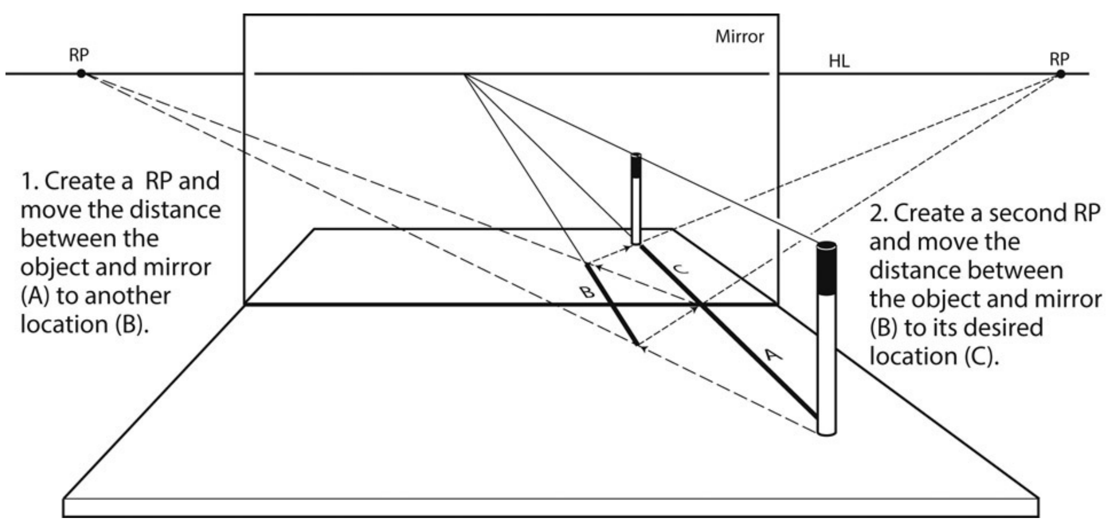


Figure 29.4 A reference point can be used to move parallel lines backward and forward in space.

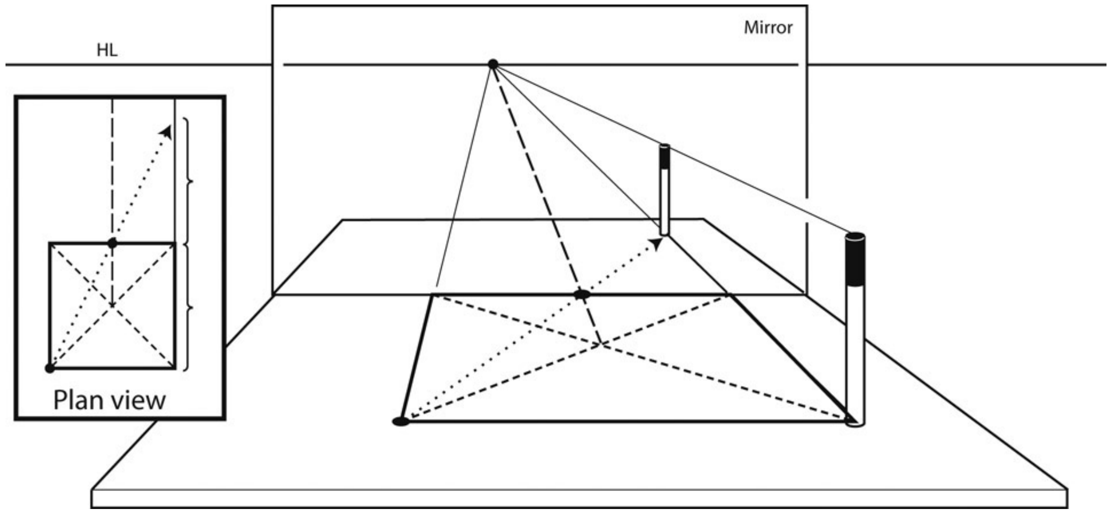


Figure 29.5 Using geometry to plot the distance of the reflection.

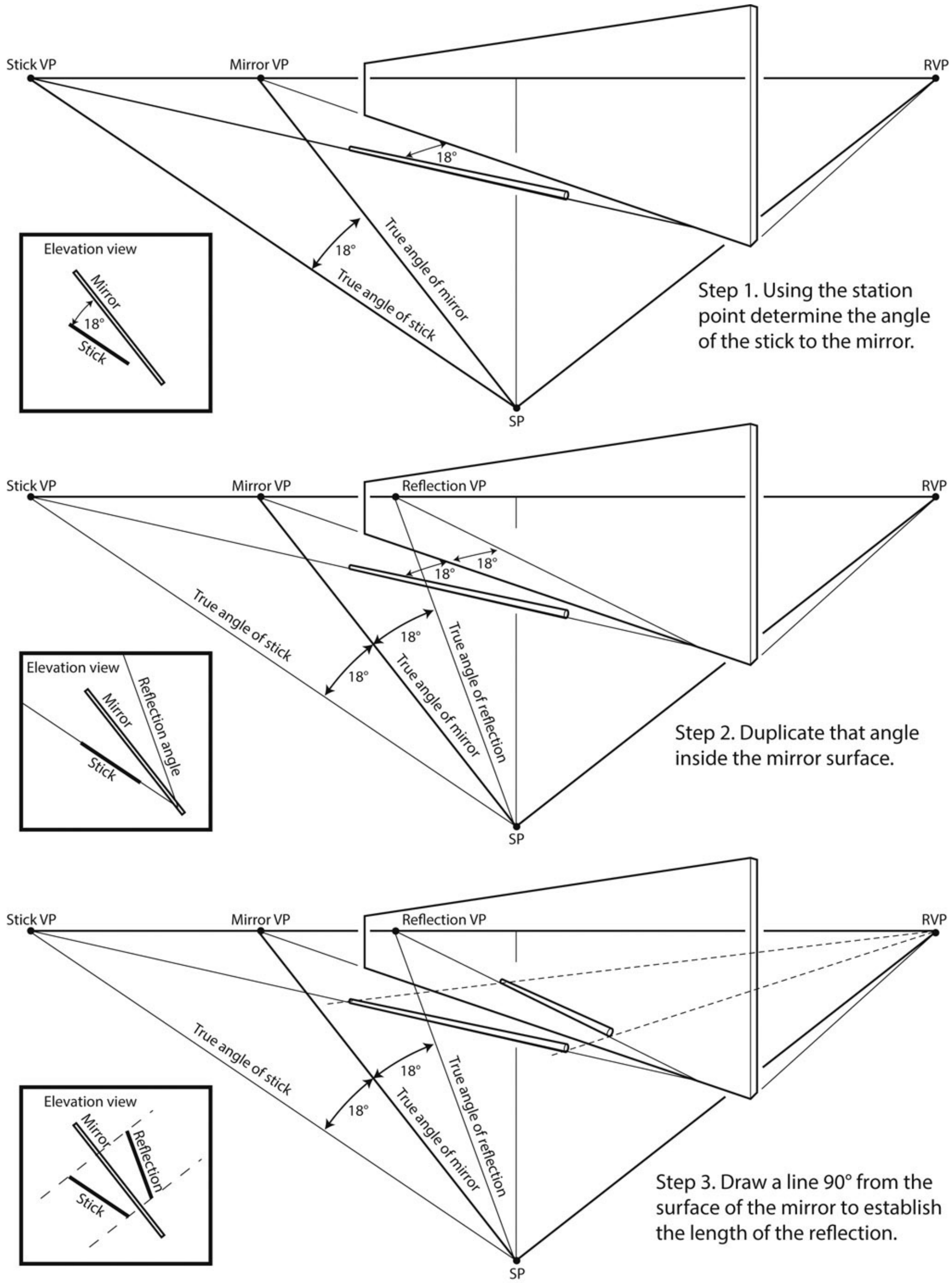


Figure 29.6 Use horizontal angles, plotted from the station point, to draw reflections angled to the mirror.

Two-Point Perspective

Reflections in vertical surfaces are foreshortened. Reflections in horizontal surfaces are not. The same procedures used to plot one-point reflections apply to two-point.

Reflective Angles

All the objects being reflected thus far have been parallel with or perpendicular to the reflective surface. If the object is angled to the mirror, the reflection is at that same angle. First calculate the angle of the object to the mirror. Then plot that same angle on the opposing side of the reflective surface (Figure 29.6). Reviewing the information on horizontal angles (Chapter 7) may be helpful.

30
Reflections on Inclined Surfaces

There are many ways to plot reflections in an inclined mirror—any of the previous methods can be used. But, understanding how angles work in perspective is the hallmark of becoming a proficient practitioner. So, in the following example, this reflection will be drawn using angles.

In this scenario, the mirror is at a 30° angle to the ground plane (Figure 30.1). First, calculate the angle of the object to the mirror (Figure 30.2). Then, duplicate that angle inside the mirror (Figures 30.3–30.4). To establish the length of the reflection, draw a 90° angle to the mirror's surface (Figure 30.5).

The reflection of a single line can be complicated. A three-dimensional object is even more so (Figure 30.6).

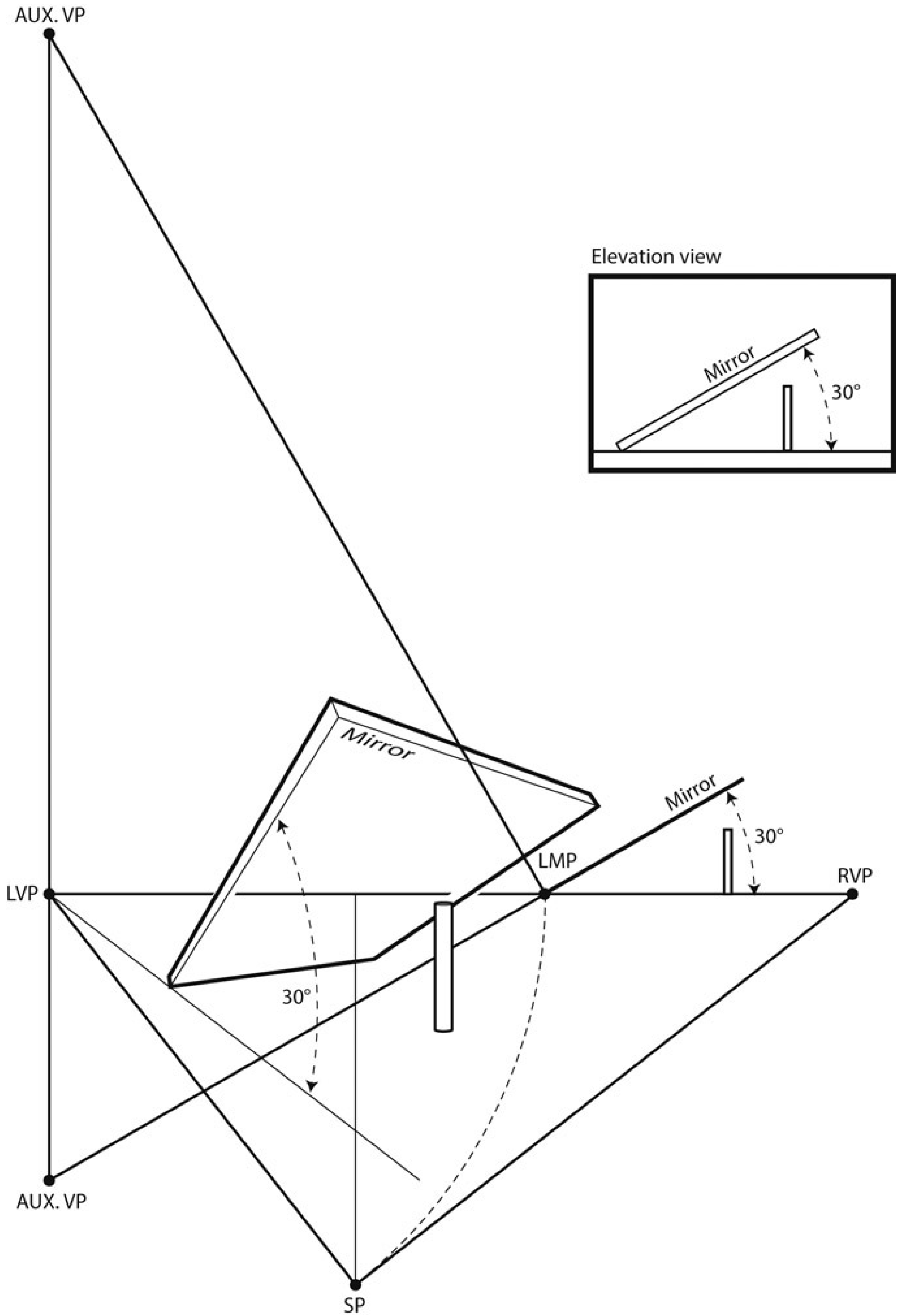
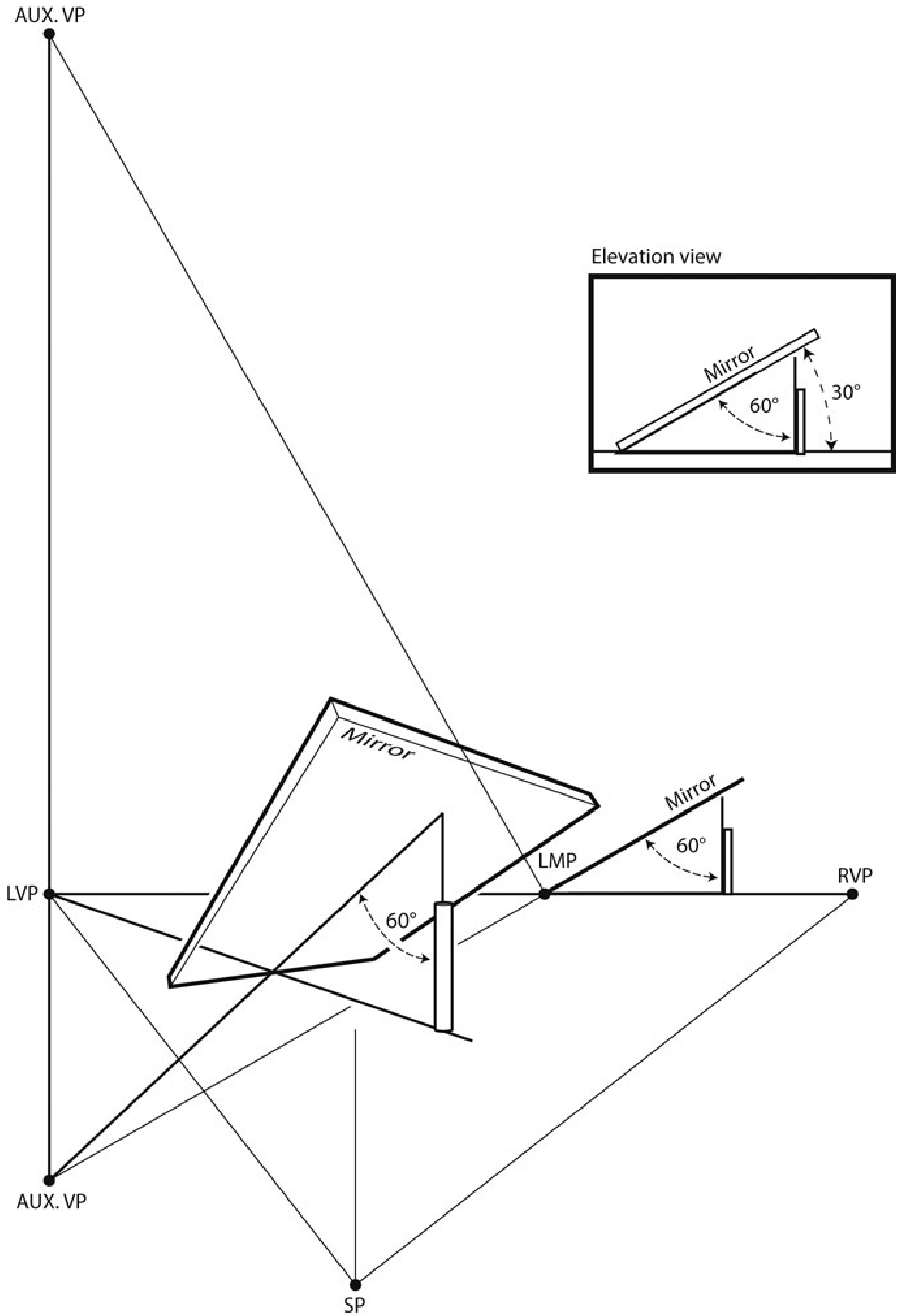


Figure 30.1 The mirror is 30° to the ground plane.



[Figure 30.2](#) **The angle of the object to the mirror is 60°.**

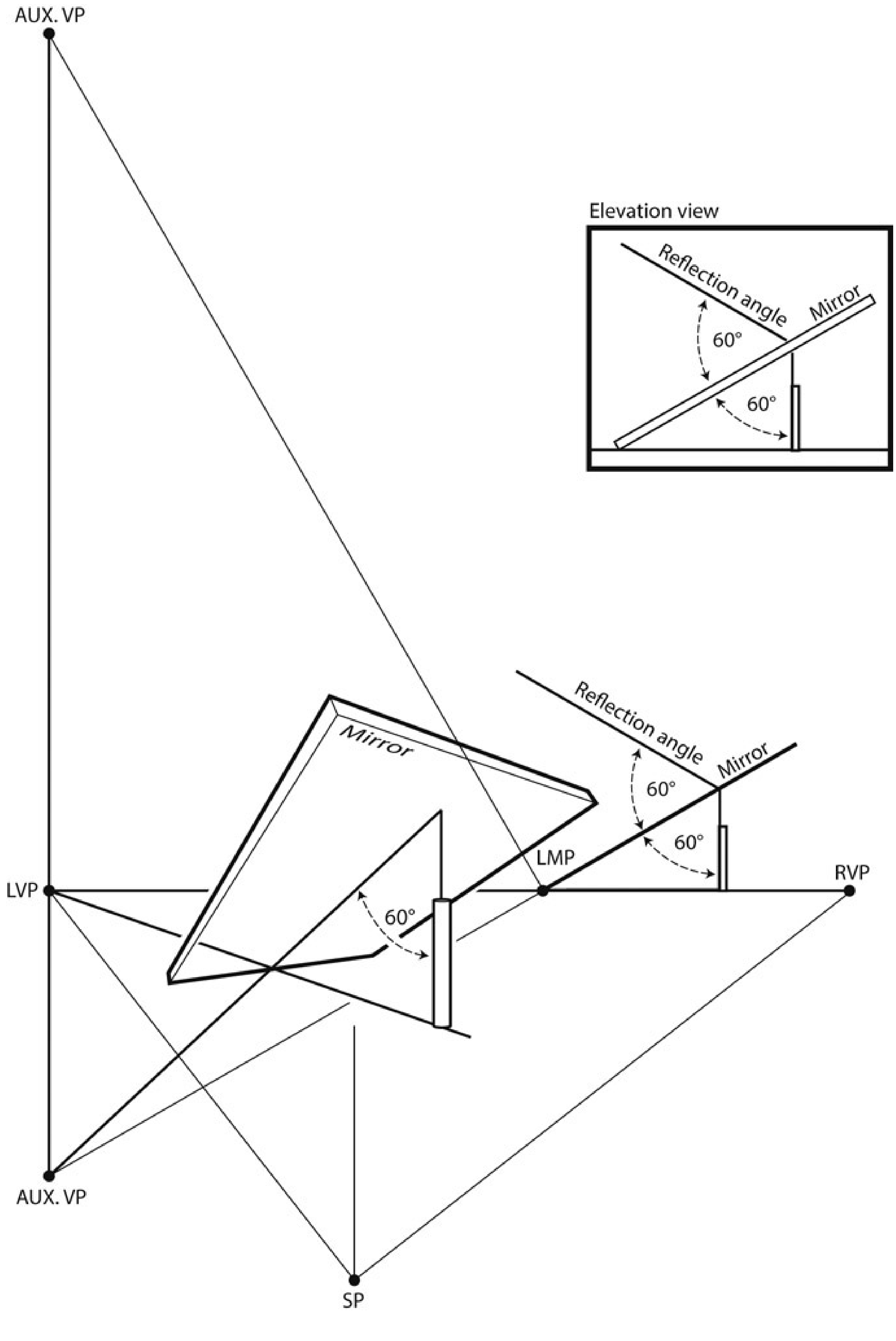

AUX. VP
Elevation view
Reflection angle
Mirror
60°
60°
Mirror
Reflection angle
Mirror
60°
60°
LMP
60°
LVP
RVP
AUX. VP
SP

Figure 30.3 If the angle of the object to the mirror is 60° then the angle of the reflection to the mirror is also 60°.

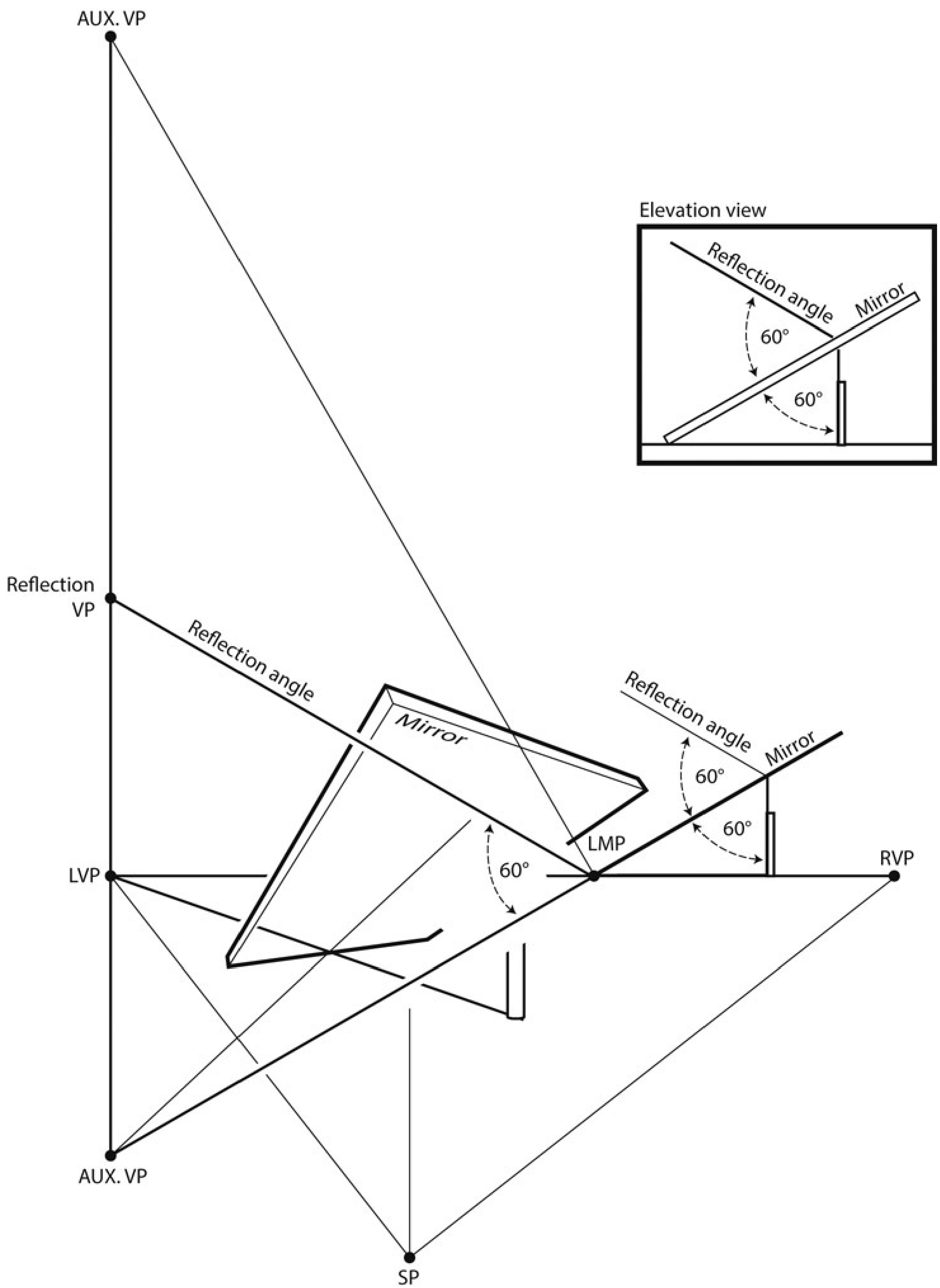


Figure 30.4 **Project a 60° angle from the mirror's surface (from the lower auxiliary vanishing point) to establish the reflection vanishing point.**

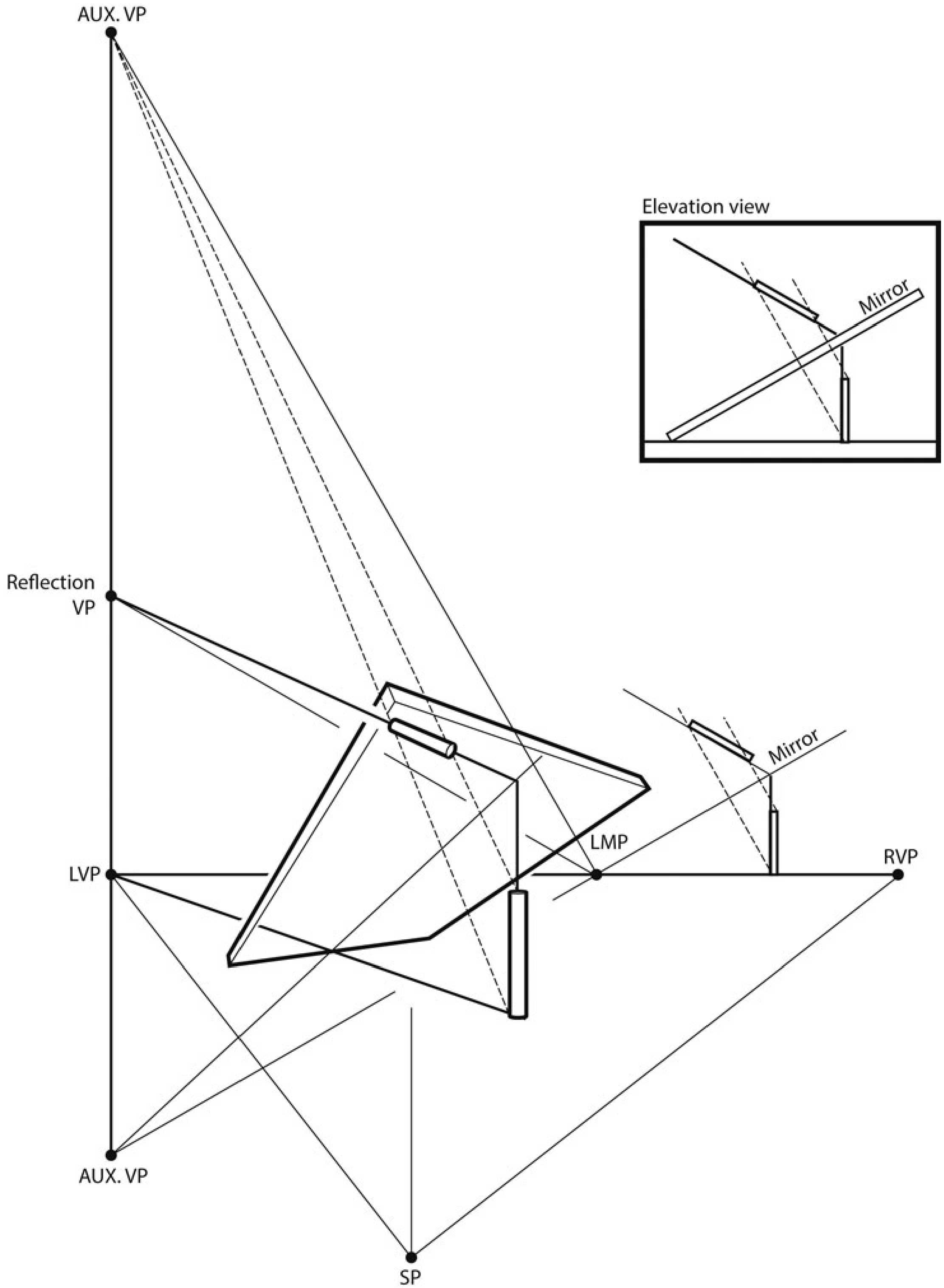


Figure 30.5 **Measure the reflection's length by drawing a line from the top and bottom of the object to a point 90° from the mirror's surface (the upper auxiliary vanishing point).**

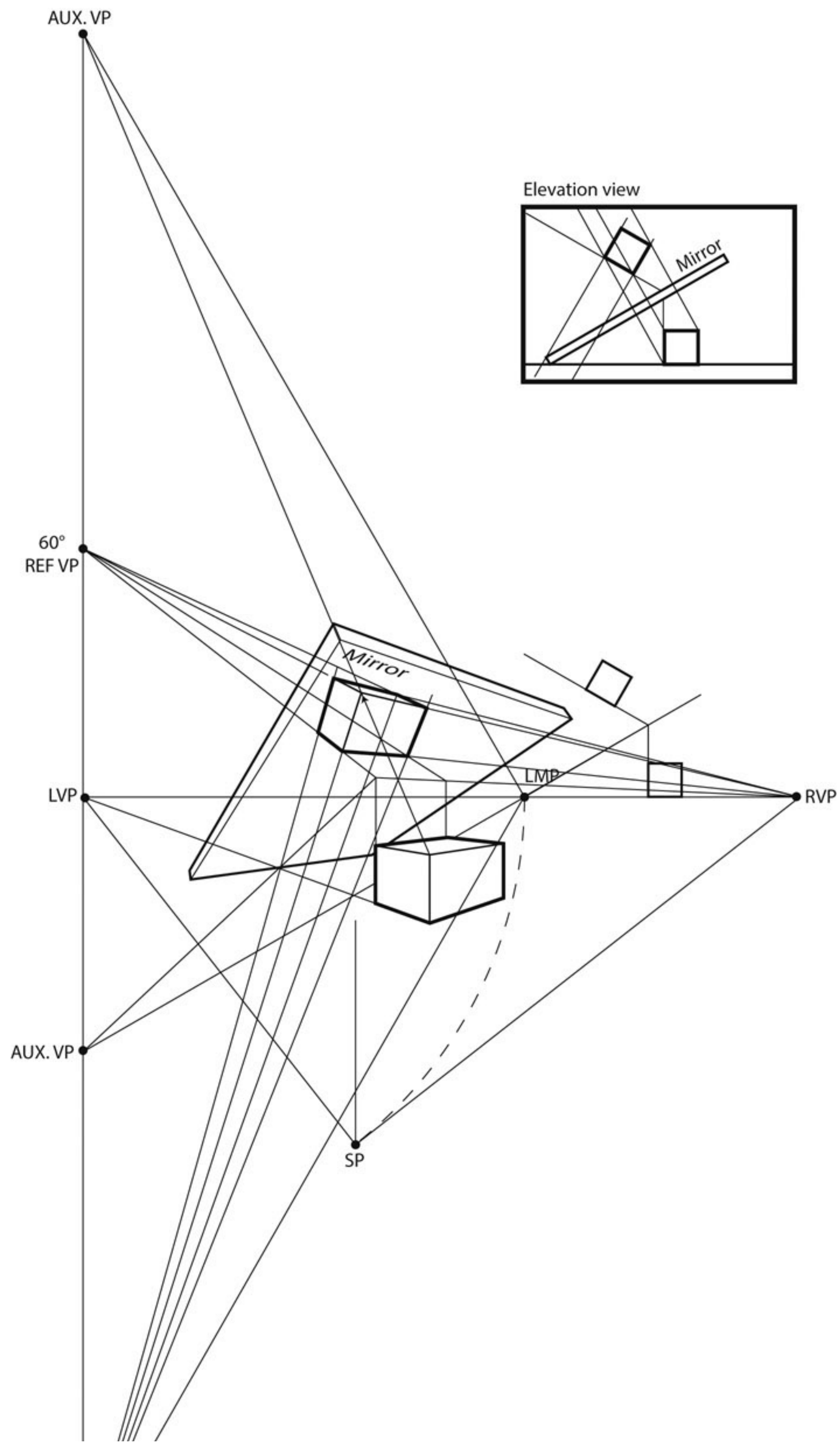


Figure 30.6 This reflection was created using the methods outlined in the previous illustrations.

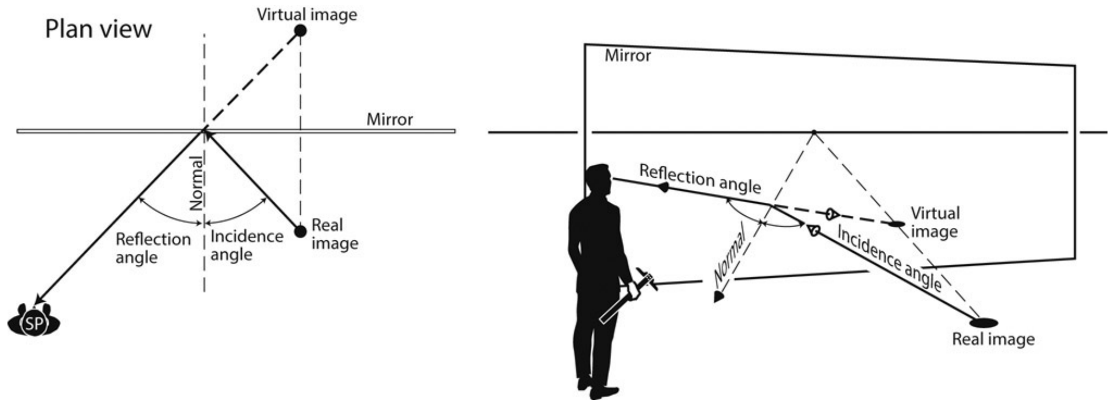
31
Reflections on Curved Surfaces

Reflections on curved surfaces are rarely discussed. The subject is usually ignored, or the advice is to "just fake it." This can be troublesome, so this subject will be explored further. But before looking at reflections on curved surfaces, reflections on flat planes will be reviewed.

Reflections are seen when light waves bounce off shiny surfaces. The wave's angle is key to the perceived position of the reflected image. Reflections appear to be inside the mirror. There is, of course, no object inside the mirror. So why is an object seen where none exists?

First, consider the science behind reflections. An angle 90° from a reflective surface is called "**normal**." Light rays hit reflective surfaces at specific angles to normal. This angle is called the incidence angle. The reflective light angle is always the same to normal as the **angle of incidence** (Figure 31.1). To put it simply: light bounces off a surface at the same angle it hits the surface. This is the law of reflection.

So, light is bouncing off the surface, but light is seen as a straight line. Because of this, two images are seen: the real object, and a virtual object (the reflection). The reflection is seen aligned with the reflection angle. In flat mirrors, reflections appear to be inside the mirror. The reflection appears at the same distance from the reflective surface as the real object (Figure 31.1).



[Figure 31.1](#) **The law of reflection states that the incidence angle to normal equals the reflection angle to normal**.

Convex Surfaces

The law of reflection works the same with curved surfaces. But, the position of the reflection is different than those on flat surfaces. The reflection's position changes with the position of the viewer. The distance from the real image to the mirror can't be measured and duplicated. This makes plotting a reflection *much* more difficult ([Figure 31.2](#)). To complicate matters, a cylinder reflects beyond 180°. Depending on the angle of incidence, objects behind the cylinder may be reflected (these reflections are thin slivers seen at the edge of the cylinder).

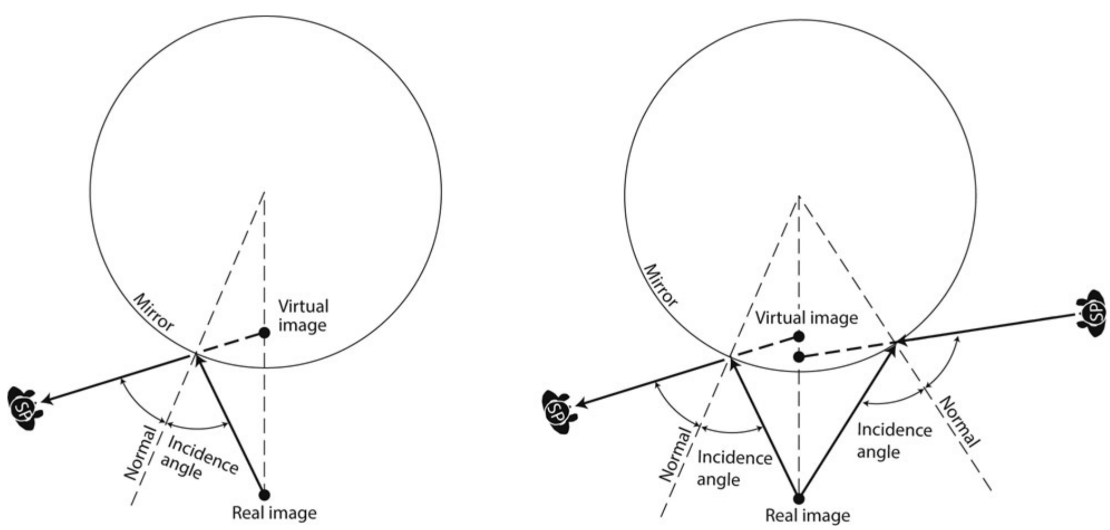
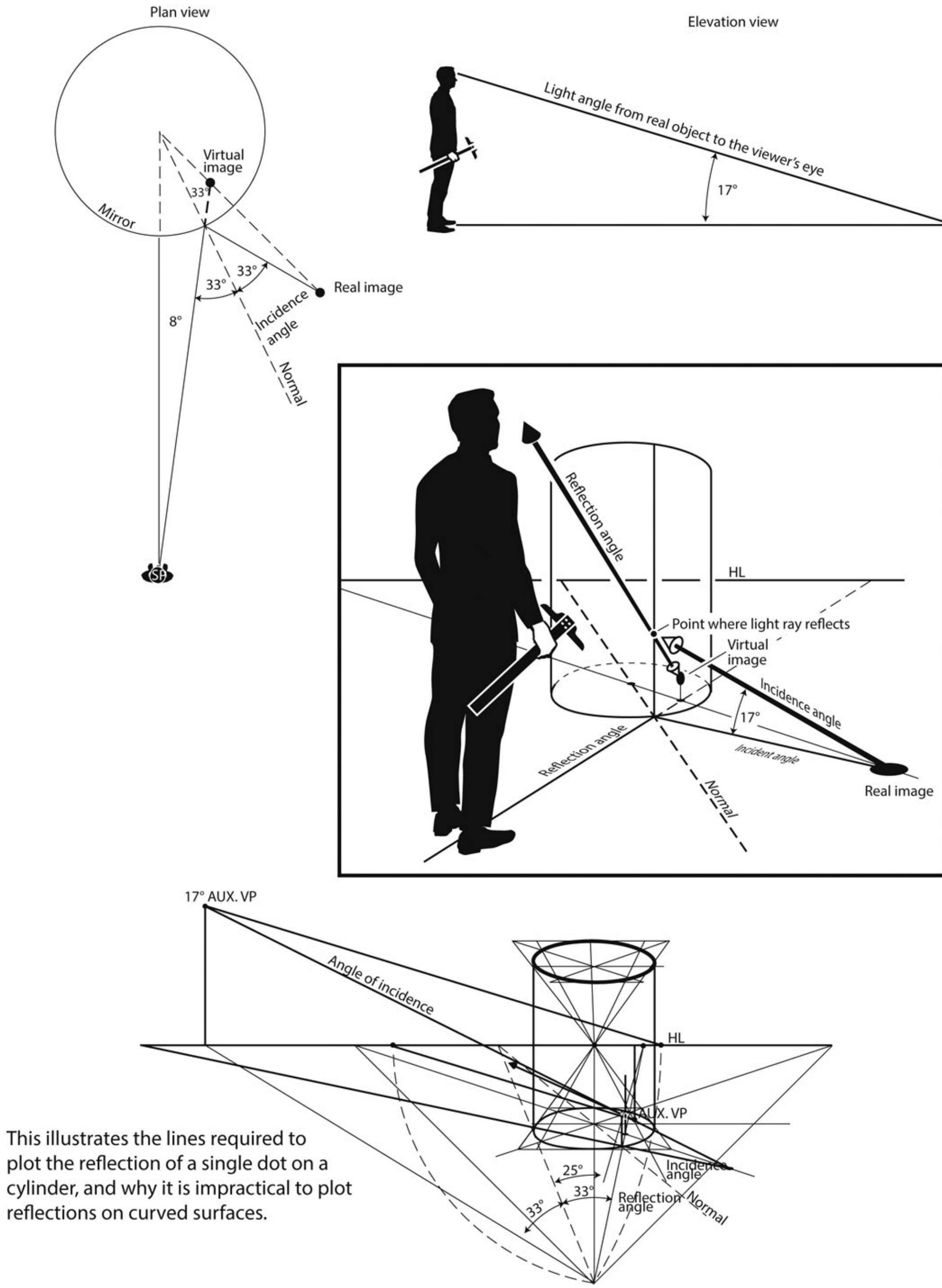


Figure 31.2 The reflection's position inside the convex mirror changes with the viewer's location.

To draw reflections on a cylinder, the incidence angle to normal needs to be plotted for each object. Although this is possible, it is impractical. While it may be attempted for "fun," it will also take a long time. Only a few points need to be plotted to understand how these reflections work (Figures 31.3–31.4). Likewise, only a few points need to be plotted to appreciate how time-consuming this exercise is.



[Figure 31.3](#) To draw a reflection on a cylinder, the angles of incidence and reflection must be plotted, which are at an incline as they travel to the viewer's eye. This is a difficult and impractical task. The bottom drawing is not designed as

an instructional guideline, but only to illustrate what is needed to plot a single dot on a reflective cylinder.

So, if it is not practical to plot reflections on curved surfaces, how are they approached? First, develop a solid understanding of the geometry. Understand the relationship between the viewer, the incidence angle, and the virtual image. Knowledge is the best tool. In addition, have at hand a collection of curved reflective objects. Metallic paper can be purchased at most art stores, and bent into many different shapes. Seeing the actual reflections brings the science to life. With an understanding of the science, and a collection of reflective objects to refer to, intelligent and convincing reflection estimations can be made.

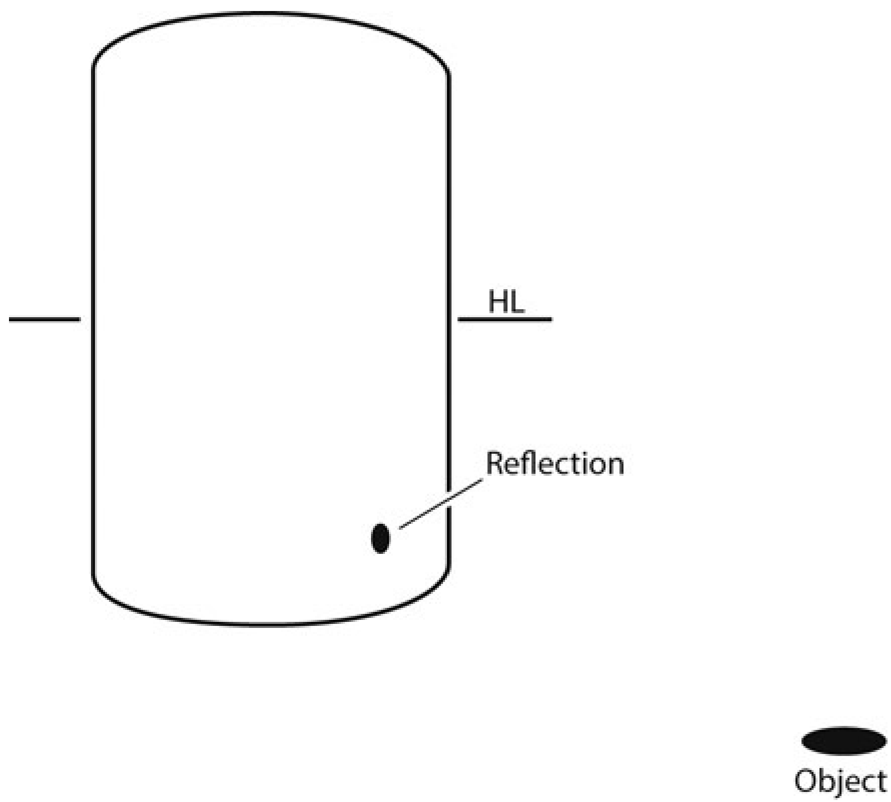


Figure 31.4 It was a great deal of work to draw this single reflection. Reflecting an entire environment in a curved surface is an unreasonably difficult task.

Concave Surfaces

There is a subject even more difficult than convex surfaces: concave surfaces. Fortunately, reflective concave surfaces are not common; they will be briefly addressed nonetheless. To fully cover the complexity of this topic would require more space than is available, so here is an overview.

Concave surfaces have a center point (the center of the reflective surface's arc) and a focus point (a point halfway between the center point and the mirror) (Figure 31.5, upper left). While the rule governing the angle of incidence and angle of reflection to normal still applies, the angle of reflection always passes through the focus point (Figure 31.5, upper right).

Four examples of reflections on concave surfaces are presented. The reflections are quite different depending on the position of the object being reflected. Sometimes the reflection is reversed; sometimes it is upright.

Reflections of objects between the viewer and the center point appear smaller, reversed, and in front of the mirror (Figure 31.5).

Reflections of objects between the focus point and center point appear larger, reversed, and in front of the real object (Figure 31.6, top).

Objects at the focus point disappear—in theory. In reality, the reflection looks like a blurred streak across the mirror surface (Figure 31.6, middle).

Objects between the focus point and the mirror are larger and not reversed, and appear behind the mirror (Figure 31.6, bottom).

The usual advice for drawing reflections on curved surfaces is to "just fake it." But if this is not understood, then the results will be far from convincing. This is not to say that plotting reflections on curved surfaces is advocated; it is extremely difficult and time-consuming. The previous information was designed to give a basic understanding of the underlying science so that it *can* be faked with confidence. Approximations should be based on educated estimations, not on random guesses.

Again, it is helpful to purchase a piece of reflective paper. This paper can be manipulated to create a variety of curved shapes, which can be a valuable reference tool.

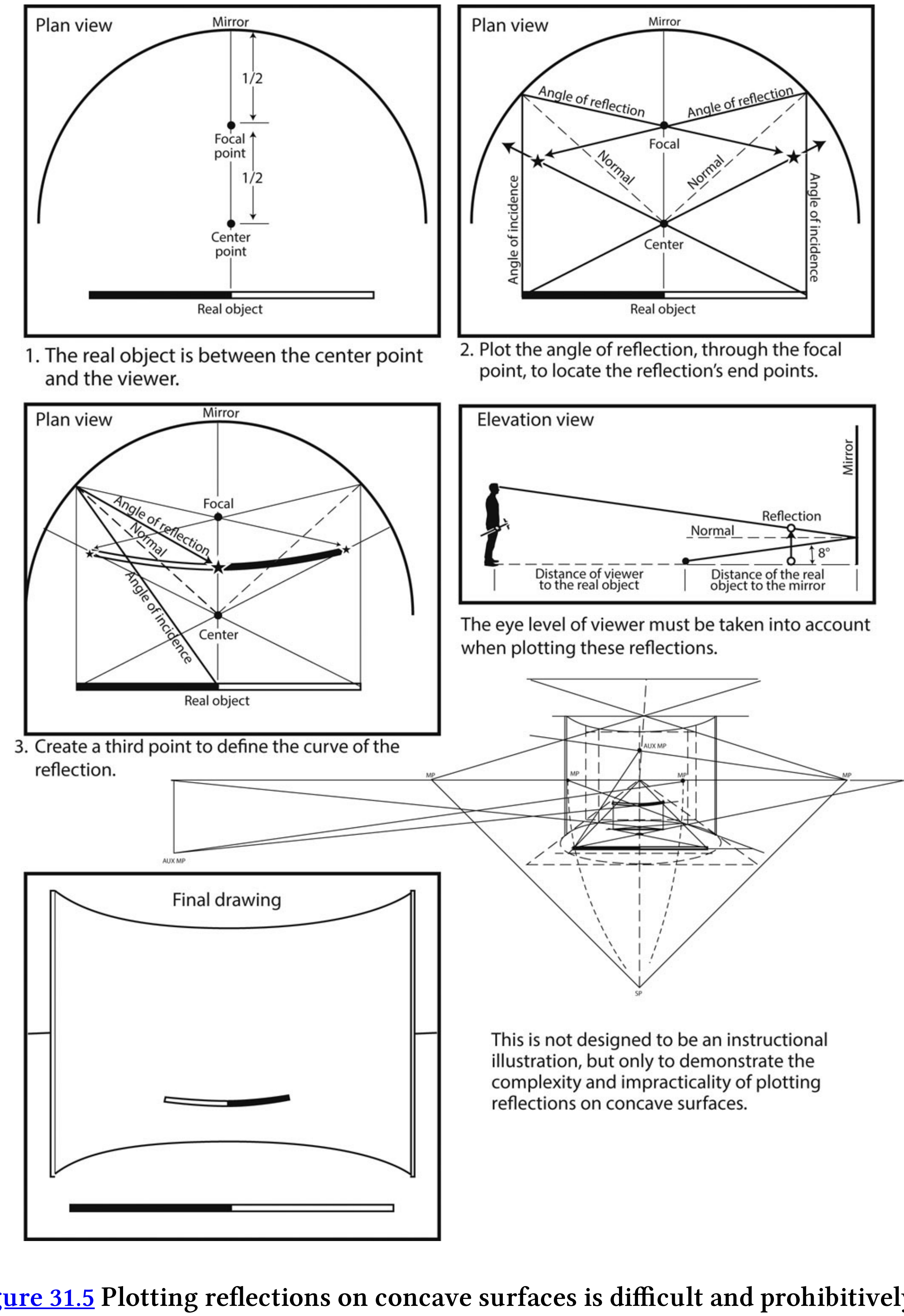
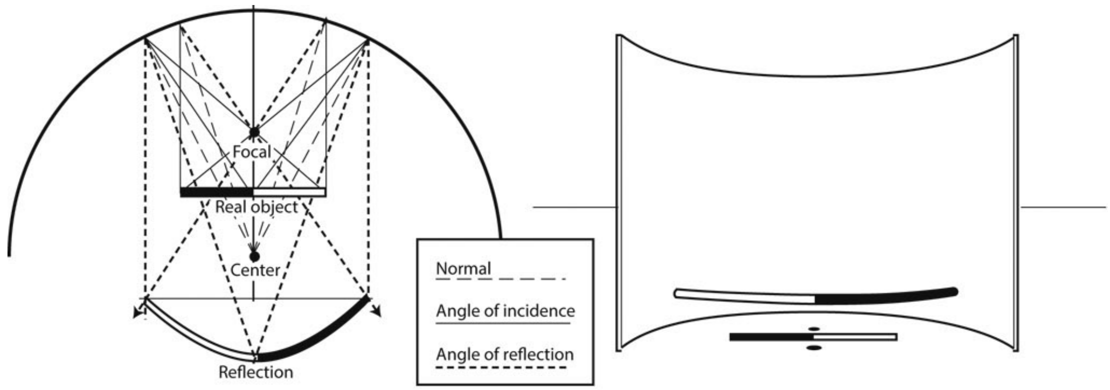
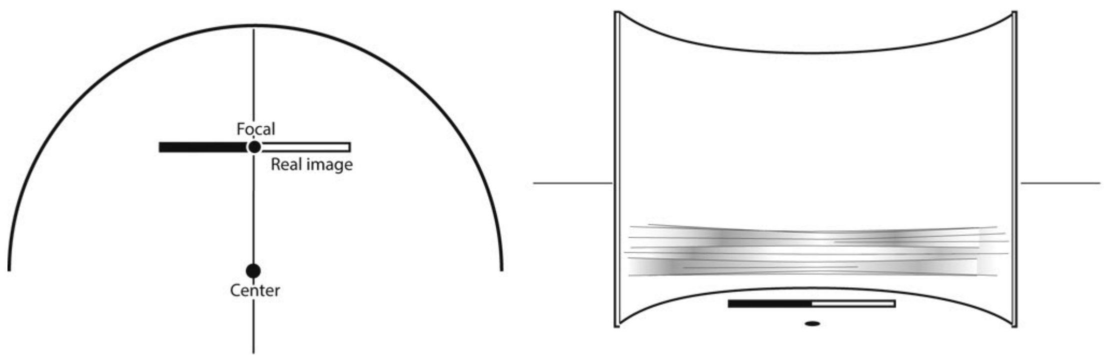


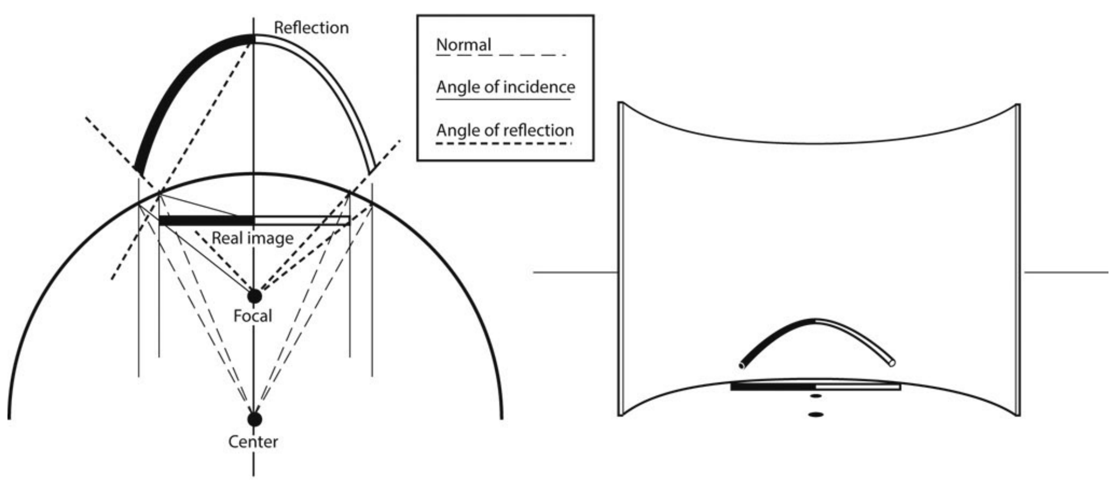
Figure 31.5 Plotting reflections on concave surfaces is difficult and prohibitively time-consuming to have a practical application.



The real object is between the center and the focal point.
The reflection is reversed, larger than real object, and projected in front of the mirror.



The real object is on the focal point.
There is no reflection, only a blured, streaked image.

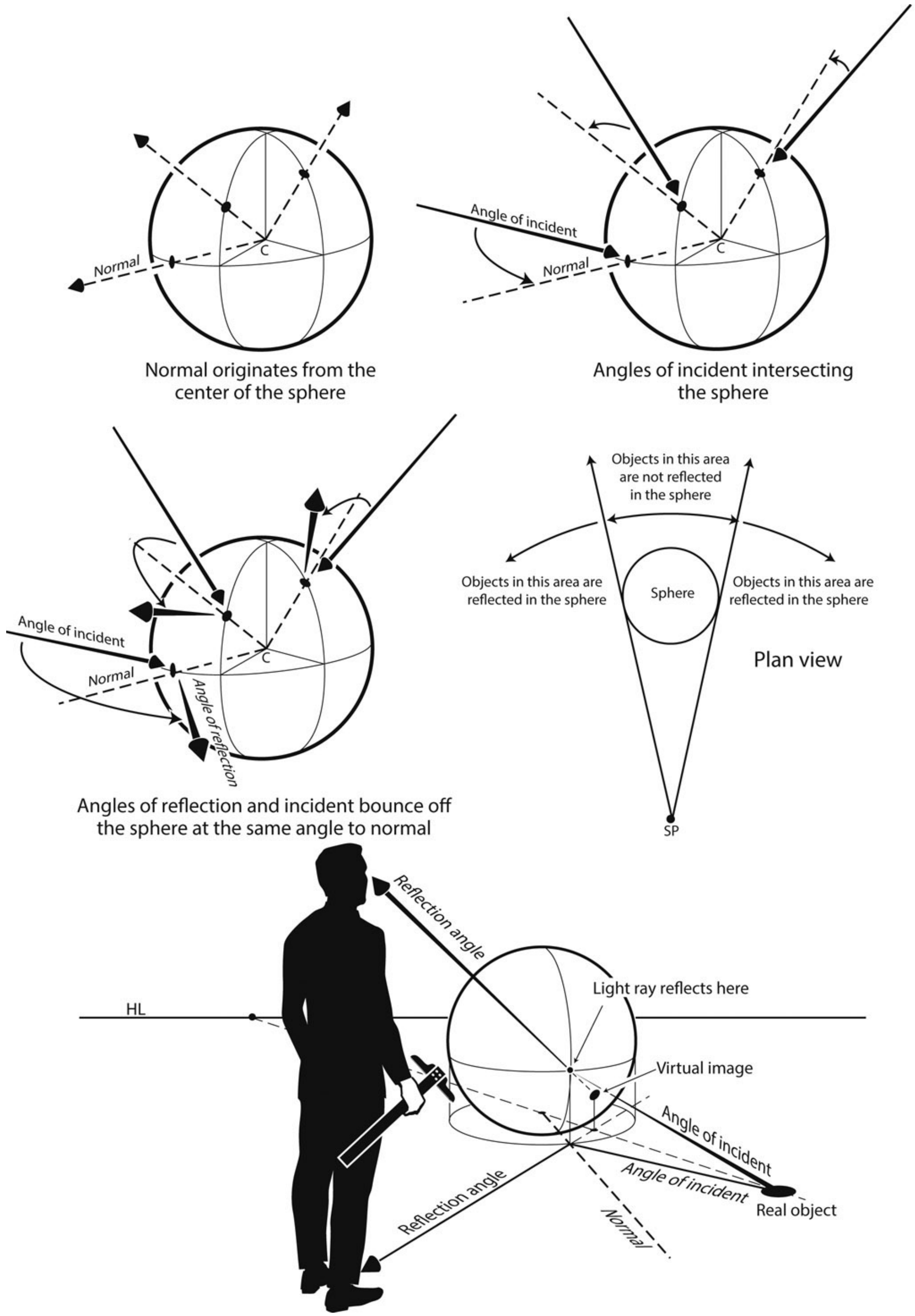


The real object is between the focal point and the mirror.
The reflection is not reversed and is projected behind the mirror.

Figure 31.6 **Reflections in convex mirrors change dramatically depending on the location of the real object.**

Sphere

Plotting reflections on a sphere requires the same tedious process. The rules of reflection still apply. Reflections on curved surfaces can be plotted by hand—but it is not recommended (Figure 31.7). Like convex and concave surfaces, the process is prohibitively time-consuming. Due to the exhausting task, most art books simply ignore the topic. While it is not practical to plot reflections on curved surfaces, it is empowering to have the knowledge—just *knowing* may be enough. To create more convincing reflections, it is best to find a reflective sphere to use as reference. Using a reflective curved surface as a guide, along with an understanding of the science behind reflections, will help you to create convincing images.



[Figure 31.7](#) **Sphere reflections are challenging, to say the least.**

32
Anamorphic Perspective

Anamorphic images came about early in the development of perspective. Artists such as Piero della Francesca and Leonardo da Vinci experimented with solutions to correct the distortion caused when looking obliquely at church frescos. As the congregation looked up at the frescos, the nearby feet looked exceptionally large compared to the distant heads (Figure 2.4). A solution to this problem was quickly formulated. The desired image was projected on the wall using a grid. This projected image corrected the distortion. Anamorphosis is the process of projecting a flat image on an oblique surface. Looking at this projected image from an angle other than the one it was projected from results in a distorted drawing—often to the point of non-recognition. But the image comes into focus when seen from the location it was projected from. Street artists use this technique with amusing effects in sidewalk chalk festivals.

Beyond its practical use, anamorphosis is sometimes used as an entertaining trick. An artist creates a mysterious skewed image; the viewer must then find the vantage point that reveals the picture (Figures 32.1–32.2).

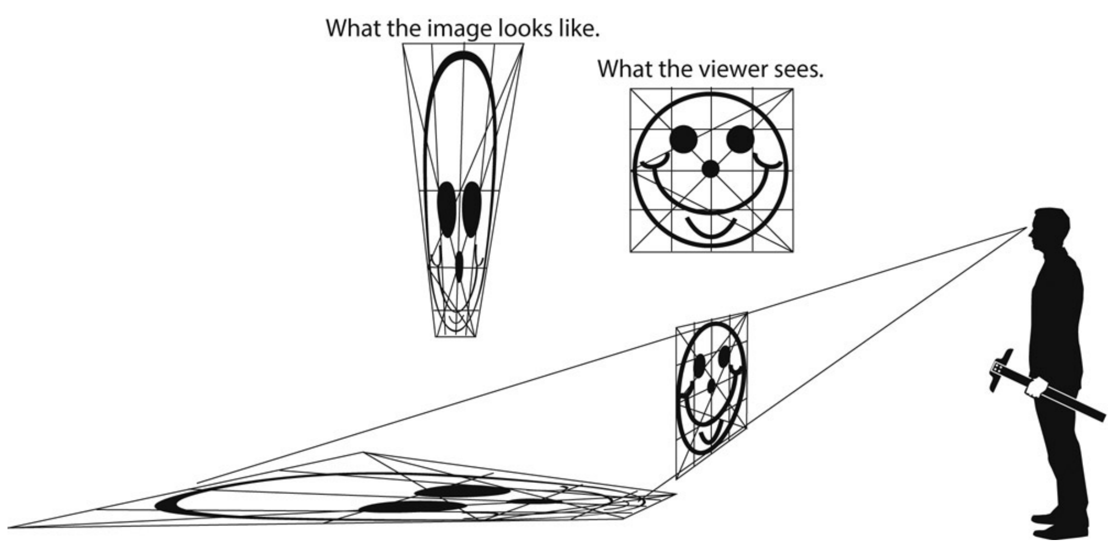
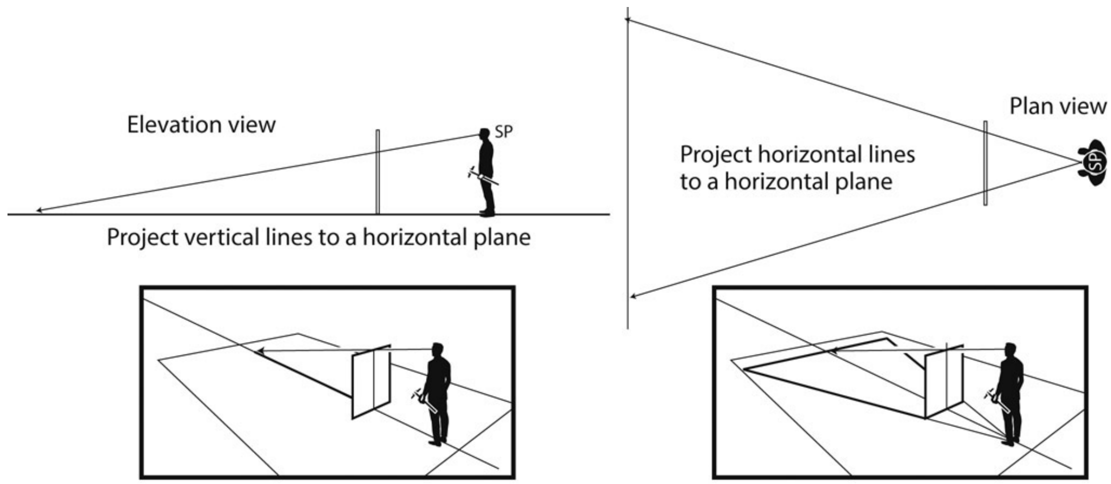


Figure 32.1 **The anamorphic image comes into focus when seen from the proper point of view**.



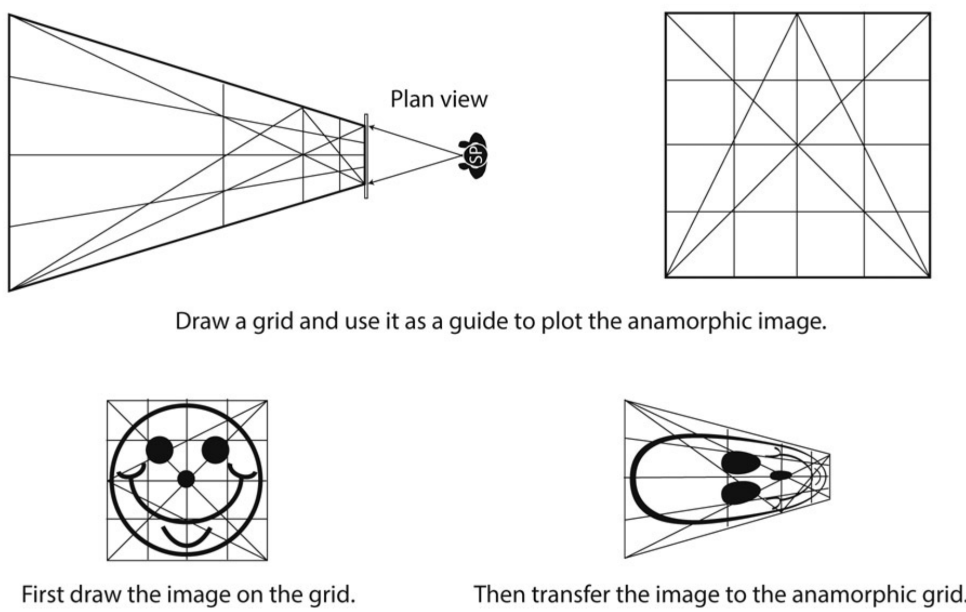


Figure 32.2 **Plotting an anamorphic grid**.

Foreshortened Image

On a related subject, this technology can be used to depict a flat image on a foreshortened surface. For example, if drawing a picture hanging on a wall, with this picture being foreshortened, this same procedure could be used in reverse. Instead of the final image being stretched, it is compressed. First, put a grid on the picture being drawn, and then draw that same grid in perspective. Use the grid as a guide to plot points (Figure 32.3).

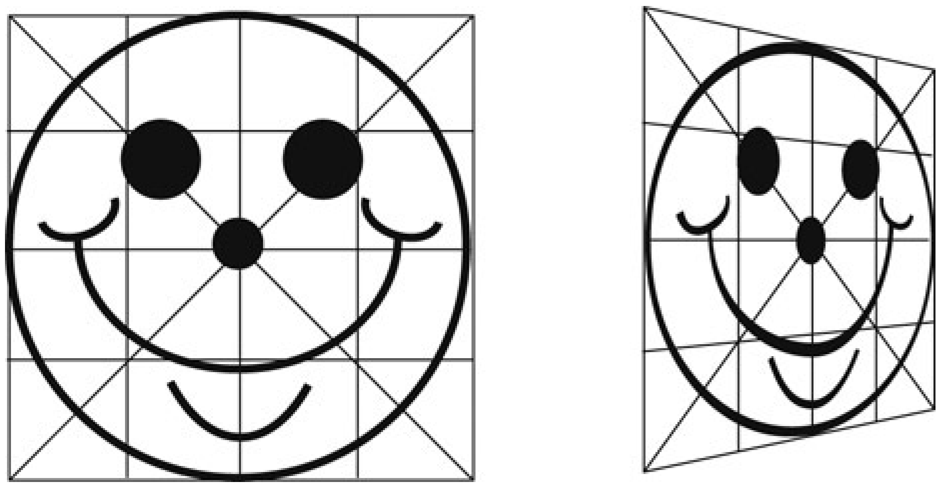


Figure 32.3 To draw a foreshortened image, draw a foreshortened grid. Be sure to adjust the thickness of the lines: they are foreshortened as well.

33
Four-Point Perspective

Objects not aligned with the center of vision become distorted. The farther they are from the center of vision the more distorted they become. Distortion occurs when the visual pyramid intersects the picture plane at an oblique angle. The more aslant the angle, the greater the distortion. But what if the picture plane is not flat? What if the picture plane is curved? The picture plane, after all, does not *need* to be flat. If the picture plane is cylindrical, the intersection of the visual pyramid will be at a right angle. The viewer's eye is at the center of the cylinder. Any line connecting to the station point will intersect the picture plane at a 90° angle to its surface. If the cylinder fully surrounds the viewer, a 360° panoramic view can be drawn without the extreme distortion caused by a flat picture plane.

Using a curved picture plane still creates distortion, it is just a different kind of distortion. With a curved picture plane, horizontal lines appear curved in the final drawing. Vertical lines, however, are parallel with the picture plane and are drawn as straight lines.

The picture plane, being circular, gives a 360° image area horizontally. But, because the walls are flat, the standard 60° cone of vision still applies to the vertical axis (Figure 33.1).

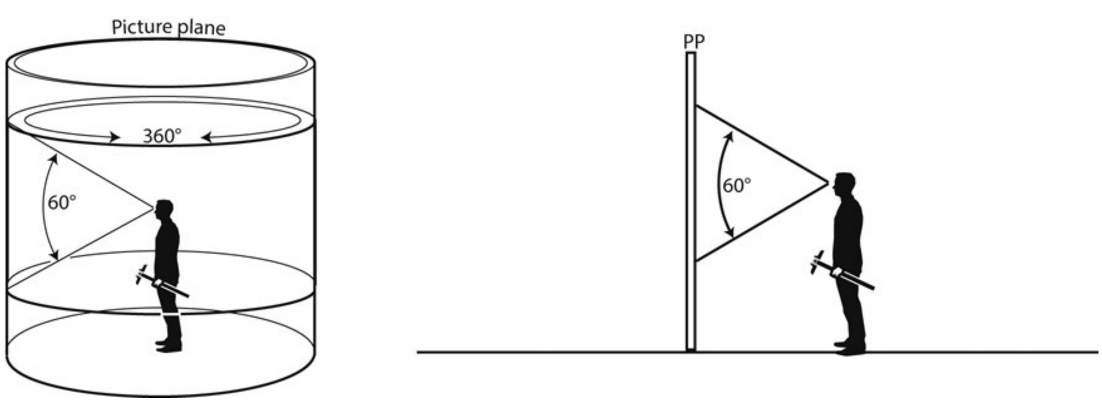


Figure 33.1 **The picture plane is cylindrical in four-point perspective**.

Four-Point Perspective Components

Vanishing Points

There are four vanishing points, each 90° apart. One at the center of vision, one 90° to the right, one 90° to the left, and one directly behind the viewer.

To illustrate how these vanishing points function, draw a symmetrical room with the viewer placed in the center. A vanishing point is centered on each wall (Figure 33.2).

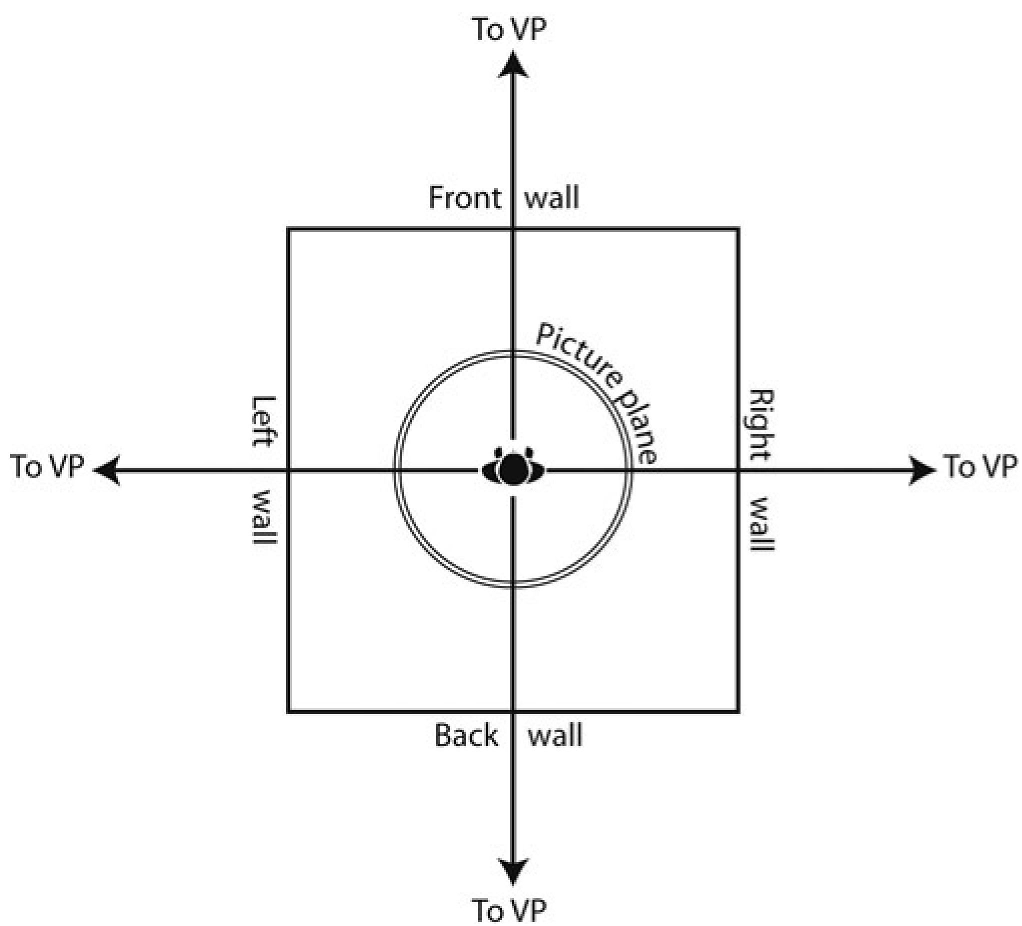


Figure 33.2 A plan view showing the viewer, the picture plane, the room, and the four vanishing points.

Image Area

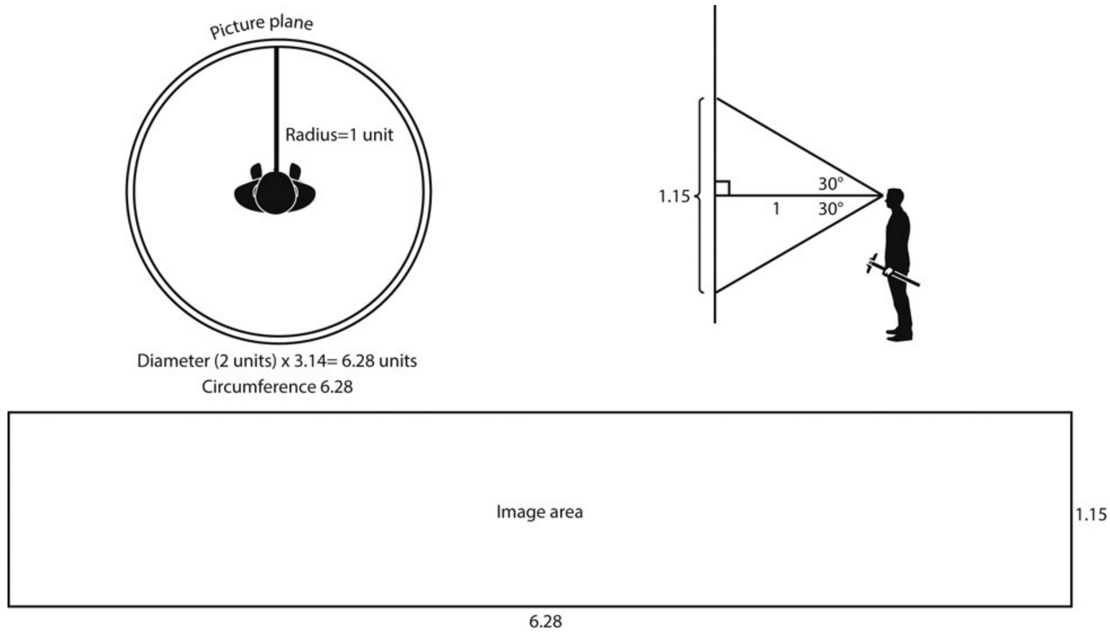
The picture plane is cylindrical, but the drawing surface is two-dimensional. So, the first step is to flatten the picture plane. The image area will be 360° long and 60° high.

Length

To calculate the length, the **circumference** of the picture plane must be known. This is a simple formula: multiply the diameter by pi (π). To do this, measure the distance from the station point to the picture plane (the radius). Multiply the radius by two (the diameter). Then multiply the diameter by 3.14 (pi). The circumference of the picture plane is the length of the panorama (Figure 33.3).

Height

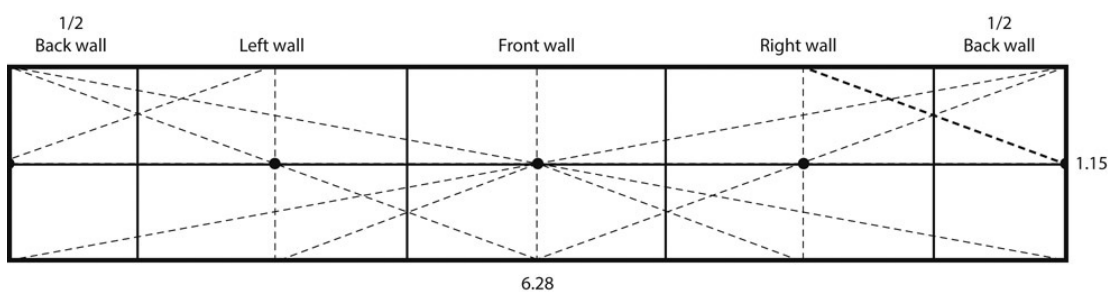
The height of the panorama remains 60° from the station point. Use the **Pythagorean theorem** to find the height. Or, draw an elevation view (to scale), with a 60° cone of vision, and measure the height with a ruler (Figure 33.3, top right).



[Figure 33.3](#) **Calculating the length and height of the panorama.**

The Drawing

The viewer is in the center of a square room. This means the walls are equally spaced and the drawing can be divided into four even quadrants. Each quadrant represents the width on the wall. Then, place a vanishing point in the center of each wall ([Figure 33.4](#)).



[Figure 33.4](#) **Divide into quadrants to find the four walls. Use a ruler, or draw an X to divide by half. Add vanishing points centered in each wall.**

Horizontal Lines

Imagine a horizontal line of infinite length, at a right angle to the line of sight. This line would disappear at the vanishing points that are 90° to the right and left of the center of vision. If this line was on the ground, and it was traced on the picture plane, it would curve up to connect with the vanishing points (Figure 33.5). Lines above the eye level would curve down. These curves are not simple compass arcs—that would be too easy.

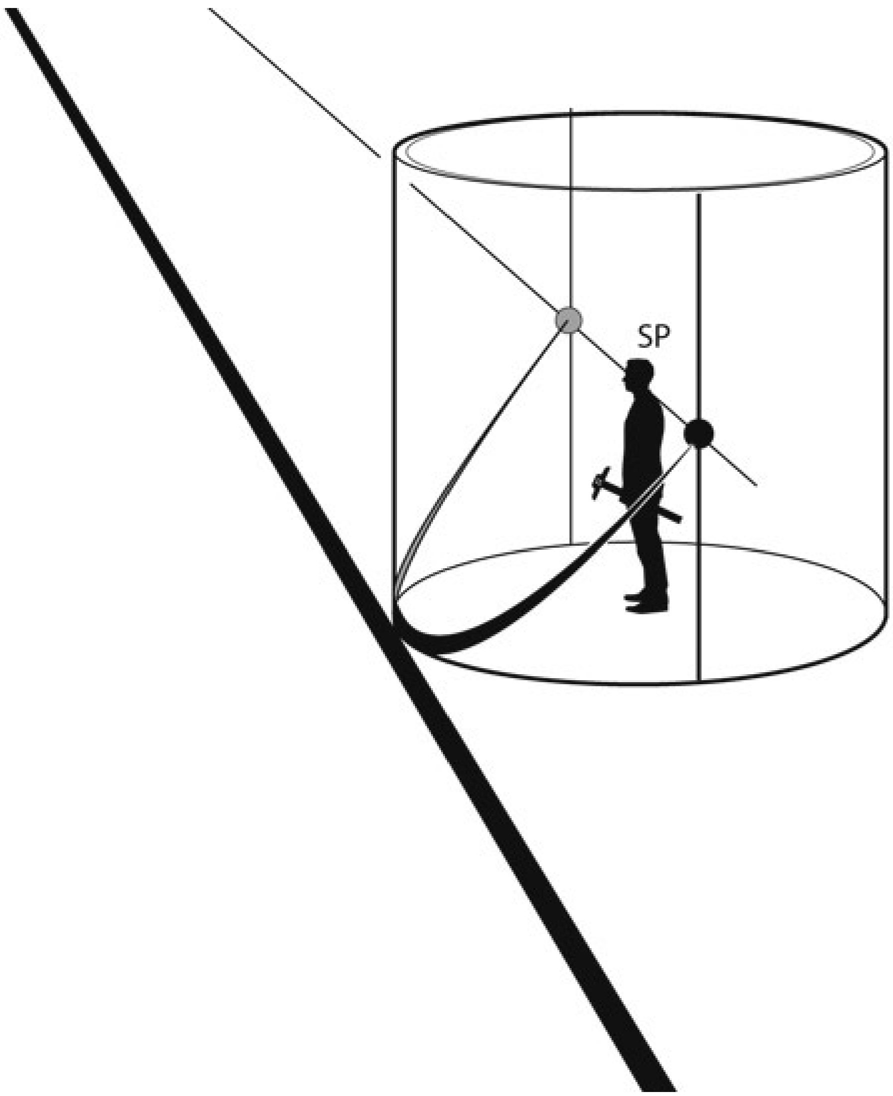


Figure 33.5 An infinite horizontal line traced on a cylindrical picture plane creates sinusoid.

To plot this curve, the viewer's eye level must be known (2 units above the ground), as well as how far the viewer is from the picture plane (1 unit) (Figure 33.6).

This line connects to the right and left vanishing points, and touches the picture plane at the ground level. This gives three known points (Figure 33.6).

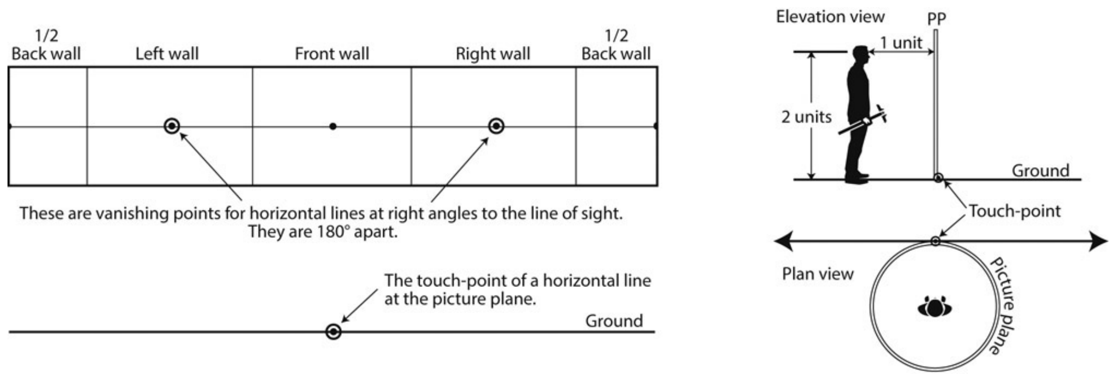


Figure 33.6 The first three points of the sinusoid are the "easy" ones to locate.

These three points can be connected to draw an arc, but three points are not enough to draw an accurate shape. At least two more points are needed.

Place these additional two points 45° on each side of the center of vision. Since the room is square, they will be aligned with the room's corner (Figure 33.7).

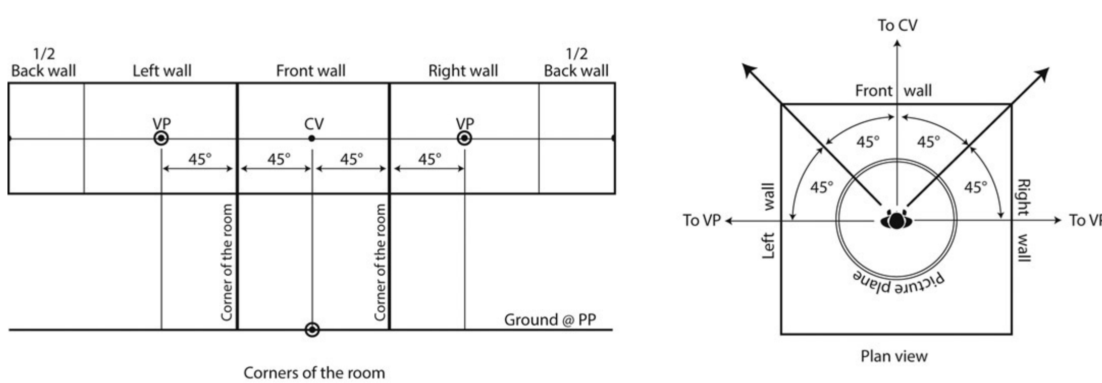


Figure 33.7 Because the room is a square, the corners are 45° from the station point.

The points that are needed indicate the distance from the line being drawn to the picture plane. This distance must be plotted. This can be accomplished using the Pythagorean theorem, but there is also a longhand method that does not require any math. Draw a plan view to scale, and with a ruler, measure the distance from the station point to the picture plane and the line being drawn (Figure 33.8, top left). Transfer those dimensions to an elevation view drawn to the same scale, then measure the distance along the picture plane (Figure 33.8, top right). Transfer that distance to the drawing (Figure 33.8, bottom). Now there are five points available to plot the curve. Connect the points to draw an arc. It is not a compass arc, but a sinusoid (Figure 33.8, bottom).

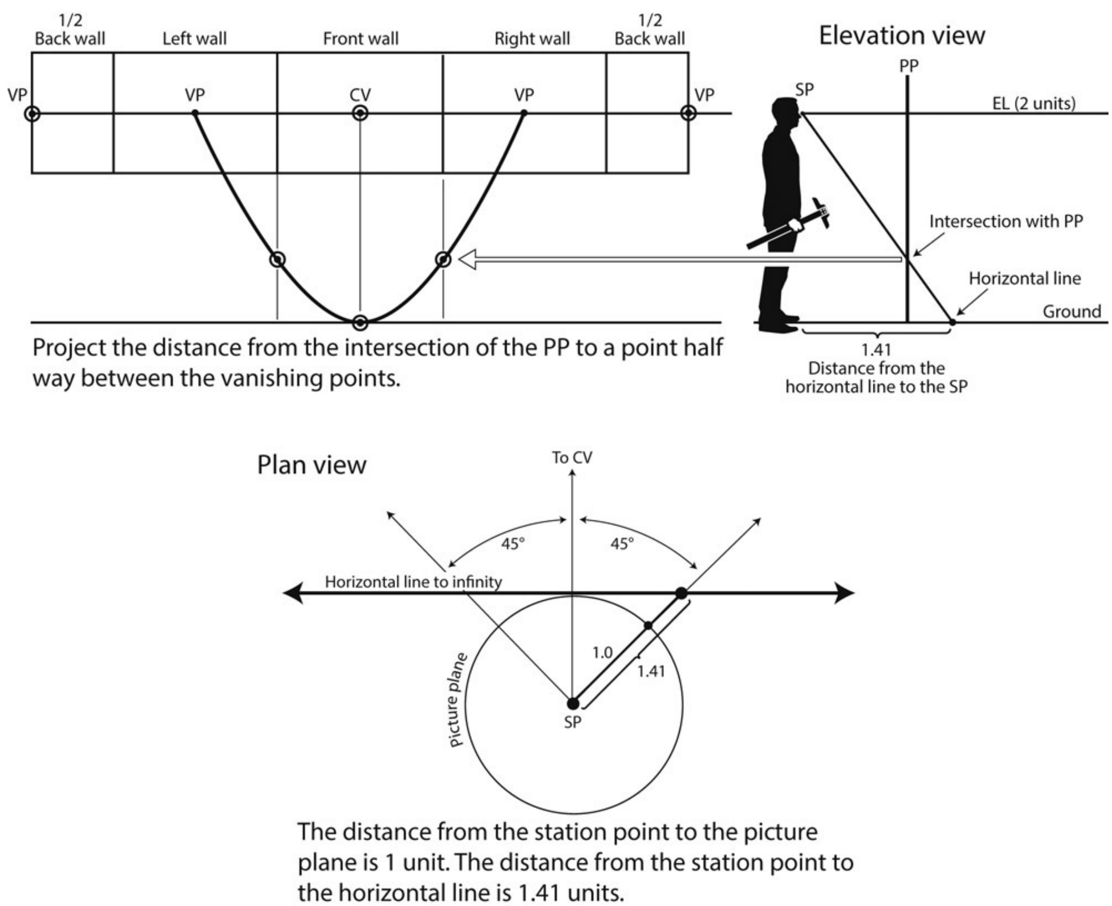
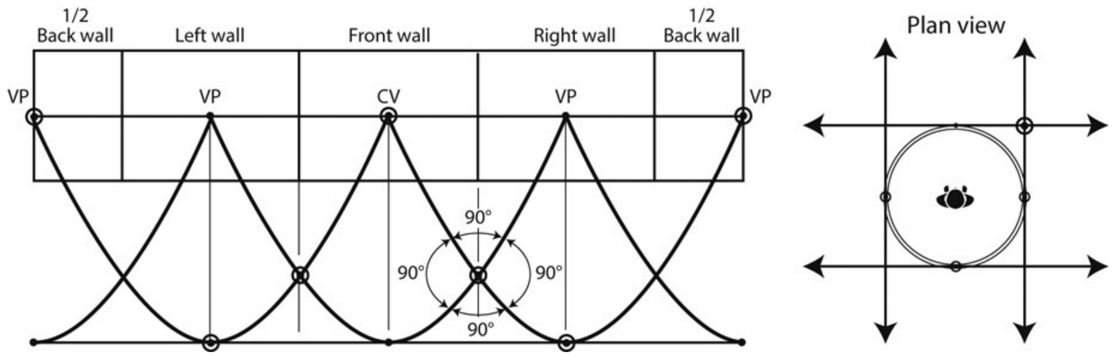


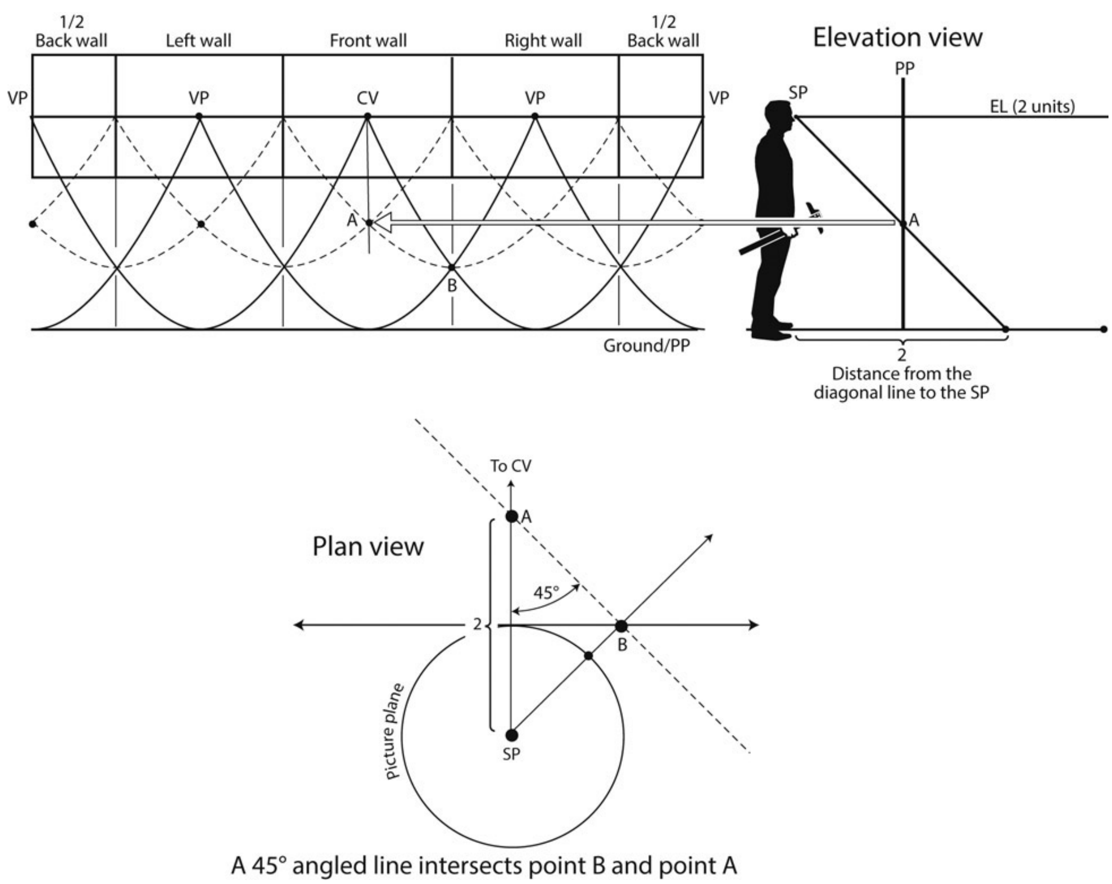
Figure 33.8 **To plot an accurate curve, two additional points are needed.**

Copy and repeat the curves, connecting each to vanishing points. The intersection of the lines creates a 90° corner (Figure 33.9).



[Figure 33.9](#) **Copy and repeat the lines, creating four orthogonal lines of infinite length**.

Copy and repeat the curves again, but this time connect each line to a point halfway between the four vanishing points. These lines are at a 45° angle ([Figure 33.10](#)). These 45° angles will later assist in drawing the grid.



[Figure 33.10](#) **Plot a 45° angle using the procedure outlined in Figure 35.8.**

Create an additional sinusoid behind the first, using the same procedure (Figure 33.11).

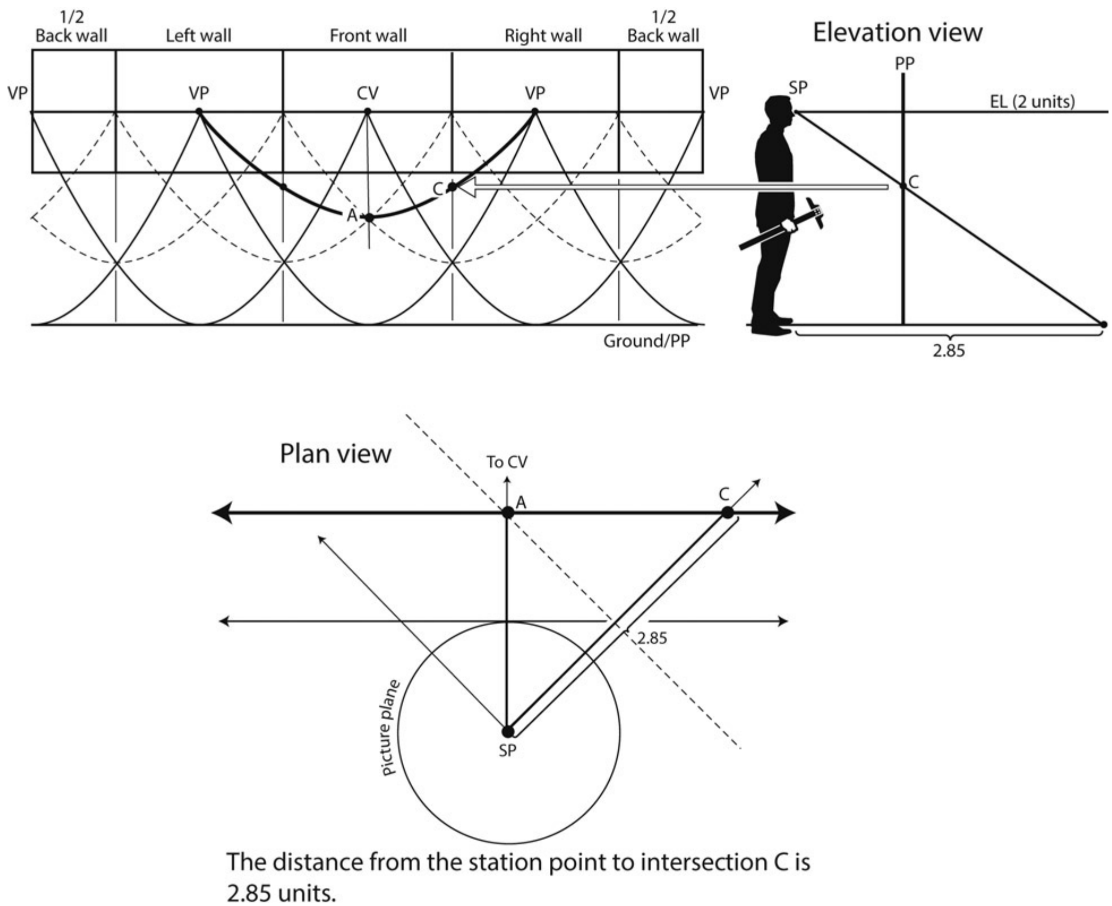


Figure 33.11 Repeat the process to plot an additional horizontal line.

Copy and repeat the sinusoids to create a series of squares (Figure 33.12).

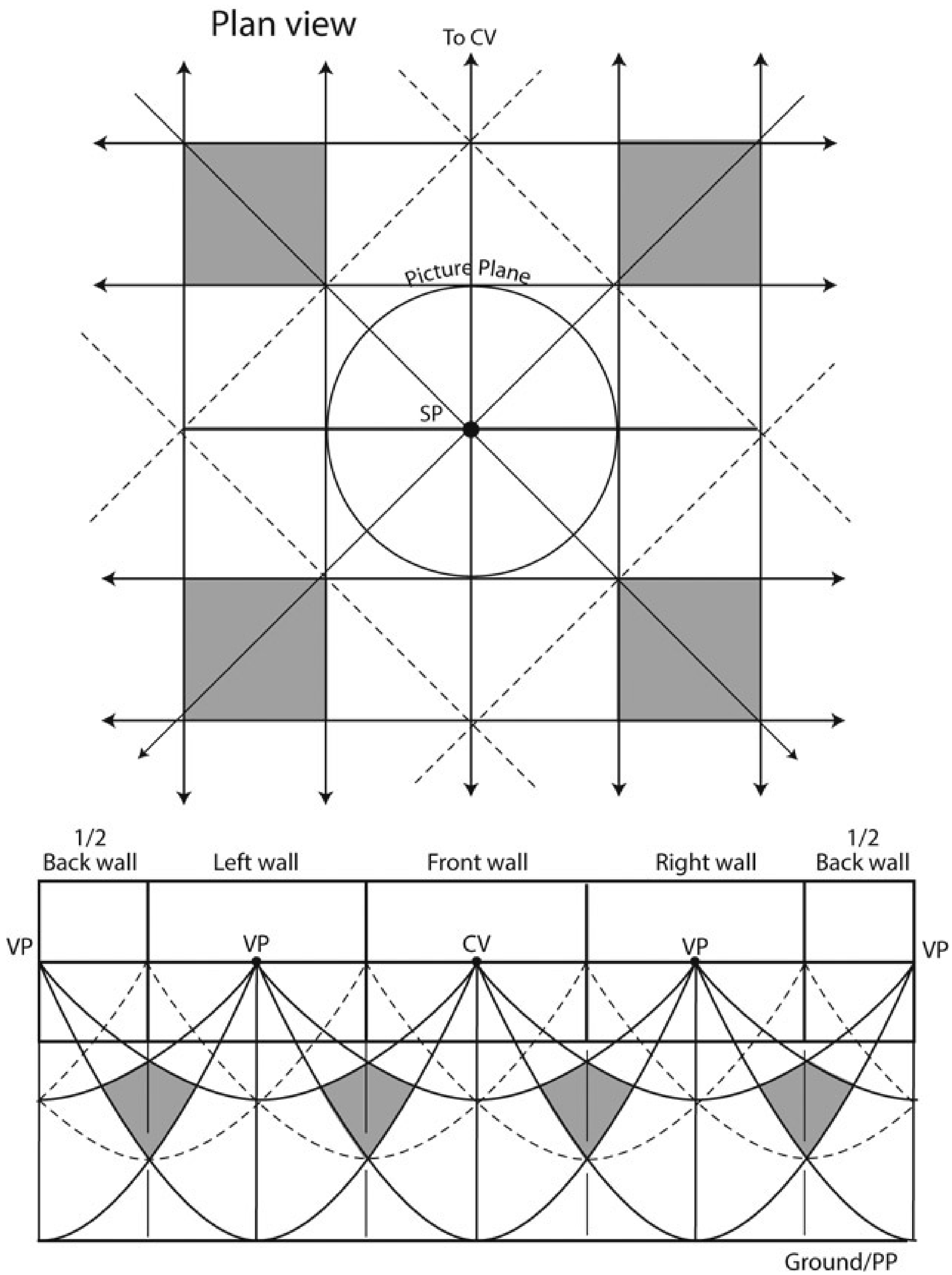
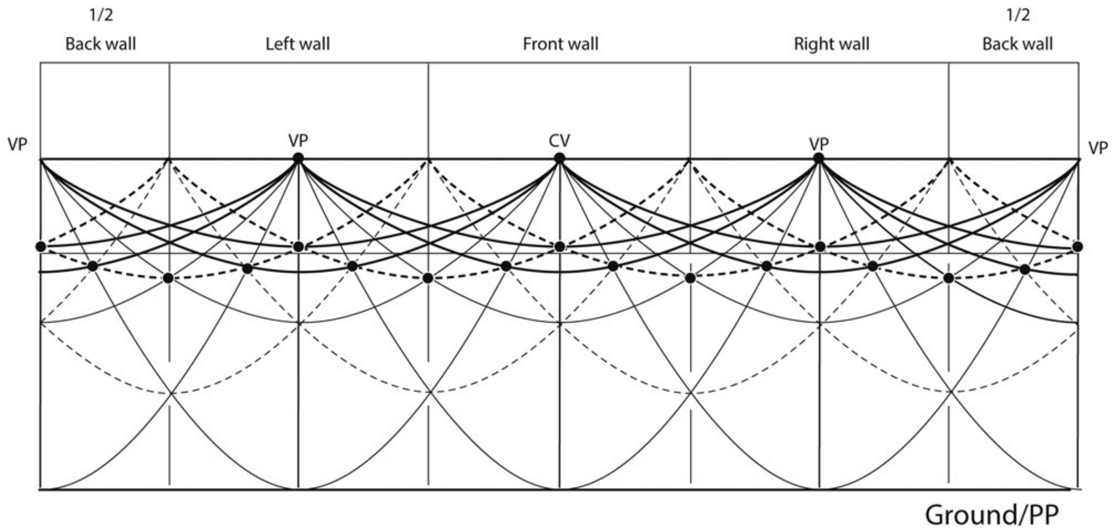


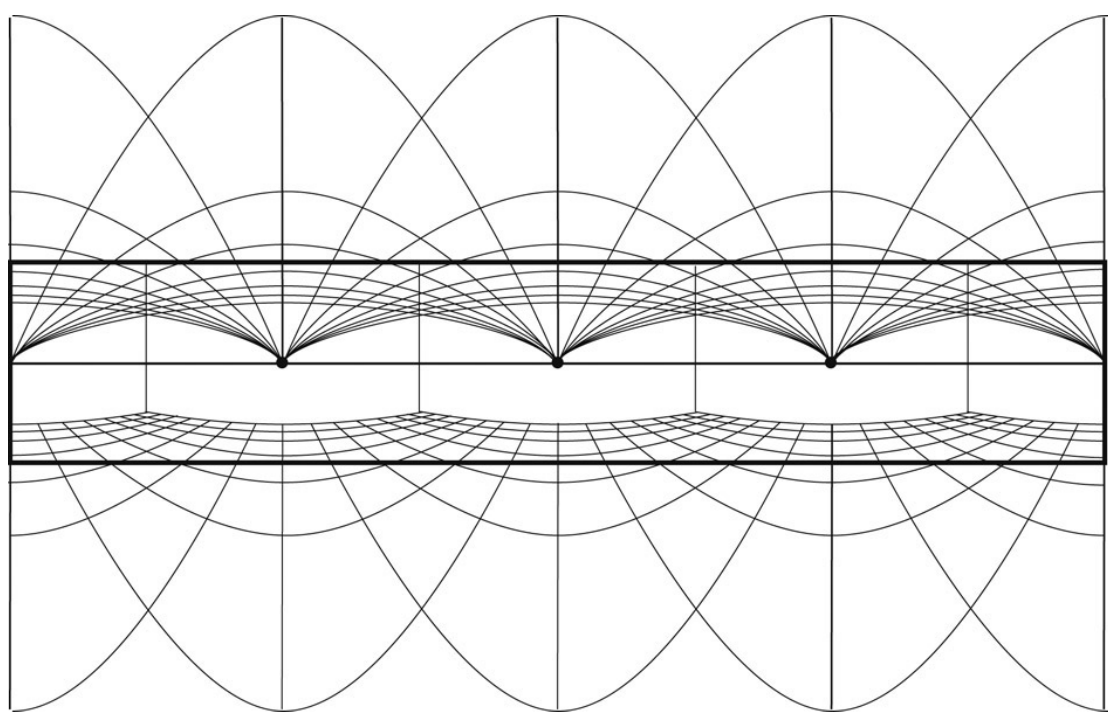
Figure 33.12 **Copy and repeat the curves to make a grid.**

Continue to copy and repeat until the four-point perspective grid is complete (Figure 33.13).



[Figure 33.13](#) **Continue to copy and repeat the curves, finding intersections to create a grid.**

Much of the grid that has been drawn is outside the cone of vision, outside of the 60° vertical limit. That part of the grid is distorted beyond what is acceptable and needs to be cropped ([Figure 33.14](#)).



[Figure 33.14](#) **The ceiling is a mirror image of the floor.**

Creating a true four-point grid is not an easy task. To expedite the job, many illustrators use a simple arc in place of the sinusoid, “eyeballing” the shapes. This is far from accurate, but it does save a great deal of time.

Vertical Lines

Vertical lines are parallel with the picture plane and are drawn as true verticals (Figure 33.15). Measure vertical lines as measured in one- or two-point perspective. Measure up from the picture plane and project the height to the desired location (Figure 33.16).

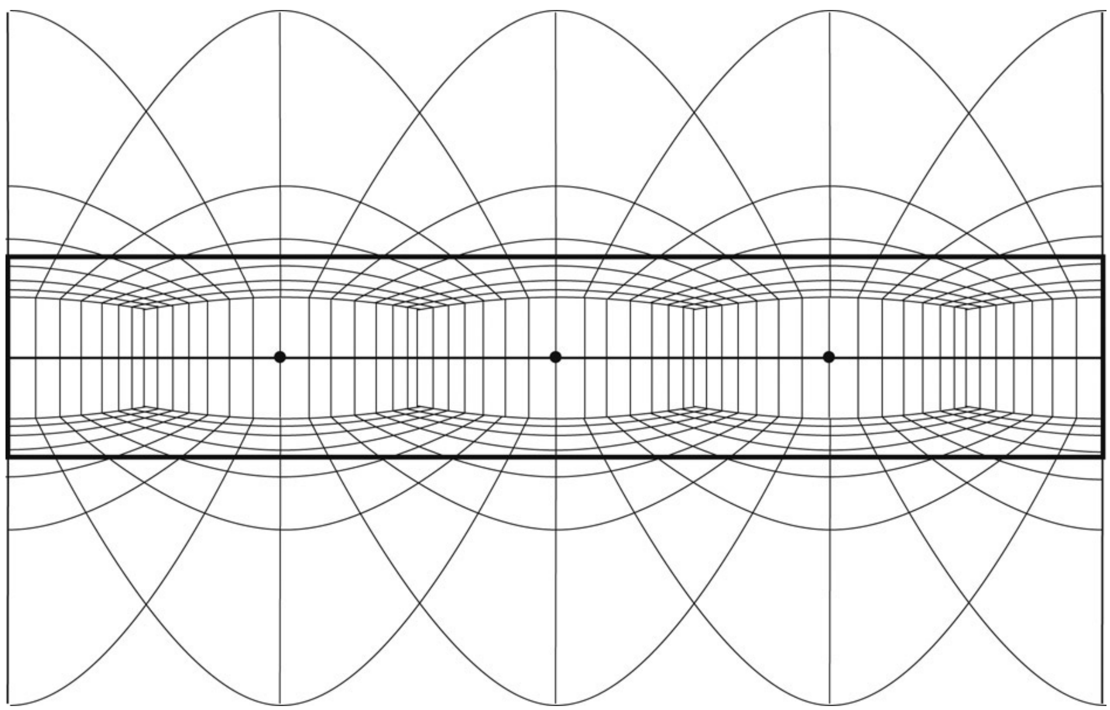


Figure 33.15 Crop the image and draw vertical lines on the walls where horizontal lines intersect.

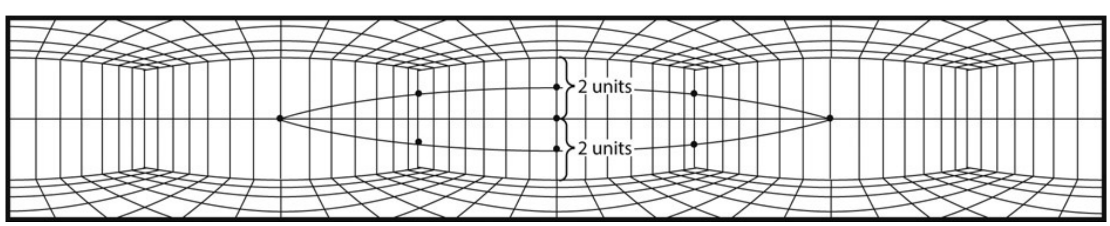


Figure 33.16 **The eye level is 2 units above the ground. Divide the center line into four even segments, creating a horizontal grid.**

Finalize the grid by connecting lines to vanishing points (Figures 33.17–33.18).

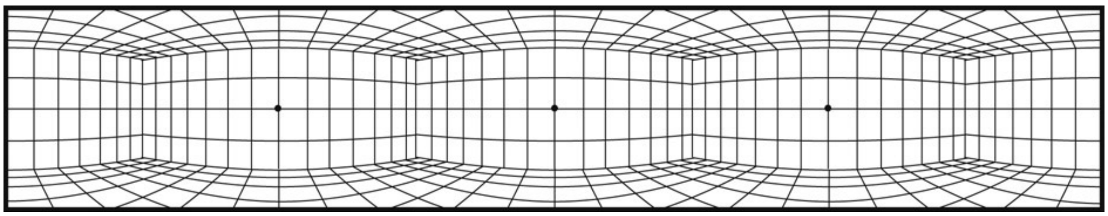


Figure 33.17 **The completed grid.**

As complicated as this diagram is, it is simpler than a diagram where the viewer is not in the center of the room and the room is not a square. In this scenario, the image can't be divided into four even spaces. The length of each wall must be plotted separately.

Length

In this example, the room is rectangular. To plot the length of each wall, start with a plan view. Project the length of each wall to the station point, intersecting the picture plane (Figure 33.18). The picture plane is cylindrical so approach width in terms of degrees.

The length of each wall is represented on the picture plane as an arc, a section of a circle. To find the length of each wall, the length of each arc must be calculated (Figure 33.19). This next step requires a little math. The formula to measure the length of an arc is $2\pi R(C/360)$ where:

C is the central angle of the arc in degrees;
R is the radius of the arc; and
π is 3.14.

There are several websites that offer calculators to solve this problem (search for "arc length circle calculator").

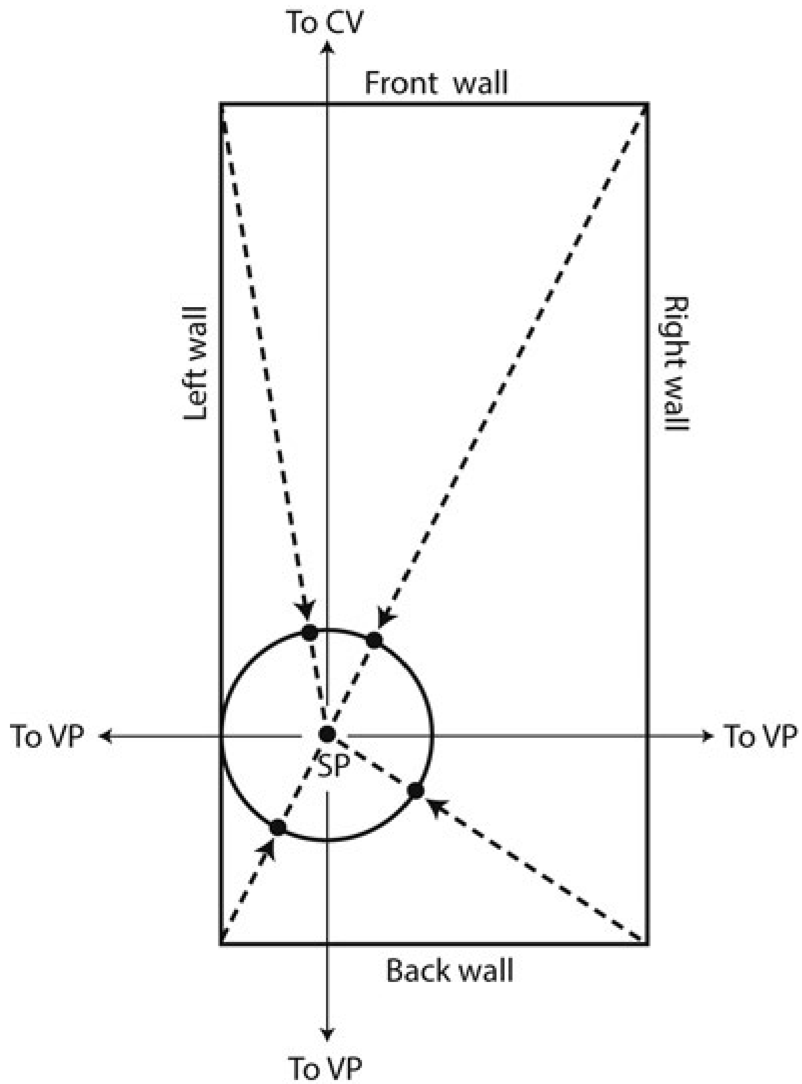


Figure 33.18 **Project the width of each wall to the picture plane**.

Unlike the previous drawing, the vanishing points are not centered within each wall. They are still 90° apart and plotted using the same methods as Figure 33.4. The grid for this asymmetrical room was drawn using the same method as the previous symmetrical room (Figure 33.20). Using this method, any environment can be accurately created and measured in four-point perspective.

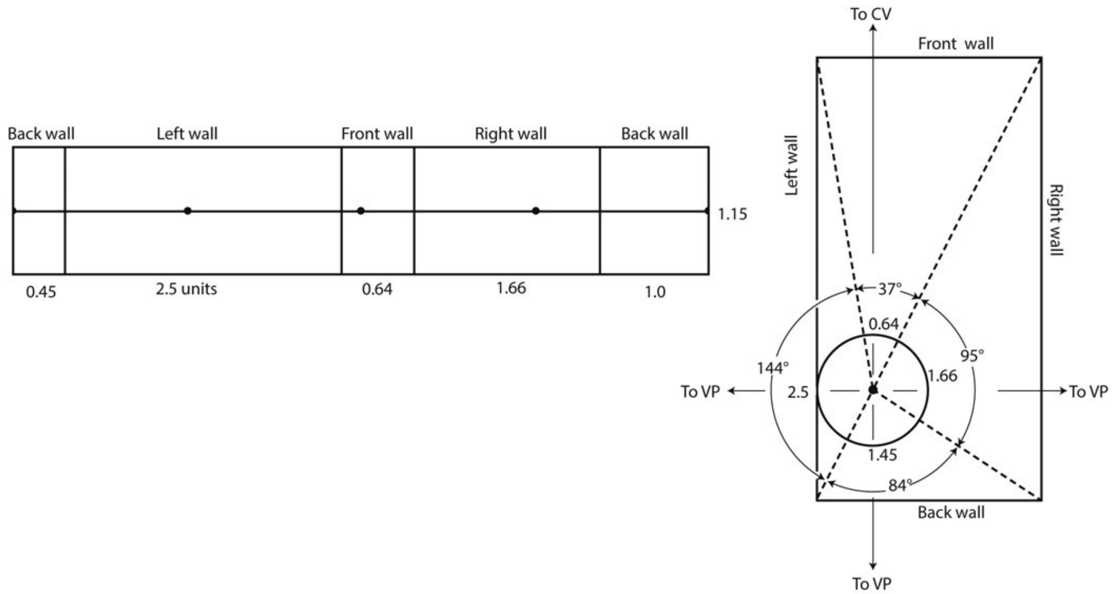


Figure 33.19 **Plot the walls and vanishing point locations.**

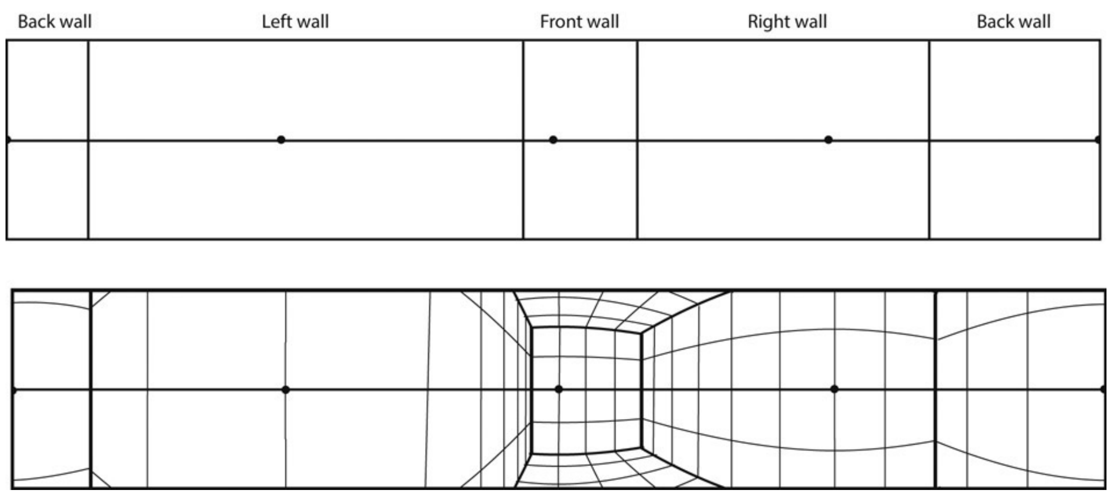


Figure 33.20 **This grid was drawn by the same process used in Figures 33.6–33.18. The left wall is touching the picture plane and the front wall is the farthest from the viewer.**

It should be noted that a 360° image is so foreign to the viewer's eyes that it is difficult for the brain to process. To create a more pleasing and comprehendible picture, it is worth cropping the image area, keeping the view within 180° (Figure 33.21).

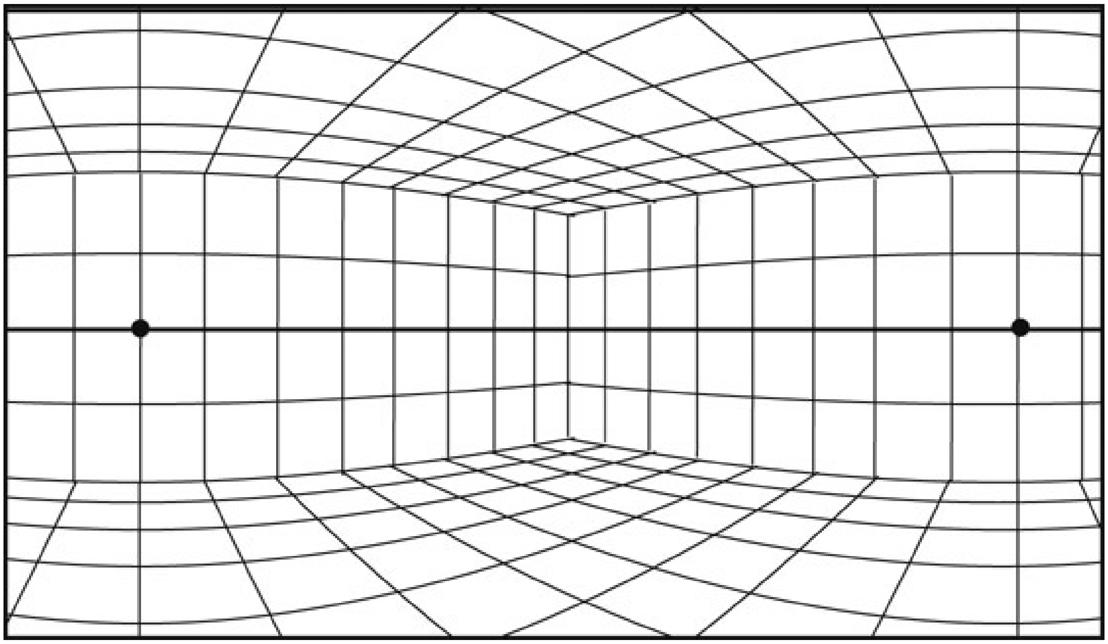
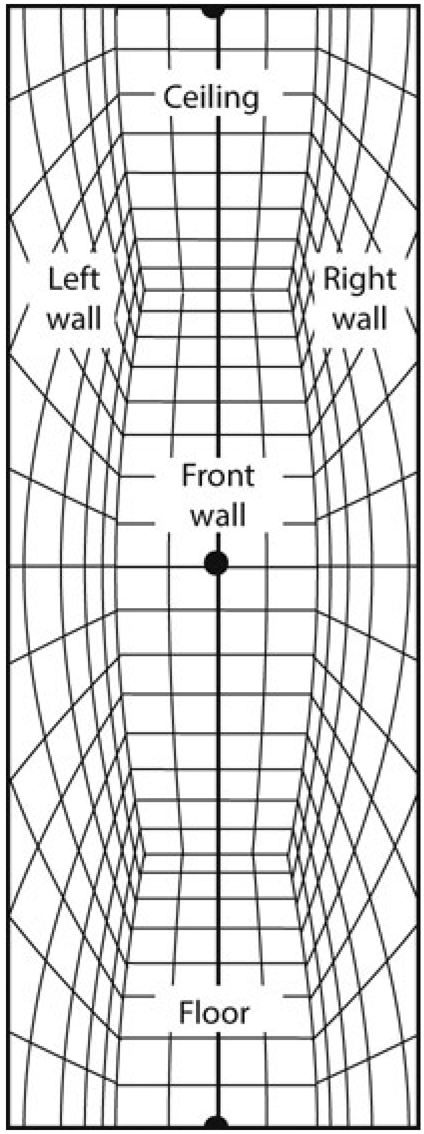


Figure 33.21 **This is a four-point perspective view, cropped a little beyond 90° (the two vanishing points are 90° apart).**

Vertical Four-Point

A four-point diagram turned sideways creates a vertical panorama. It combines a bird's-eye view with a worm's-eye view. The view includes what is below, in front, and above. This includes the floor, the wall facing the viewer, and the ceiling above them. Vertical lines extend to vertical vanishing points: one at zenith, and one at nadir. Horizontally, however, the **field of vision** remains at 60° (Figure 33.22).



[Figure 33.22](#) **A vertical four-point perspective view cropped at 180°.**

34
Five-Point Perspective

Five-point perspective is a fisheye view, displaying everything from east to west and from north to south. Everything in front of the viewer is depicted. The picture plane is a hemisphere. Horizontal, vertical, and angled lines projected on a hemisphere behave differently than when projected on a cylinder. Before drawing a five-point image, how these lines appear when projected on a sphere must be understood.

Five-Point Perspective Components

Horizontal Lines (*y*-Axis)

A horizontal line of infinite length would connect to vanishing points on the horizon line: one at the extreme left and the other at the extreme right of the sphere. The horizon line, being at the viewer's eye level, is drawn as a straight line. Horizontal lines not aligned with the eye level are curved. The curve is a great circle (Figure 34.1).

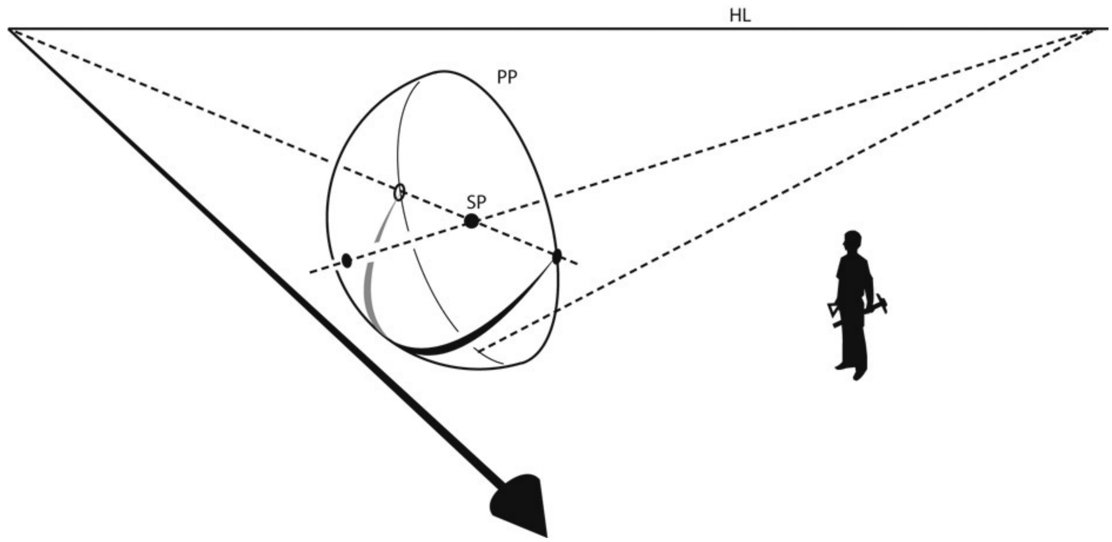


Figure 34.1 Horizontal lines projected on a hemisphere connect to vanishing points on horizon line 90° from the center of vision.

Horizontal Lines (*x*-Axis)

Horizontal lines parallel with the line of sight are drawn as a great circle, with one vanishing point at the center of vision and the other directly behind the viewer (Figure 34.2).

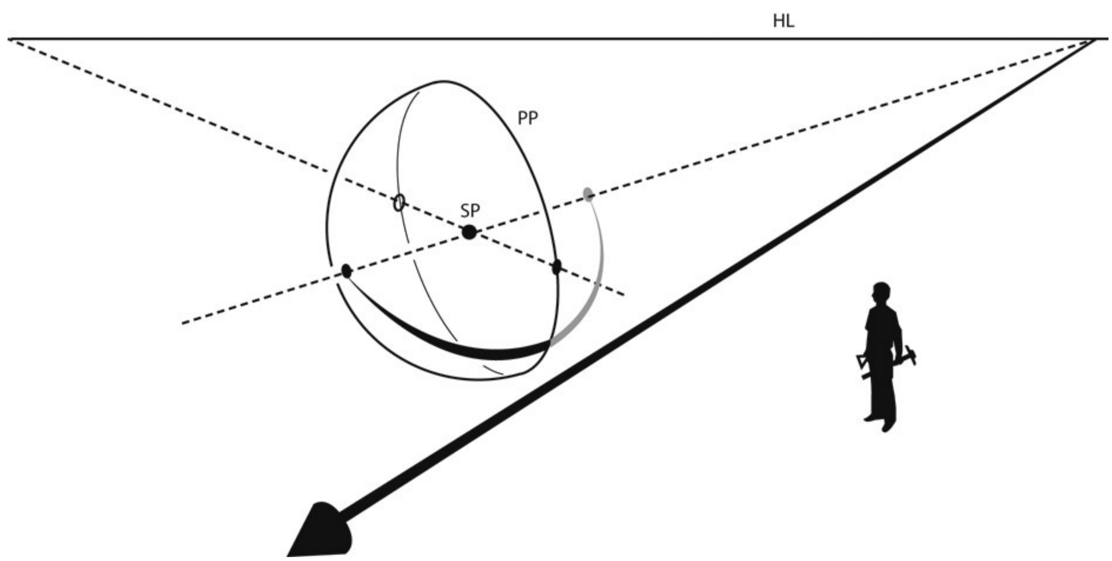


Figure 34.2 Horizontal lines projected on a hemisphere are half a great circle, one vanishing point being the center of vision, the other behind the viewer (not

shown).

Vertical Lines (*z*-Axis)

Vertical lines are drawn as a great circle and have vanishing points at zenith and nadir (Figure 34.3).

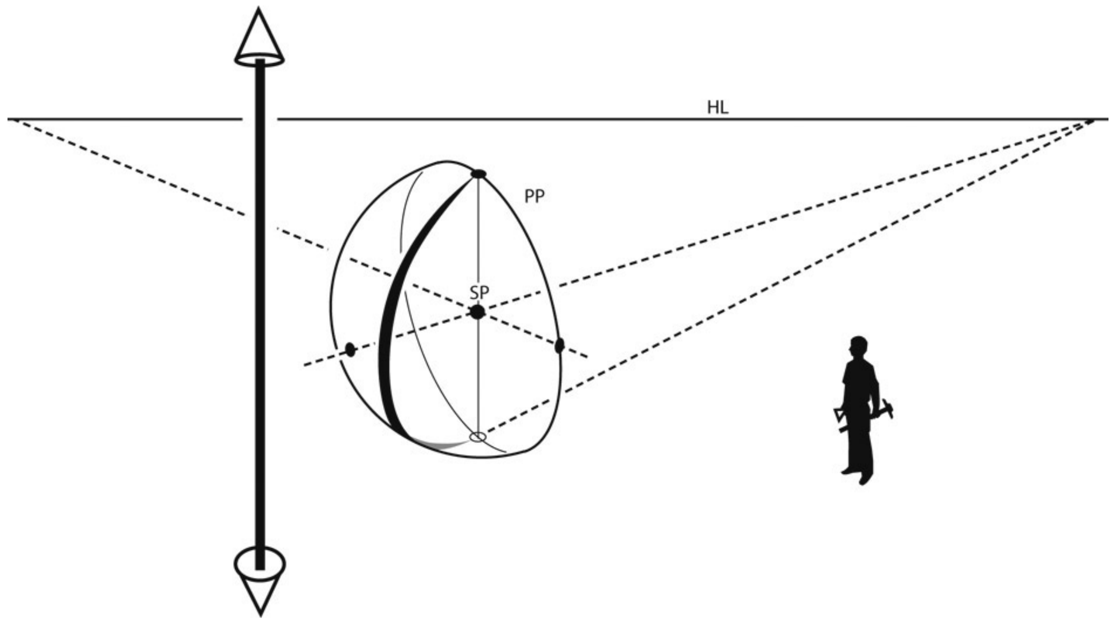
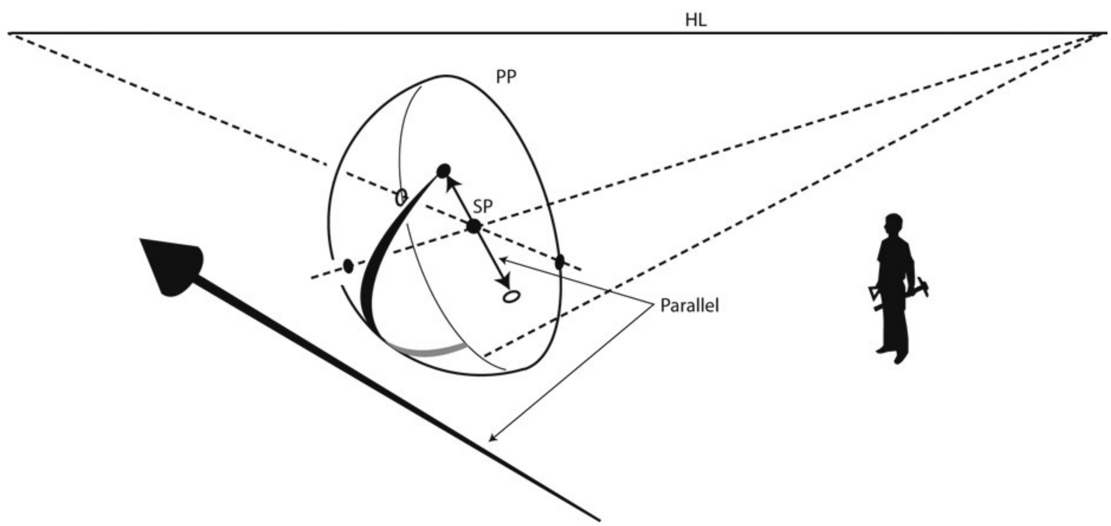


Figure 34.3 Vertical lines projected on a hemisphere are a great circle, with vanishing points at the top and bottom of the picture plane.

Angled Lines

Angled lines have vanishing points at an angle corresponding to the line being drawn. They are also drawn as a great circle (Figure 34.4).



[Figure 34.4](#) **The vanishing points for angled lines are at the same angle as the line being drawn, with vanishing points 180° apart.**

Flattening A Sphere

Now that the theory of how lines appear when projected on a hemisphere is understood, how are they drawn on a flat surface? The paper has only two dimensions; the hemisphere has three. So, before these lines can be plotted on the paper, the hemisphere must first be flattened. Flattening a round surface is a problem that has plagued cartographers for thousands of years. It is a problem that still exists today; it is a problem that cannot be solved. Flattening a spherical surface, without distortion, is impossible. If a ball was cut in half, and then flattened, the only way to accomplish this task would be to tear, stretch, or fold the ball. No matter the approach, the shape of the ball will suffer. A cone or cylinder can be flattened without tearing, folding, or stretching. This is called a developable surface. A sphere is not a developable surface. Any flat representation of a sphere will, by its nature, be distorted.

Over the centuries, cartographers and mathematicians have found a variety of ways to limit this distortion. There are well over fifty methods to project a curved surface on a flat plane. The methods developed often diminish distortion in one aspect, while increasing it in others. Each method leads to different results. Depending on the application, one method is often more advantageous than another.

The best method for drawing purposes is an azimuthal projection. There are many different azimuthal projections; among them are gnomonic, orthographic, stereographic, and equidistant.

Gnomonic Projection

The gnomonic projection is one of the oldest, dating to the sixth century BC. While the center of the image has little distortion, the edges of the sphere are greatly stretched. Additionally, this method cannot display a true 180°. The outside edge of the sphere is at infinity (Figure 34.5).

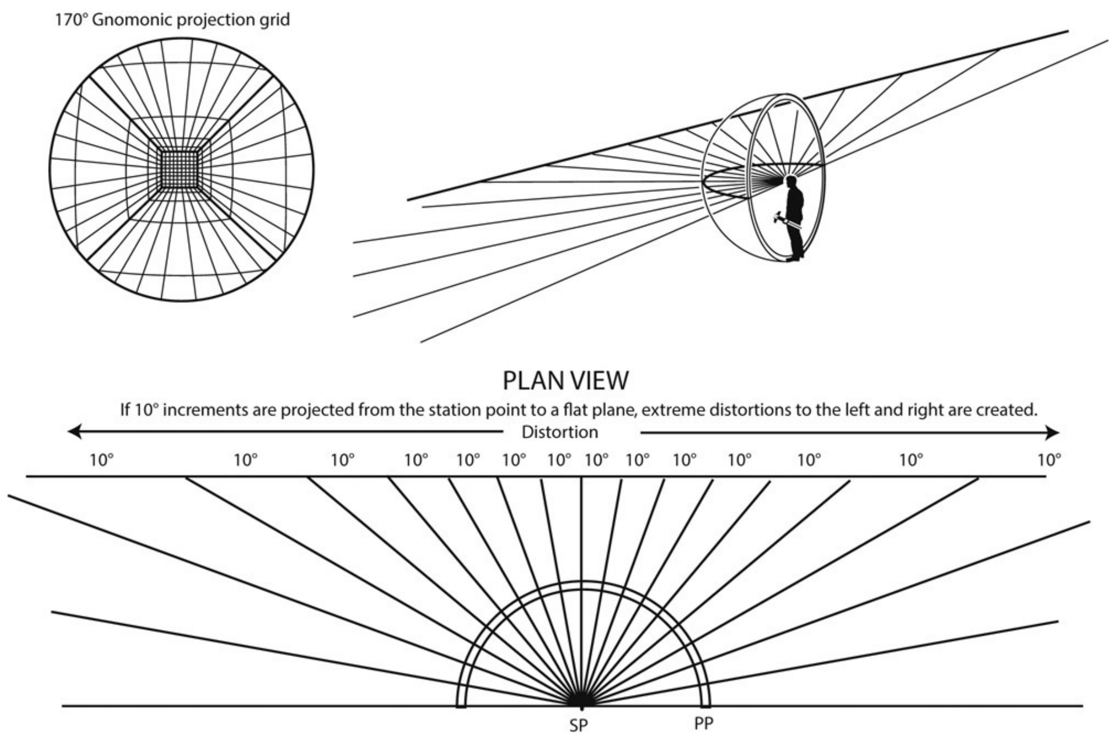


Figure 34.5 Gnomonic projection.

Orthographic Projection

An orthographic projection puts the station point at infinity. As with gnomonic projection, the orthographic projection is more accurate toward the center of the sphere. But while the gnomonic projection stretches the edges of the sphere, the orthographic projection compresses them. An orthographic projection is like a photograph of the earth from space. The land and water masses are foreshortened as the surface of the sphere recedes and becomes more oblique to the viewer (Figure 34.6).

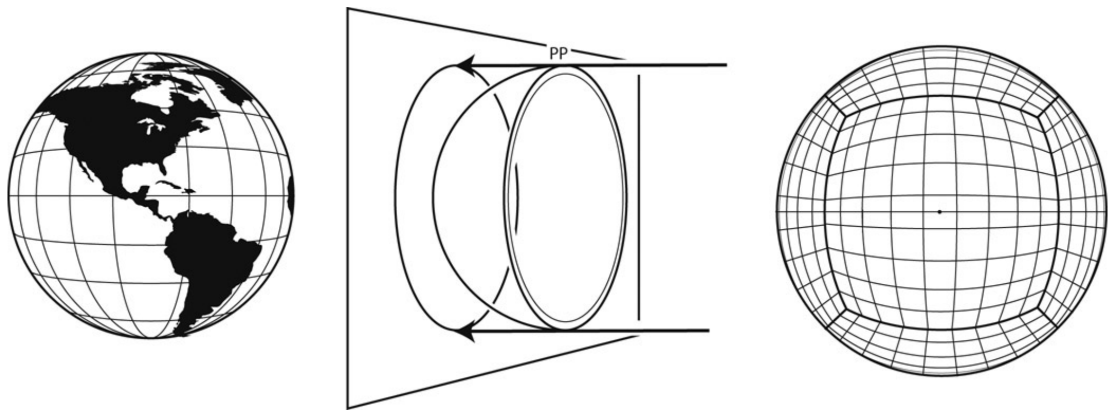


Figure 34.6 Orthographic projection.

Stereographic Projection

The stereographic projection places the station point at the far edge of the sphere. The image is still distorted; the projected circumference of the sphere is much too large. But this is closer to the desired result (Figure 34.7).

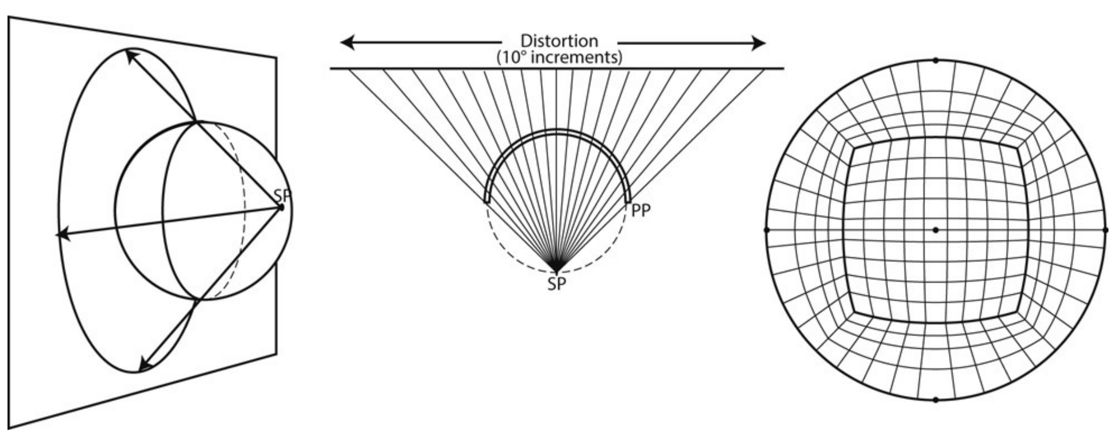


Figure 34.7 **Stereographic projection**.

Azimuthal Equidistant

The azimuthal equidistant projection preserves the circumference of the sphere. In addition, all lines from the center point are straight and have correct angles. There is still noteworthy distortion, as the diameter of the sphere is considerably smaller than the diameter of the projection. For drawing purposes, however, this is the best projection possible (Figure 34.8).

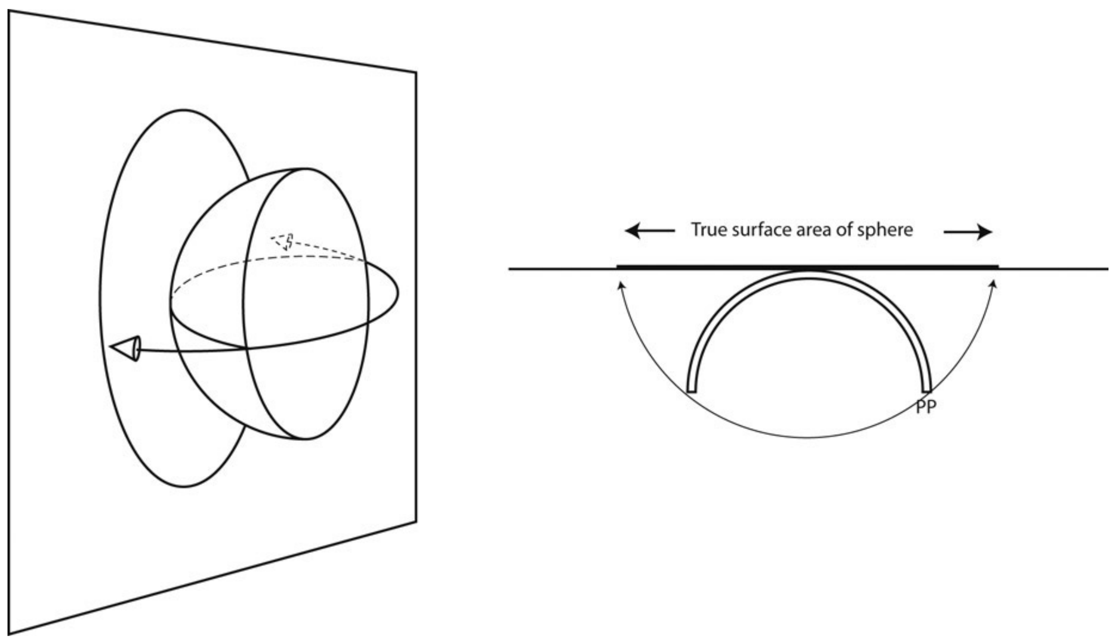


Figure 34.8 **Azimuthal equidistant projection**.

Five-Point Grid

After examining how lines of various orientations look when plotted on a hemisphere, and discussing the best sphere projection possible, combine this information to plot a five-point perspective grid.

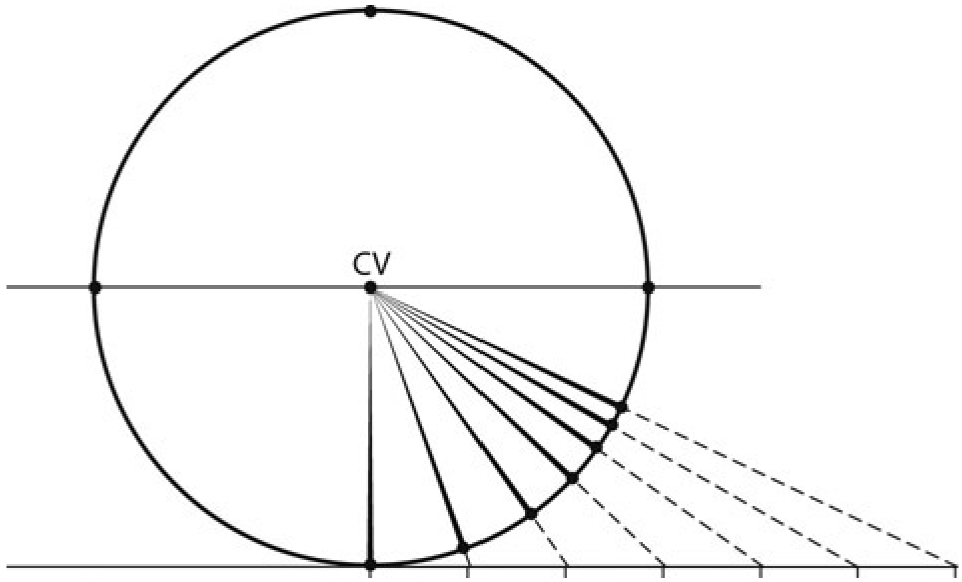


Figure 34.9 Evenly spaced horizontal lines projected to the hemisphere, then connected to the center of vision.

Horizontal Lines (*x*-Axis)

Lines parallel with the viewer's line of sight, lines along the *x*-axis, appear as true straight lines receding to the center of vision. To create a series of evenly spaced lines receding in space, first draw a horizontal line along the ground plane, divide the line into even spaces, then project each increment to the center of vision (Figure 34.9).

Lines on the *z*- and *y*-Axis

Vertical and horizontal lines on these axes are curved. They are a great circle when drawn on the picture plane, but not quite a great circle on the drawing. There is some distortion due to the equidistant projection method. Considering this image is not a developable surface—and, therefore, can never be truly accurate—it saves a great deal of time and effort simply to use a compass arc. The shape of a compass arc is very close to the distorted arch.

Placement of Vertical Lines (*z*-Axis)

Think in terms of degrees when placing vertical lines in specific locations. There are 180° between the left and right vanishing points. Measure angles in degrees, along the horizon line. For example, if using a line 45° to the right, place a dot centered between the right vanishing point and the center of vision (Figure 34.10, left). Vertical lines are drawn with a compass, connecting to the vanishing points at zenith and nadir (Figure 34.10, right).

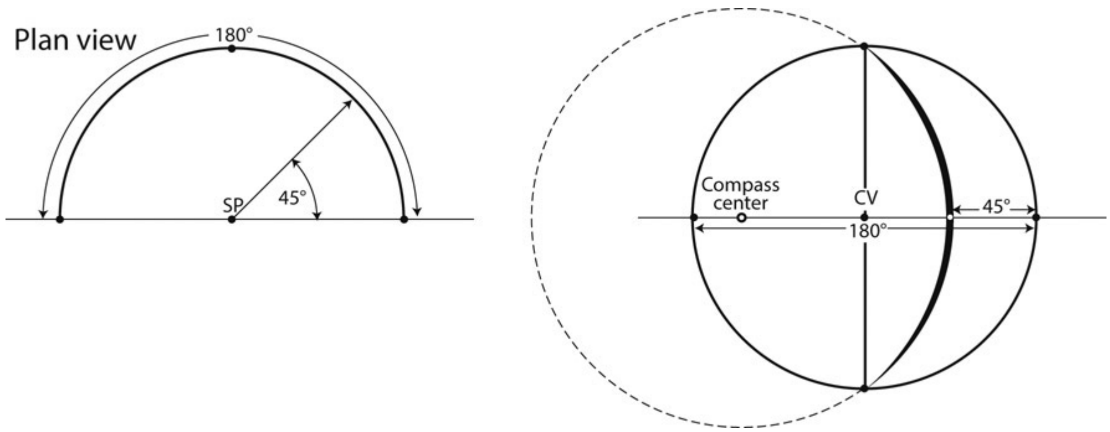
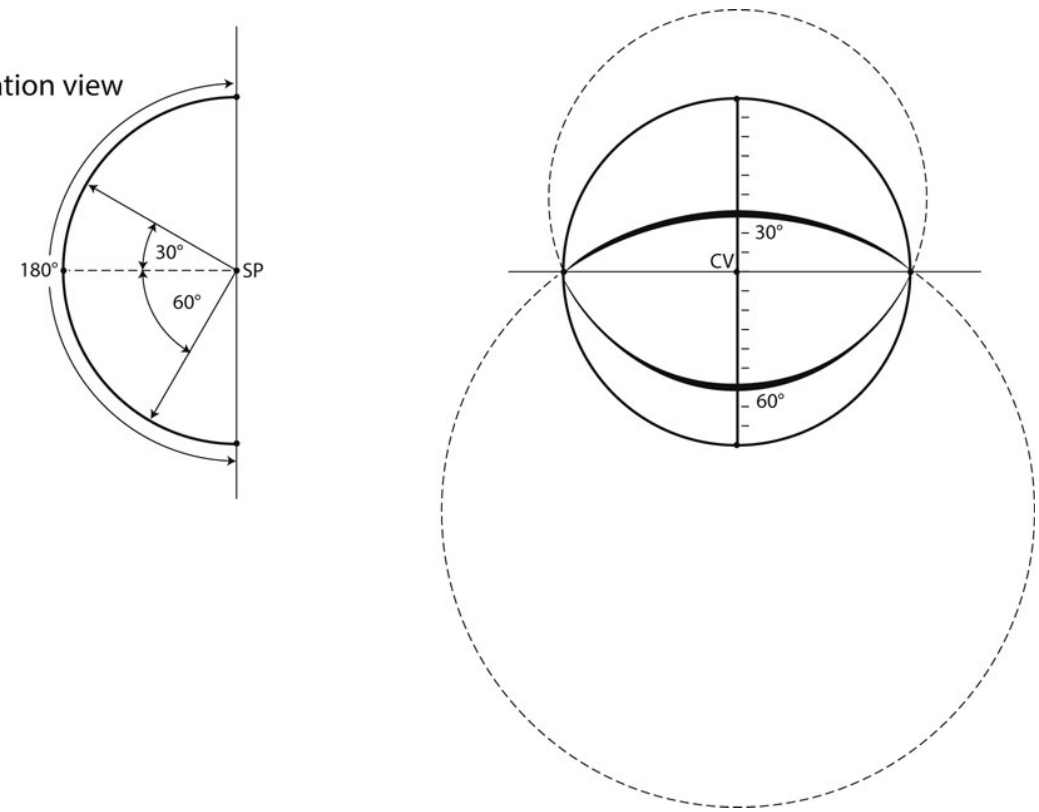


Figure 34.10 **A vertical line 45° to the right of the center of vision**.

Placement of Horizontal Lines (*y*-Axis)

Horizontal lines are approached in the same way as vertical lines. Measure in degrees, above or below the horizon line, to determine their placement. Use a compass to connect to the left and right vanishing points (Figure 34.11).



[Figure 34.11](#) **Placement of horizontal lines are determined in degrees. The drawing above illustrates a horizontal line 30° above the horizon, and a horizontal line 60° below the horizon.**

Create the Grid

Use a 45° vanishing point to create a series of squares (Figure 34.12). Repeat the process to create a grid (Figure 34.13).

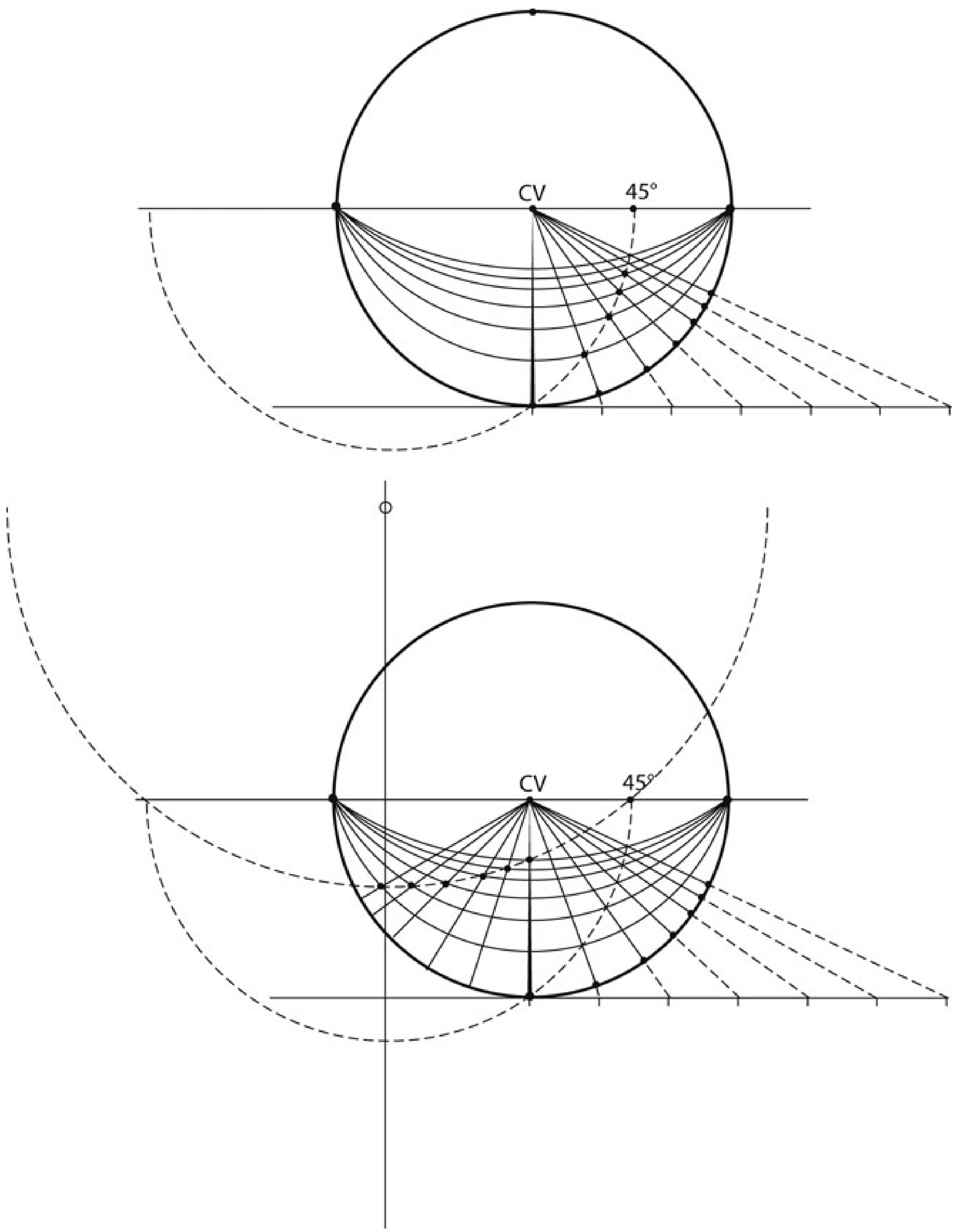


Figure 34.12 Create a series of squares using the 45° vanishing point.

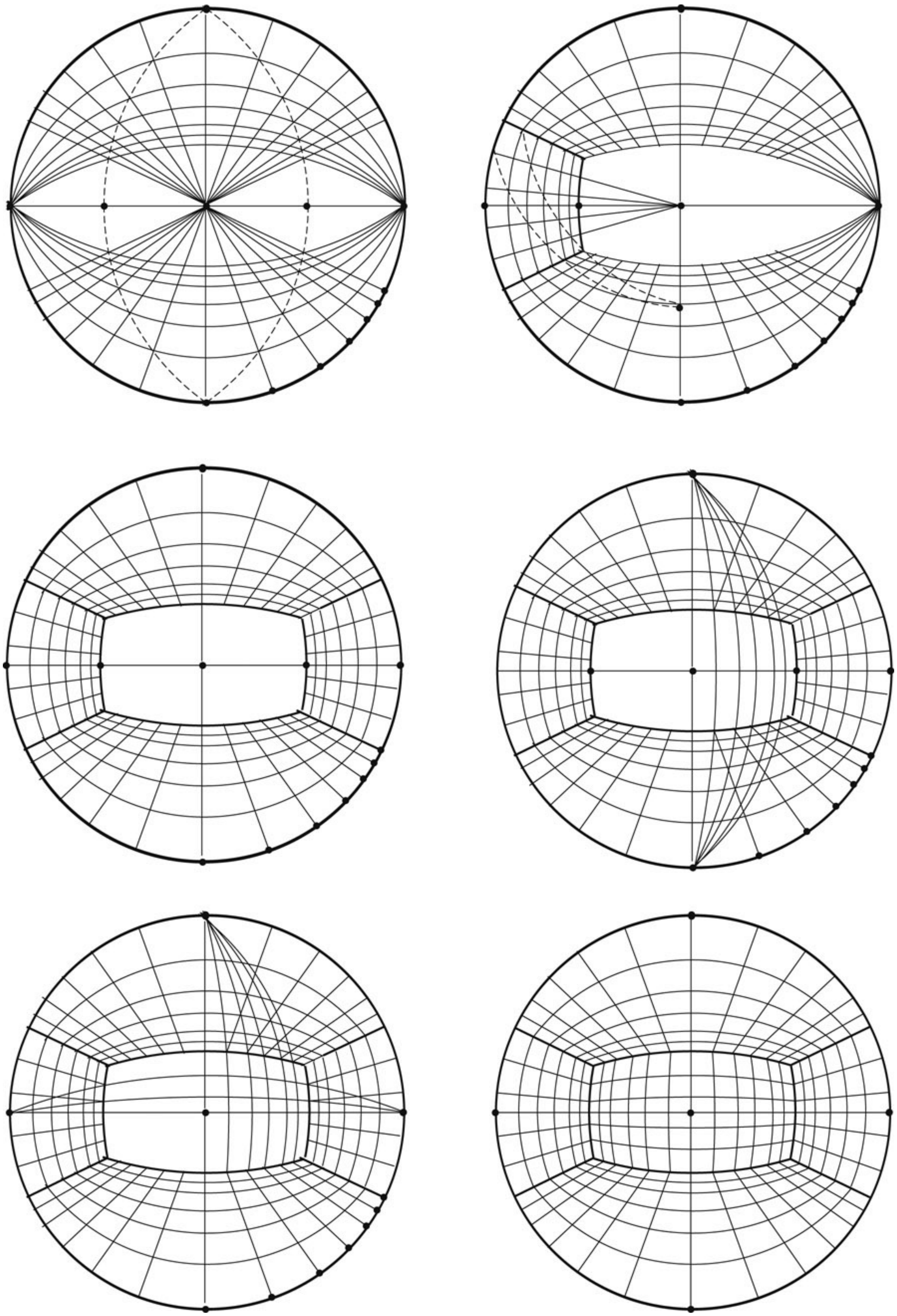


Figure 34.13 Repeat the process to complete the grid.

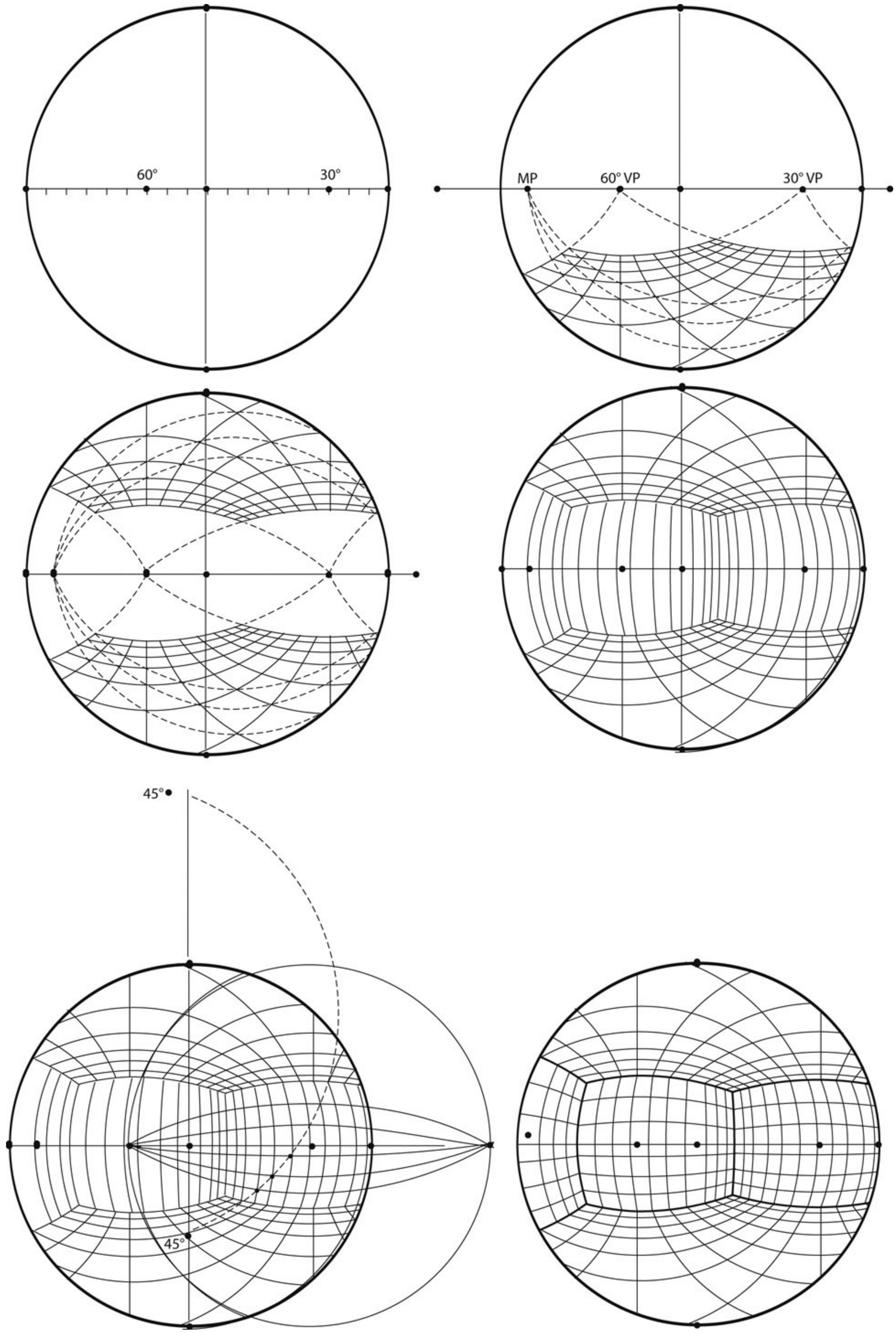

60°
30°
MP
60° VP
30° VP
45°
45°

Figure 34.14 A five-point grid turned 60°/30°.

Angled Grid

In the previous five-point example, the room's front wall was oriented perpendicular to the line of sight (equivalent to a one-point perspective view). This room can be drawn at any angle by relocating the vanishing points along the horizon line, but the vanishing points must remain 90° apart. For example, to create a grid that is angled 60°/30° to the viewer, place a point 60° from the far left vanishing point, and another point 30° from the far right vanishing point (Figure 34.14, top left).

To measure squares, it is helpful to draw 45° angles in perspective. These angles are found from points placed on the horizon line, 45° from the vanishing points (Figure 34.14, top right).

Create a vertical grid by using vertical measuring points (Figure 34.14, bottom left).

35
Six-Point Perspective

Five-point perspective displays 180° of information, but it is only half of the viewer's environment. Six-point perspective adds the other half. A six-point diagram is a full 360° view—everything surrounding the viewer is displayed.

The picture plane is a sphere, cut in half, and opened to show both ends. The viewer sees the two halves. One half displays what is in front of the viewer, and the other what is behind. The two halves can be displayed stacked or side by side. There are six vanishing points, one in front of the viewer and one behind, one above the viewer and one below, one to the left of the viewer and one to the right. The two halves share top, bottom, left, and right vanishing points (Figure 35.1).

Creating a six-point grid is the same as creating a five-point grid, except it is done twice.

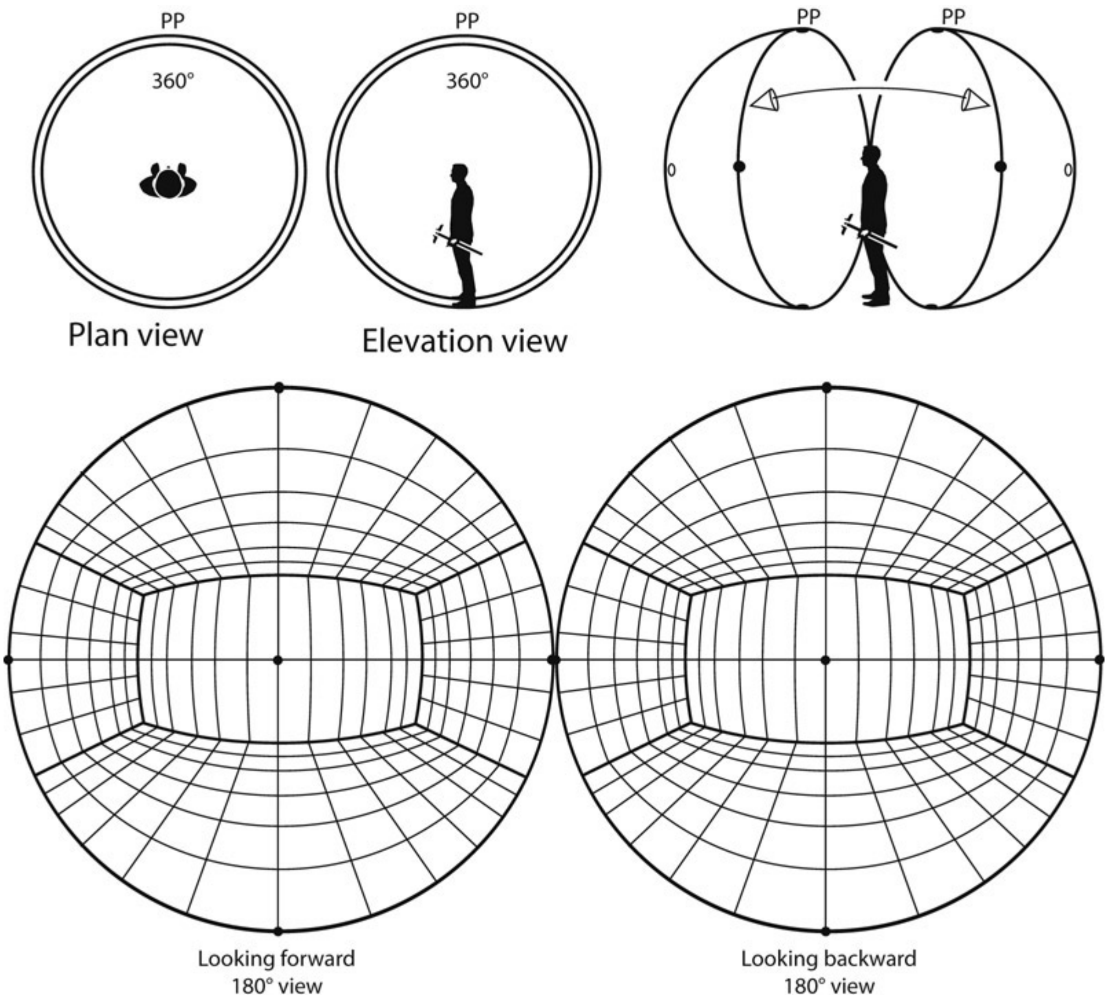


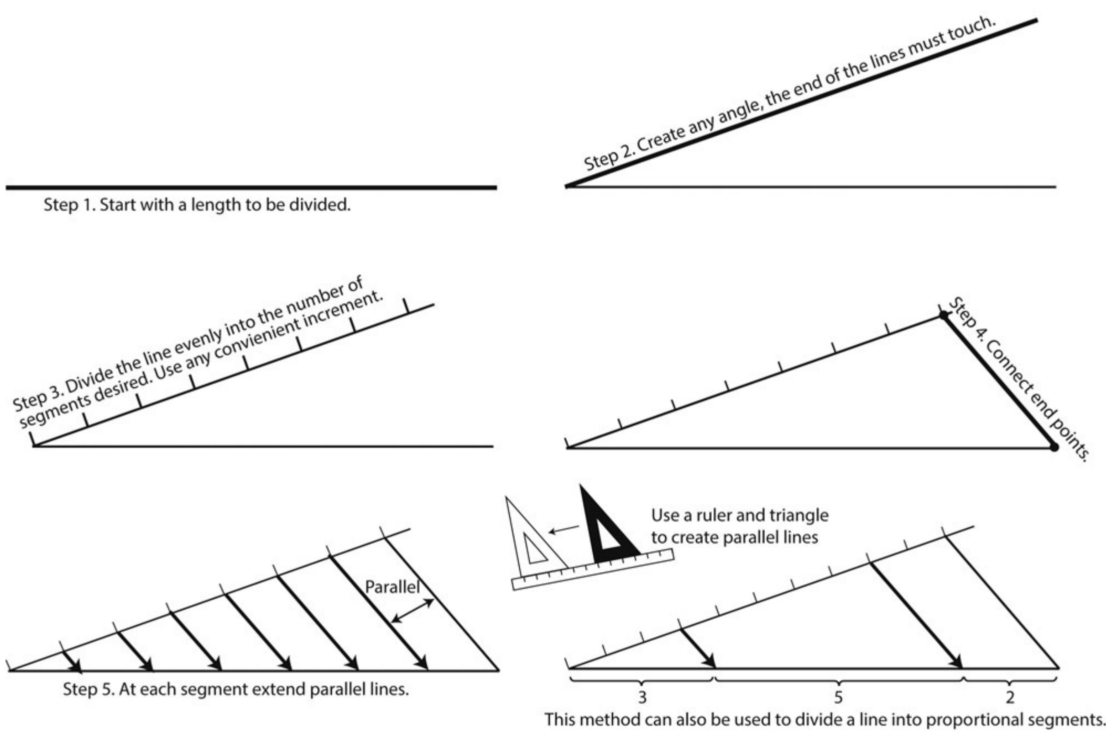
Figure 35.1 **Six-point perspective.**

36 Miscellaneous

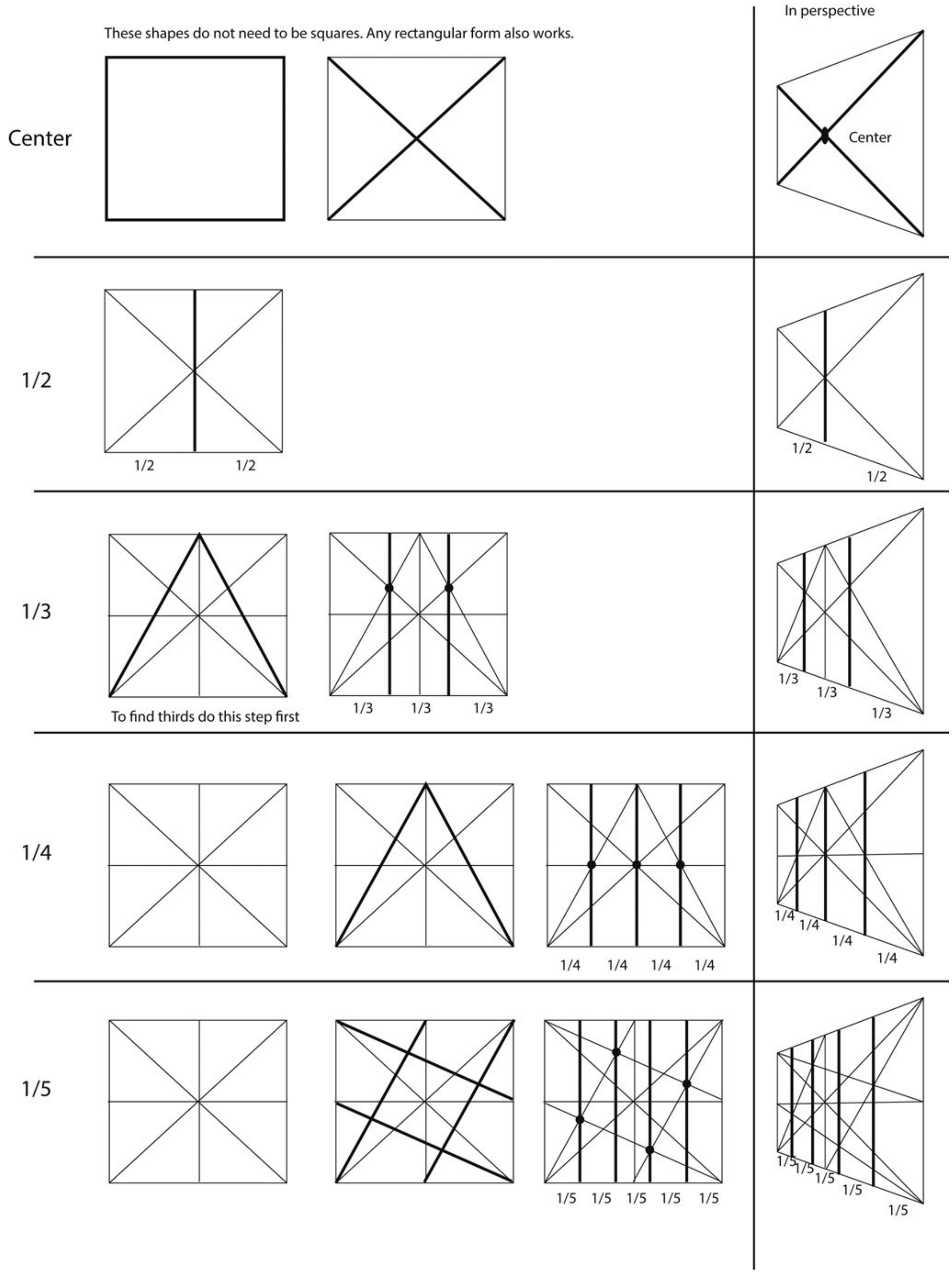
Dividing Lengths

Dividing a length into a given number of spaces can be done with a ruler and a little math. A better method involves using a ruler and a triangle. It is fast, accurate, and no calculator is needed (Figure 36.1). This technique only works on lines parallel with the picture plane. It does not work on lines that are foreshortened. It is an excellent tool to plot evenly spaced segments.

Another technique (that does work with foreshortened lines) uses a variety of methods to divide squares or rectangles into even segments. There are several methods, and each result in a different number of divisions (Figure 36.2).



[Figure 36.1](#) **Dividing a length into evenly spaced increments.**



[Figure 36.2](#) **Dividing a rectangle or square into evenly spaced increments.**

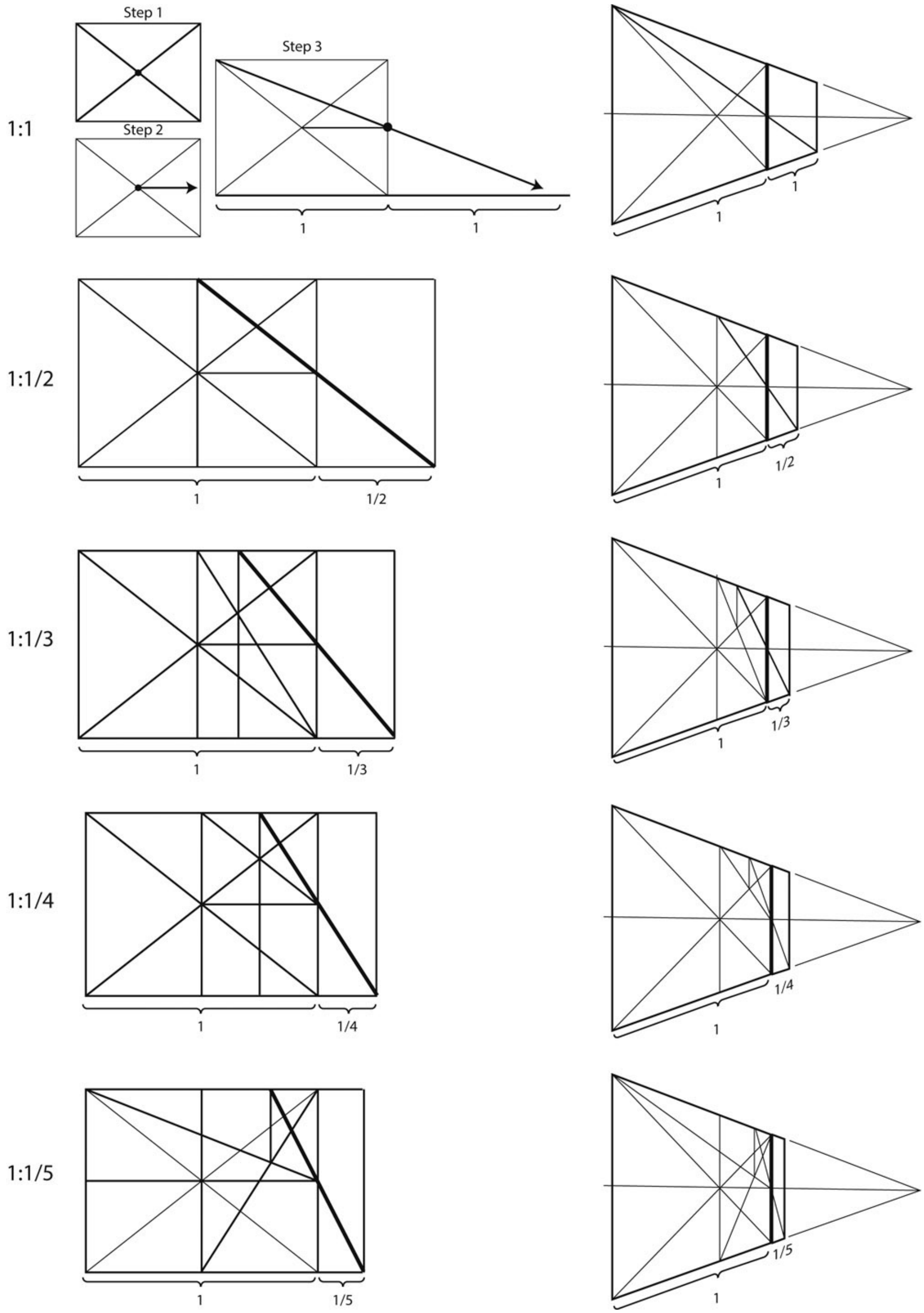


Figure 36.3 Techniques for multiplying distances.

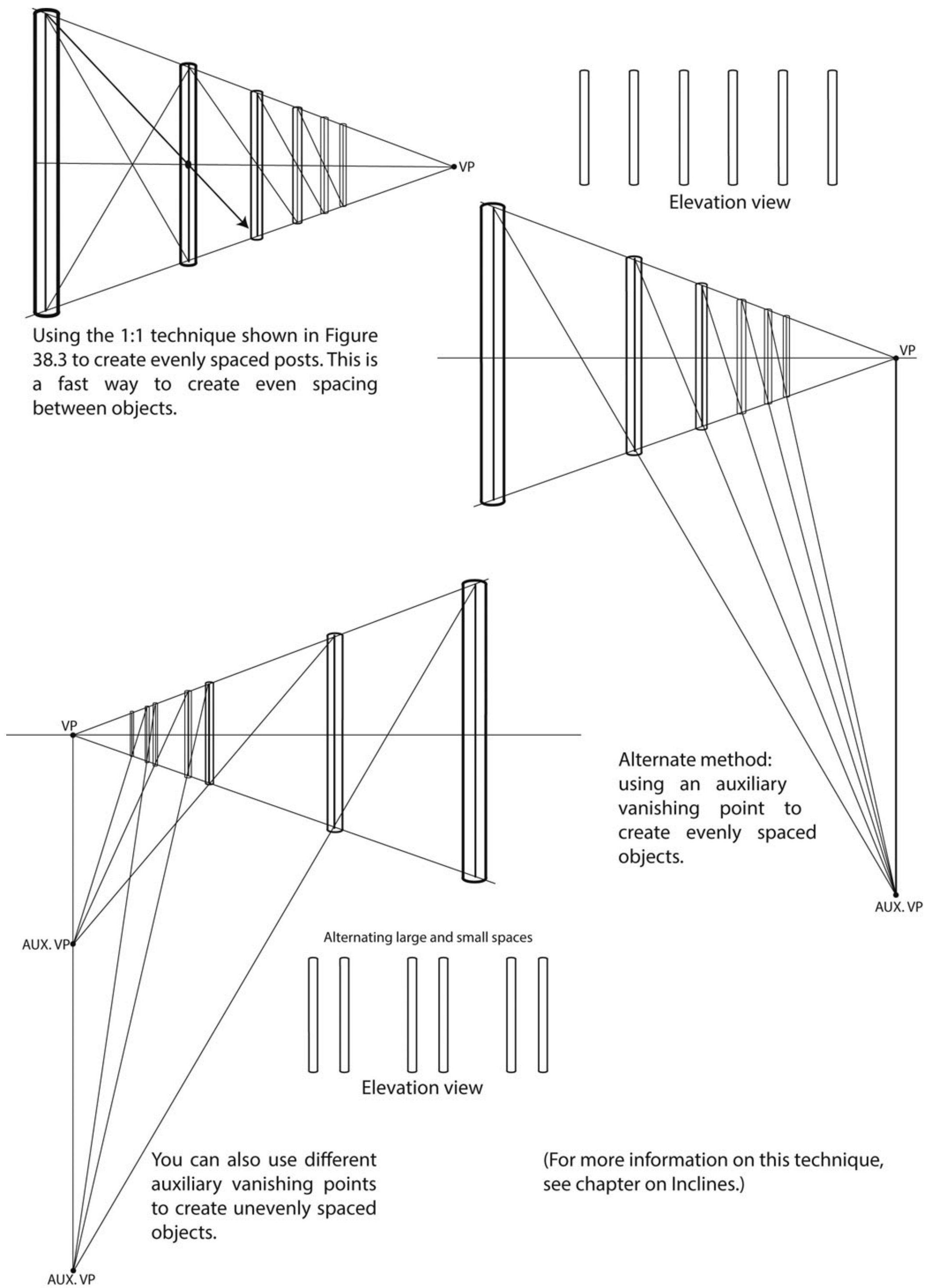


Figure 36.4 An auxiliary vanishing point is a useful tool to create evenly spaced divisions.

Multiplying Lines

If an existing line needs to be made longer by a given percentage, there are a few methods that will assist in this endeavor (Figure 36.3).

Using an auxiliary vanishing point is an alternative method to create evenly spaced segments (Figure 36.4).

Intersecting Forms

The shape created by two intersecting forms can be complex. Dozens of examples can be presented, each looking quite different, but each solved using the same basic method—cross-sections. Cross-sections can be used to find the intersection of one form with another. For example, a dormer is the intersection of one **prism** with another. To find this intersection, draw two cross-sections. First bisect the roofline at the ridge (Figure 36.5, top left). Then draw another cross-section, at a right angle to the first, along the centerline of the dormer (Figure 36.5, top right). The junction of the dormer's ridgeline to the building's roofline is the intersection of the two shapes (Figure 36.5, bottom left).

Drawing the intersection of two curved shapes (or, for that matter, any two shapes) uses the same method. Drawing more cross-sections creates more intersections, and a more accurate shape (Figure 36.6).

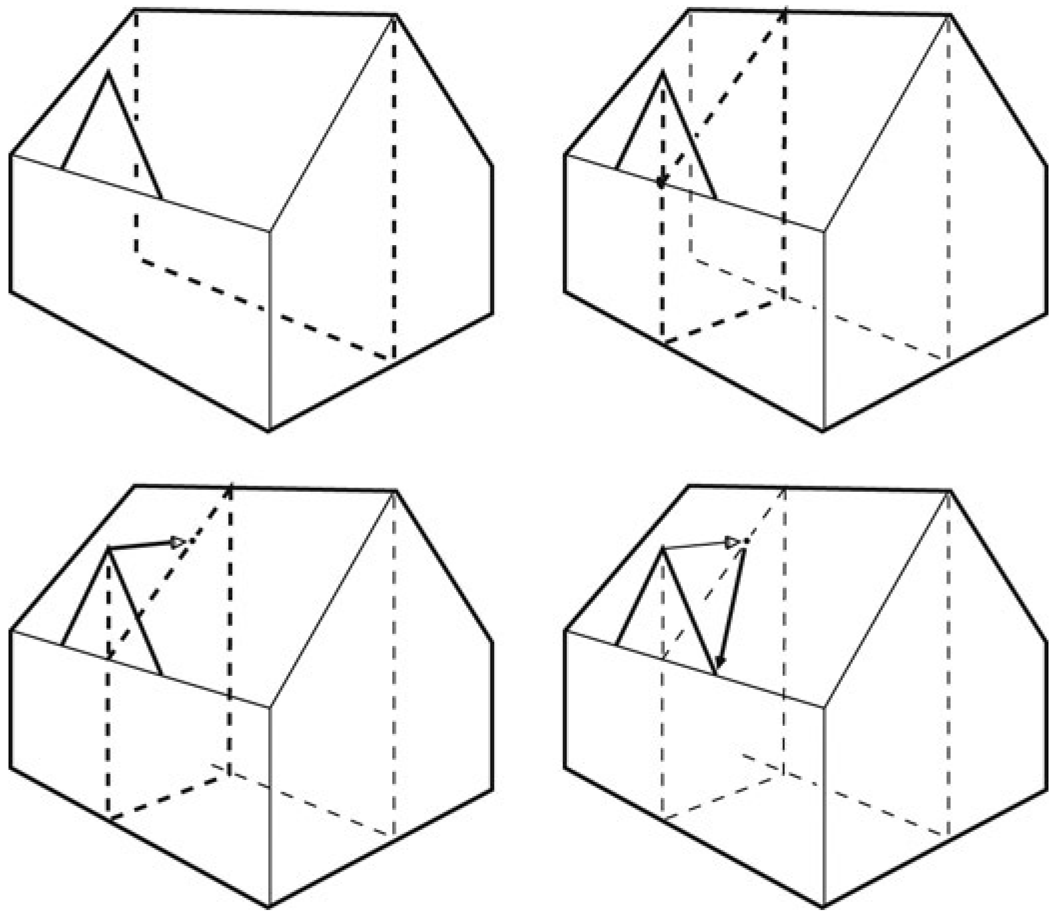


Figure 36.5 Drawing the intersection of a dormer with a roof line.

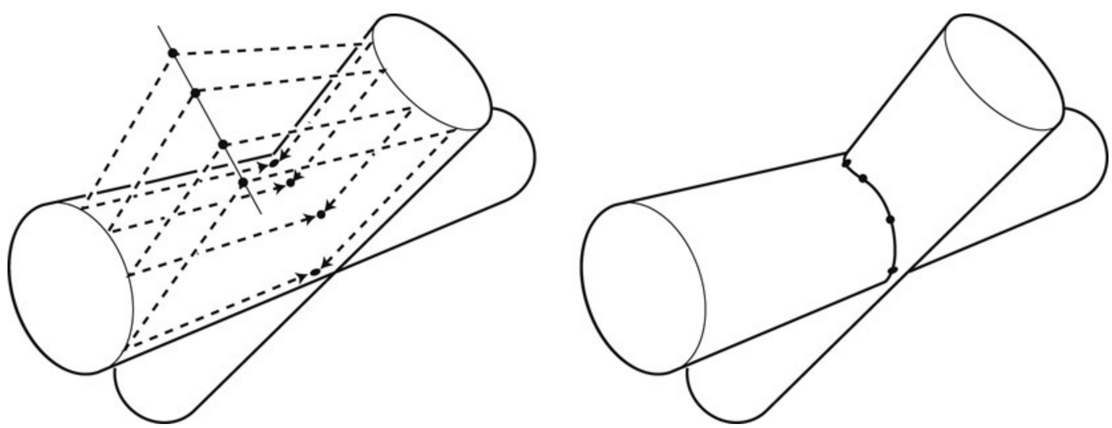


Figure 36.6 Using cross-sections to draw curved intersecting forms.

Dividing A Circle

Dividing a circle into even segments has many applications (revolving doors, spiral staircases, spoked wheels, etc.).

Decide on the number of segments needed, and divide that number by 360. For example, if dividing a circle into twelve even segments, each will be 30° (360 ÷ 12 = 30). Transfer the segments from a plan view to a perspective view (Figure 36.7).

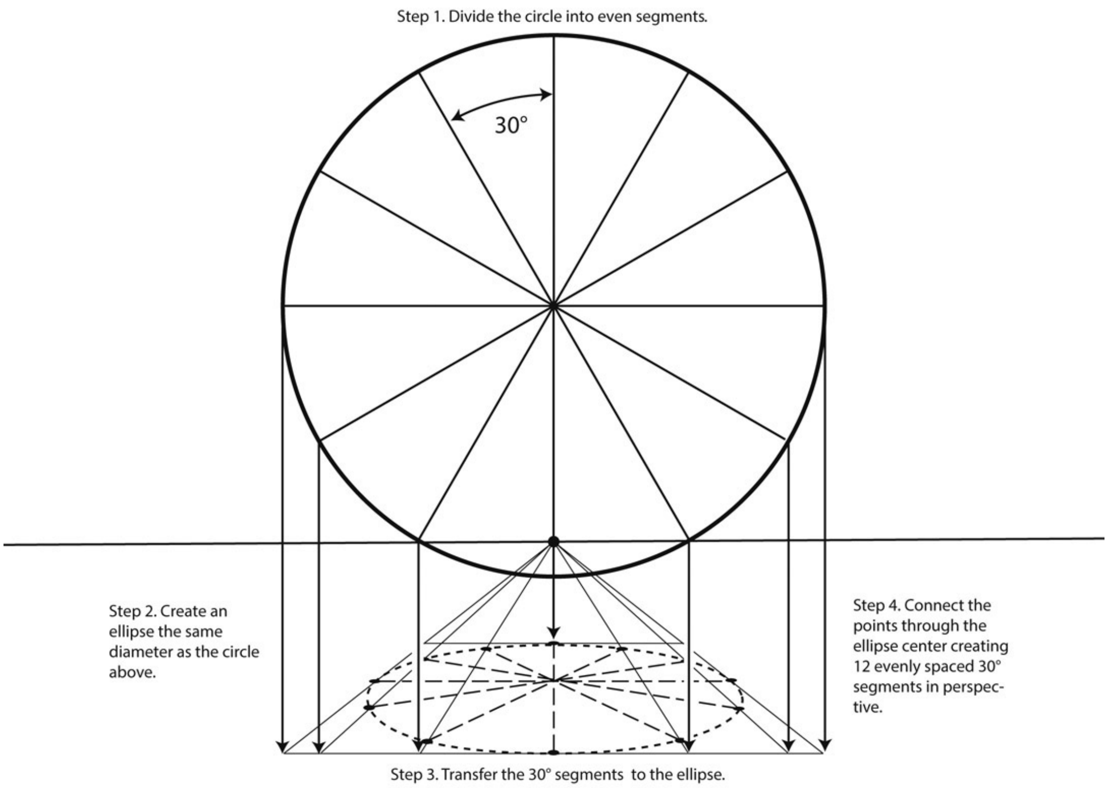


Figure 36.7 Transfer evenly spaced segments from a plan view to an ellipse.

Glossary

Angle of Incidence. The angle between the light ray and normal.

Arc. A section of the circumference of a circle.

Artificial Light. Light from a source other than the sun or moon.

Bird's-Eye View. An image where the viewer is looking down.

Bisect. Divide into two equal parts.

Center of Vision (CV). Where the viewer is looking. The focal point.

Circumference. The perimeter of a circle.

Cone of Vision. A 60° cone emanating from the viewer's eye, intersecting the picture plane, creating a circle around the center of vision. Objects drawn outside this circle become noticeably distorted.

Congruent. Having the same angles or measurements.

Converging Light. Shadows resulting from an artificial light source.

Converging Lines. Parallel lines that connect to a single vanishing point.

Cross-section. The intersection of a three-dimensional form by a plane.

Cube. A prism with six square sides meeting at a right angle.

Cuboid. A rectangular prism with each face meeting at a right angle.

Cylinder. A three-dimensional form with parallel sides and two circular ends.

Diameter. A line that passes through the center of a circle with both ends touching its circumference.

Diminution. The appearance of an object getting smaller as it moves further away from the viewer.

Elevation View. A side view showing height and depth, or a front view showing height and width. There is no perspective, no diminution, no foreshortening.

Eye Level (EL). The distance from the ground to the viewer's eye. The horizon line is always at the viewer's eye level.

Field of Vision. The area that can be seen without the viewer turning their head.

Five-Point Perspective. A 180° view of the world. The picture plane is half a hemisphere.

Foreshortening. The apparent reduction in length of an angle due to the position from which it is viewed.

Four-Point Perspective. A panorama view up to 360°. The picture plane is a cylinder.

Great Circle. A cross-section of a sphere that creates the largest diameter possible. The circle intersects the center of the sphere.

Ground Line (GL). Generic: a line drawn on the ground plane. Shadows: the angle of a shadow cast from a vertical line on a horizontal surface.

Ground Plane. The horizontal surface of the ground.

Hemisphere. Half a sphere.

Horizon Line (HL). The edge of the earth; the line that separates sky from land.

Hypotenuse. The longest side of a right-angled triangle.

Isosceles Triangle. A triangle where two sides are of equal length.

Linear Perspective. A system using the rules of geometry to depict 3-D space on a 2-D surface.

Light Angle (LA). The angle of the light ray to the ground plane.

Line of Sight. An imaginary line indicating the direction the viewer is looking.

Major Axis. The longest distance across an ellipse.

Measuring Point (MP). A point that transfers the distance of a foreshortened line to a line parallel with the picture plane, creating an isosceles triangle.

Minor Axis. The axis through the center of the ellipse, 90° to the elliptical plane.

Natural Light. Light from the sun or moon.

Negative Shadows. When a natural light source is located behind the viewer.

Normal. A line at right angles to a reflective surface.

One-Point Perspective. Vertical and horizontal dimensions are parallel with the picture plane. Foreshortened lines converge to the center of vision.

Orthogonal Lines. Lines at right angles.

Orthographic. Representing a three-dimensional form using two-dimensional (plan and elevation) views.

Parallel Light. When a natural light source is 90° to the line of sight.

Peripheral Vision. The visual area outside the cone of vision.

Perpendicular. Two lines intersecting at 90° angles.

Plan View. A top view showing width and depth. There is no perspective, no diminution, no foreshortening.

Plane. A flat surface.

Picture Plane (PP). A transparent plane between the viewer and the world.

Polygon. A closed plane formed by three or more line segments.

Positive Light. When a natural light source is in front of the viewer.

Prism. A three-dimensional form with two parallel and congruent bases.

Pythagorean Theorem. The square of the hypotenuse is equal to the sum of the squares of a right-angled triangle's other two sides.

Radius. The distance from the center of a circle to the circumference.

Reference Point (RP). A point used to move objects forward or backward in space.

Right Angle. A straight line that is 90° at its point of intersection with another straight line.

Six-Point Perspective. A 360° view of the world. The picture plane is a sphere.

Sphere. A three-dimensional form where every point of its surface is an equal distance from its center.

Station Point (SP). The viewer's eye.

Three-Point Perspective. No sides of the object are parallel with the picture plane. The center of vision is above or below the horizon line.

Two-Point Perspective. Vertical dimensions are parallel with the picture plane. Horizontal dimensions are foreshortened and connect to right and left vanishing points.

Vanishing Point (VP). A point at infinity where objects disappear.

Worm's-Eye View. An image where the viewer is looking up.

Bibliography

Alberti, Leon Battista, and Rocco Sinisgalli. *Leon Battista Alberti: On Painting: A New Translation and Critical Edition*. Cambridge, UK: Cambridge UP, 2011. Print.

Belting, Hans, and Deborah Lucas Schneider. *Florence and Baghdad: Renaissance Art and Arab Science*. Cambridge, MA: Belknap of Harvard UP, 2011. Print.

Boyer, Carl B., and Uta C. Merzbach. *A History of Mathematics*. Hoboken, NJ: Wiley, 2011. Print.

Brannan, David A., Matthew F. Esplen, and Jeremy J. Gray. *Geometry*. Cambridge, UK: Cambridge UP, 2011. Print.

Campbell, Lorne. *The Fifteenth Century Netherlandish Paintings*. London: National Gallery, 2000. Print.

"Cartesian Coordinate System." *Cartesian Coordinate System—New World Encyclopedia*. www.newworldencyclopedia.org/entry/Cartesian_coordinate_system. (Accessed 2017).

Chelsea, David. *Extreme Perspective! for Artists: Learn the Secrets of Curvilinear, Cylindrical, Fisheye, Isometric, and Other Amazing Drawing Systems That Will Make Your Drawings Pop off the Page*. New York: Watson-Guptill, 2011. Print.

Chelsea, David. *Perspective! for Comic Book Artists: How to Achieve a Professional Look in Your Artwork*. New York: Watson-Guptill Publications, 1997. Print.

Clegg, Brian. *Light Years: The Extraordinary Story of Mankind's Fascination with Light*. London: Piatkus, 2002. Print.

Damisch, Hubert, and John Goodman. *The Origin of Perspective*, Cambridge, MA: MIT, 2000. Print.

Fenna, Donald. *Cartographic Science: A Compendium of Map Projections, with Derivations*. Boca Raton, FL: CRC, 2007. Print.

Flocon, Albert, and AndreÍ Barre. *Curvilinear Perspective: From Visual Space to the Constructed Image*. Berkeley, CA: University of California, 1987. Print.

"Fundamentals of Mapping." *The Intergovernmental Committee on Surveying and Mapping*. www.icsm.gov.au/mapping/map_projections.html. (Accessed 2017).

Geometry Transformations. www.emathematics.net/transformations.php?def=reflection. (Accessed 2017).

Grafton, Anthony. *Leon Battista Alberti: Master Builder of the Italian Renaissance*. Cambridge, MA: Harvard UP, 2000. Print.

"History of Mirrors—Mirrors in Ancient World." *History of Mirrors—The Earliest Man Made Mirrors*. www.mirrorhistory.com/mirror-history/history-of-mirrors/. (Accessed 2017).

"Image Characteristics for Concave Mirrors." *The Physics Classroom*. www.physicsclassroom.com/class/refln/Lesson-3/Image-Characteristics-for-Concave-Mirrors. (Accessed 2017).

Kemp, Martin. *The Science of Art: Optical Themes in Western Art from Brunelleschi to Seurat*. New Haven, CT: Yale UP, 1992. Print.

"The Law of Reflection." *The Physics Classroom*. www.physicsclassroom.com/class/refln/Lesson-1/The-Law-of-Reflection. (Accessed 2017).

"Law of Reflection." *Reflection and Fermat's Principle*. http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/Fermat.html#c1. (Accessed 2017).

Linde, Sharon. "Peripheral Vision: Definition & Problems." http://study.com/academy/lesson/peripheral-vision-definition-problems.html. (Accessed 2017).

Massey, Lyle. *The Treatise on Perspective: Published and Unpublished*. Washington, DC: National Gallery of Art, 2003. Print.

Mathforum.org. http://mathforum.org/sarah/hamilton/ham.1side.1angle.html. (Accessed 2017).

McIlwain, James T. *An Introduction to the Biology of Vision*. Cambridge, UK: Cambridge UP, 1998. Print.

Millodot, Michel. *Dictionary of Optometry and Visual Science*. Elsevier Health Sciences UK, 2008. Print.

Montague, John. *Basic Perspective Drawing: A Visual Approach*. Hoboken, NJ: John Wiley & Sons, 2010. Print.

Niceron, Jean Francois, and Marin Mersenne. *La Perspective Curieuse*. Paris: J. Du Puis, 1663. Print.

Panofsky, Erwin. *Perspective as Symbolic Form*. New York: Zone, 2012. Print.

"Projective Geometry." *Wolfram MathWorld*. http://mathworld.wolfram.com/ProjectiveGeometry.html. (Accessed 2017).

Rashed, Roshdi, and Nader El-Bizri. *A History of Arabic Sciences and Mathematics*. London: Routledge, 2012. Print.

Sabra, A. I. *Theories of Light: From Descartes to Newton*. Cambridge, UK: Cambridge UP, 1981. Print.

Sinusoid. *Encyclopedia of Mathematics*. www.encyclopediaofmath.org/index.php?title=Sinusoid&oldid=35602

"The Six Functions." *The Six Functions (Trig without Tears Part 2)*. https://brownmath.com/twt/sixfunc.htm#SideLength. (Accessed 2017).

Snyder, John P. *Flattening the Earth: Two Thousand Years of Map Projections*. Chicago, IL: University of Chicago, 1998. Print.

"Spring Measurements and Specifications." *Spring Measurements and Specifications— Quality Spring, Affordable Prices*. www.acxesspring.com/spring-measurements-specifications.html/. (Accessed 2017).

Stidwill, David, and Robert Fletcher. *Normal Binocular Vision: Theory, Investigation, and Practical Aspects*. Chichester, UK: Blackwell, 2011. Print.

Strasburger, Hans, Ingo Rentschler, and Martin Jüttner, (2011). "Peripheral Vision and Pattern Recognition: A Review". *Journal of Vision 11* (5): 13, pp. 1–13. Web.

Watson, Ernest W. *Creative Perspective for Artists and Illustrators*. New York: Dover, 1992. Print.

Weisstein, Eric W. "Gnomonic Projection." *Wolfram MathWorld*. http://mathworld.wolfram.com/GnomonicProjection.html. (Accessed 2017).

Wheatstone, Charles. *Contributions to the Physiology of Vision. Part the First: On Some Remarkable and Hitherto Unobserved Phenomena of Binocular Vision*. London: Royal Society of London, 1838. Print.

Winkel Tripel Projections. www.winkel.org/other/Winkel%20Tripel%20Projections.htm. (Accessed 2017).

Index

Page numbers in *italics* indicate an illustration